CLASS: 9TH

SUBJECT: MATHEMATICS

SESSION 2024 - 2025

Unit 2nd

Polynomials

Exercise 2.1

1 Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer

i $4x^2 - 3x + 7$

ii $y^2 + \sqrt{2}$

iii $3\sqrt{t} + t\sqrt{2}$

iv $y + 2/y$

v $x^{10} + y^3 + t^{50}$

Sol. ***i*** Given expression is a polynomial in one variable, as it degree is whole

ii Given expression is a polynomial in one variable, as it degree is whole

iii Given expression is not a polynomial , as it degree is not whole

iv Given expression is not a polynomial , as it degree is not whole

v Given expression is a polynomial in three variable, as it degree is whole

2. Write the coefficients of x^2 in each of the following:

(i) $2 + x^2 + x$

(ii) $2 - x^2 + x^3$

(iii) $\frac{\pi}{2}x^2 + x$

(iv) $\sqrt{2}x - 1$

Sol. **i** Given polynomial is $2 + x^2 + x$.

Hence, the coefficient of x^2 in given polynomial is equal to 1.

ii Given polynomial is $2 - x^2 + x^3$.

Hence, the coefficient of x^2 in given polynomial is equal to -1.

iii Given polynomial is $\frac{\pi}{2}x^2 + x$

Hence, the coefficient of x^2 in given polynomial is equal to $\frac{\pi}{2}$

iv Given polynomial is $\sqrt{2}x - 1$.

In the given polynomial, there is no x^2 term.

Hence, the coefficient of x^2 in given polynomial is equal to 0.

3 Give one example each of a binomial of degree 35 and of a monomial of degree 100.

Sol. A binomial of degree 35 can be $x^{35} + 7$

A monomial of degree 100 can be $2x^{100}$

4 . Write the degree of each of the following polynomials

i $5x^3 + 4x^2 + 7x$

ii $4 - y^2$

iii $5t - \sqrt{7}$

iv 3

Solution: i) Given polynomial is $5x^3 + 4x^2 + 7x$

Hence, the degree of given polynomial is equal to 3.

ii) Given polynomial is $4 - y^2$

Hence, the degree of given polynomial is equal to 2.

iii) Given polynomial is $5t - \sqrt{7}$

Hence, the degree of given polynomial is 1.

iv) Given polynomial is 3.

Hence, the degree of given polynomial is 0

5. Classify the following as linear, quadratic and cubic polynomials

i $x^2 + x$

ii $x - x^3$

iii $y + y^2 + 4$

iv $1 + x$

v $3t$

vi r^2

vii $7x^3$

Solution:

i) Given polynomial is $x^2 + x$

It is a quadratic polynomial as its degree is 2.

ii) Given polynomial is $x - x^3$.

It is a cubic polynomial as its degree is 3.

iii) $y + y^2 + 4$

It is a quadratic polynomial as its degree is 2.

iv) Given polynomial $1 + x$

It is a linear polynomial as its degree is 1

v) Given polynomial 3t

It is a linear polynomial as its degree is 1

vi) Given polynomial is r^2

It is a quadratic polynomial as its degree is 2.

vii) Given polynomial is x^3

It is a cubic polynomial as its degree is 3.

EXERCISE 2.2

1. Find the value of the polynomial $5x - 4x^2 + 3$ at

i $x = 0$

ii $x = -1$

iii $x = 2$

Sol.

i) Given polynomial is $5x - 4x^2 + 3$

Value of polynomial at $x = 0$ is $5(0) - 4(0)^2 + 3 = 0 - 0 + 3 = 3$

(ii) Given polynomial is $5x - 4x^2 + 3$

Value of given polynomial at $x = -1$ is $5(-1) - 4(-1)^2 + 3$

$= -5 - 4 + 3 = -6$

iii) Given polynomial is $5x - 4x^2 + 3$

Value of given polynomial at $x = 2$ is $5(2) - 4(2)^2 + 3$

$= 10 - 16 + 3 = -3$

2 Find P(0), P(1) and P(2) for each of the following polynomials.

(i) $P(y) = y^2 - y + 1$

(ii) $P(t) = 2 + t + 2t^2 - t^3$

(iii) $P(x) = x^3$

(iv) $P(x) = (x - 1)(x + 1)$

Sol.

i Given polynomial is $P(y) = y^2 - y + 1$

$P(0) = (0)^2 - 0 + 1 = 1$

$P(1) = (1)^2 - 1 + 1 = 1$

$P(2) = (2)^2 - 2 + 1 = 4 - 2 + 1 = 3$

ii Given polynomial is $P(t) = 2 + t + 2t^2 - t^3$

$P(0) = 2 + 0 + 2.(0)^2 - (0)^3 = 2$

$P(1) = 2 + 1 + 2(1)^2 - (1)^3 = 4$

$P(2) = 2 + 2 + 2.(2)^2 - (2)^3 = 4$

iii Given polynomial is $P(x) = x^3$

$P(0) = (0)^3 = 0$

$P(1) = (1)^3 = 1$

$P(2) = (2)^3 = 8$

iv Given polynomial is $p(x) = (x - 1)(x + 1)$

$P(0) = (0 - 1)(0 + 1) = (-1)(1) = -1$

$P(1) = (1 - 1)(1 + 1) = (0)(2) = 0$

$P(2) = (2 - 1)(2 + 1) = (1)(3) = 3$

3. Verify whether the following are zeroes of the polynomial, indicated against them.

Sol i $P(x) = 3x + 1$

$X = -1/3$

$P(-1/3) = 3(-1/3) + 1 = -1 + 1 = 0$

Hence $^{-1/3}$ is a zero of polynomial $P(x) = 3x + 1$.

ii Given polynomial is $P(x) = 5x - \pi$

$X = 4/5$

$P(4/5) = 5(4/5) - \pi$

$= 4 - \pi$

Hence $x = 4/5$ is not a zero of polynomial $5x - \pi$

iii Given polynomial is $P(x) = x^2 - 1$

At $x = 1$

$P(1) = (1)^2 - 1 = 0$

And $x = -1$

$P(-1) = (-1)^2 - 1 = 1 - 1 = 0$

Hence $x = 1, -1$ are zeroes of polynomial $x^2 - 1$.

iv. Given polynomial is $P(x) = (x + 1)(x - 2)$

At $x = -1$,

$P(-1) = (-1 + 1)(-1 - 2)$

$= (0)(-3) = 0$

And $x = 2$,

$P(2) = (2 + 1)(2 - 2)$

$= (0)(3) = 0$

Hence $x = -1, 2$ are zeroes of polynomial $(x + 1)(x - 2)$

v. Given polynomial is $P(x) = x^2$

AT $X = 0$

$P(0) = (0)^2 = 0$

Hence $x = 0$ is zero of polynomial x^2

vi) Given polynomial is $P(x) = lx + m$

At $x = -\frac{l}{m}$

$P(-\frac{l}{m}) = l(-\frac{l}{m}) + m$

$= -m + m = 0$

Hence $x = -\frac{l}{m}$ is zero of polynomial $lx + m$ { vii and viii same as previous parts}

4. Find the zero of the polynomials in each of the following cases.

Solution:

i. Given polynomial is $P(x) = x + 5$

Now, $P(x) = 0$

$\Rightarrow x + 5 = 0$

$\Rightarrow x = -5$

Hence x = −5 is zero of polynomial P(x) = x + 5

ii. Given polynomial is P(x) = x −5

Now, $P(x) = 0$

$\Rightarrow x - 5 = 0$

$\Rightarrow x = 5$

Hence x = 5 is zero of polynomial P(x) = x − 5.

iii. Given polynomial is P(x) = 2x + 5

Now, P(x) = 0

$\Rightarrow 2x + 5 = 0$

$\Rightarrow X = -5/2$

Hence x = −5/2 is zero of polynomial P(n) = 2x + 5.

iv. Given polynomial is P(x) = 3x – 2

Now, P(x) =0

$\Rightarrow 3x - 2 = 0$

$\Rightarrow X = 2/3$

Hence x =2/3 is zero of polynomial P(x) = 3x – 2

vi) Given polynomial is P(x) = ax

Now, P(x) = 0

$\Rightarrow ax = 0$

$\Rightarrow a = 0$ or $x = 0$

But given that $a \neq 0$

Hence $x = 0$ is zero of polynomial $P(x) = ax$. [V and vii same as previous parts]

Exercise: 2.4

1. Determine which of the following polynomials has (x + 1) as factor:

Solution:

i) Given polynomial is $p(x) = x^3 + x^2 + x + 1$

Let $g(x) = x+1$

Put $g(x) = 0$

$\Rightarrow X+1 = 0$

$\Rightarrow x = -1$

Hence, remainder $= p(-1) = (-1)^3 + (-1)^2 + (-1) + 1$

$= -1 + 1 - 1 + 1$

$= 0.$

Hence $x + 1$ is a factor of polynomial $x^3 + x^2 + x + 1$

ii. Given polynomial is $p(x) = x^4 + x^3 + x^2 + x + 1$

Let $g(x) = x+1$

Put $g(x) = 0$

$\Rightarrow X+1 = 0$

$\Rightarrow X = -1$

Hence, remainder $= p(-1) = (-1)^4 + (-1)^3 + (-1)^2 + (-1) + 1$

$= 1 - 1 + 1 - 1 + 1$

$= 1$

As remainder $\neq 0$,

Hence $x + 1$ is not a factor of polynomial $x^4 + x^3 + x^2 + x + 1$.

iii. Given polynomial is $p(x) = x^4 + 3x^3 + 3x^2 + x + 1$.

Let $g(x) = x+1$

Put $g(x) = 0$

$\Rightarrow X+1 = 0$

$\Rightarrow X = -1$

Hence, remainder $= p(-1) = (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1$

$= 1 - 3 + 3 - 1 + 1 = 1$

As remainder $\neq 0$.

Hence $(x + 1)$ is not a factor of polynomial $x^4 + 3x^3 + 3x^2 + x + 1$

iv. Given polynomial is $p(x) = x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

Let $g(x) = x+1$

Put $g(x) = 0$

$\Rightarrow X+1 = 0$

$\Rightarrow X = -1$

$p(-1) = (-1)^3 - (-1)^2 - (2 + \sqrt{2})(-1) + \sqrt{2} = 2\sqrt{2}$

As remainder $\neq 0$,

Hence $(x + 1)$ is not a factor of polynomial $-x^2 - (2 + \sqrt{2})x + \sqrt{2}$.

2. Use the factor theorem to determine whether $g(x)$ is a factor of $p(x)$ in each of the following cases:

Solution: **i)** $p(x) = 2x^3 + x^2 - 2x - 1$, $g(x) = x + 1$

put $g(x) = 0$

$\Rightarrow x + 1 = 0$

$\Rightarrow x = -1$

Hence, remainder $= p(-1) = 2(-1)^2 + (-1)^3 - 2(-1) - 1$

$= -2 + 1 + 2 - 1 = 0.$

As remainder when polynomial $p(x)$ is divided by polynomial $g(x)$ is equal to zero, then polynomial $g(x) = x + 1$ is a factor of polynomial

$p(x) = 2x^3 + x^2 - 2x - 1$

ii. $p(x) = x^3 + 3x^2 + 3x + 1$, $g(x) = x + 2$

put $g(x) = 0$

$\Rightarrow x + 2 = 0$

$\Rightarrow x = -2$

Hence, remainder $= p(-2) = (-2)^3 + 3(2)^2 + 3(-2) + 1$

$= -8 + 12 - 6 + 1$

$= -1$

Since remainder $\neq 0$, the polynomial $g(x) = x + 2$ is not a factor of polynomial $p(x) = x^3 + 3x^2 + 3x + 1$.

iii. $p(x) = x^3 - 4x^2 + x + 6$, $g(x) = x - 3$

put $g(x) = 0$

$\Rightarrow x - 3 = 0$

$\Rightarrow x = 3$

Hence, remainder $= p(3) = (3)^3 - 4(3)^2 + 3 + 6$

$= 27 - 36 + 9$

$= 0$

Since reminder $= 0$, the polynomial $g(x) = x - 3$ is factor of polynomial $p(x) = x^3 - 4x^2 + x + 6$.

3. Find the value of k if x-1 is the Factor of p(x) in each of the following cases:

Solution: **(i)** (x) = x^2+ x +k and g(x)= x-1

put g(x)= 0

⇒ x-1= 0

⇒ x= 1

p(1)= 0 since, x-1 is the factor of x^2+ x +k

⇒ $(1)^2$+ 1 +k=0

⇒ 1+1+k= 0

⇒ 2+k= 0

⇒ K= -2

K= -2- √2=-(2+√2) {part ii same as i}

(iii) p(x) = kx^2– √2x +1 and g(x)= x-1

put g(x)= 0

⇒ x-1= 0

⇒ x= 1

p(1)= 0 since, x-1 is the factor of p(x)

⇒ k $(1)^2$- √2 (1)+1=0

⇒ k -√2= -1

⇒ k= -1+√2

(iv) p(x) = kx^2– 3x +k and g(x)= x-1

put g(x)= 0

⇒ x-1= 0

⇒ x= 1

p(1)= 0 since, x-1 is the factor of p(x)

⇒ k $(1)^2$- 3 (1)+k=0

⇒ 2k = 3

⇒ k= 3/2

4. Factorise;

Sol. (i) Given polynomial is $12x^2 - 7x + 1$

$= 12x^2 - 7x + 1$

$= 12x^2 - 4x - 3x + 1$

$= 4x(3x - 1) - 1(3x - 1)$

$= (4x - 1)(3x - 1)$

$12x^2 - 7x + 1 = (3x - 1)(4x - 1)$

(ii) Given polynomial is $2x^2 + 7x + 3$

$= 2x^2 + 7x + 3$

$= 2x^2 + 6x + x + 3$

$= 2x(x + 3) + 1(x + 3)$

$= (2x + 1)(x + 3)$

$2x^2 + 7x + 3 = (2x + 1)(x + 3)$

(iii) Given polynomial is $6x^2 + 5x - 6$

$= 6x^2 + 9x - 4x - 6$

$= 3x(2x + 3) - 2(2x + 3)$

$= (2x + 3)(3x - 2)$

$6x^2 + 5x - 6 = (3x - 2)(2x + 3)$

(iv) Given polynomial is $3x^2 - x - 4$

On splitting middle term

$= 3x^2 - 4x + 3x - 4$

$= x(3x - 4) + 1(3x - 4)$

$= (x + 1)(3x - 4)$

$3x^2 - x - 4 = (x + 1)(3x - 4)$

5 Factorise

i) $x^3 - 2x^2 - x + 2$

sol. $x^3 - 2x^2 - x + 2$

$= x^2 (x-2) - 1(x-2)$

$= (x-2)(x^2 - 1)$

$= (x-2)(x+1)(x-1)$

ii) $x^3 - 3x^2 - 9x - 5$

sol. $x^3 - 3x^2 - 9x - 5$

$= x^3 + x^2 - 4x^2 - 4x - 5x - 5$

$= x^2 (x+1) - 4x(x+1) - 5(x+1)$

$= (x+1)\{(x^2 - 4x - 5)\}$

$= (x+1)\{(x^2 - 5x + x - 5)\}$

$= (x+1)\{x(x-5) + 1(x-5)\}$

$= (x+1)(x-5)(x+1)$

iii. $x^3 + 13x^2 + 32x + 20$

sol. $x^3 + 13x^2 + 32x + 20$

. $= x^{3+} x^2 + 12x^2 + 12x + 20x + 20$

$= x^2 (x+1) + 12x(x+1) + 20(x+1)$

$= (x+1)\{(x^2 + 12x + 20)\}$

$= (x+1)\{(x^2 + 10x + 2x + 20)\}$

$= (x+1)\{x(x+10) + 2(x+10)\}$

$= (x+1)(x+10)(x+2)$

iv. $2y^3 + y^2 - 2y - 1$

Sol. $2y^3 + y^2 - 2y - 1$

$= y^2 (2y+1) - 1(2y + 1)$

$= (y^2 - 1)(2y+1)$

$= (y+1)(y-1)(2y+1)$

Exercise: 2.5

1. Use suitable identities to find the following products

Solution: (i) $(x + 4)(x + 10)$

We know that

$(x + a)(x + b) = x^2 + (a + b)x + ab$

Given polynomial is $(x + 4)(x + 10)$

Here, $a = 4, b = 10$

$(x + 4)(x + 10) = x^2 + (4 + 10)x + 40$

$= x^2 + 14x + 40$

(ii) $(x + 8)(x - 10)$

We know that

$(x + a)(x + b) = x^2 + (a + b)x + ab$

Given polynomial is $(x + 8)(x - 10)$

Here $a = 8, b = -10$

$(x + 8)(x - 10) = x^2 + (8 - 10)x - 80$

$= x^2 - 2x - 80.$

Rest parts are same as part (iii)(iv)(v)

2. Evaluate the following products without multiplying directly

Solution:

(i) $103 \times 107 = (100 + 3) \times (100 + 7)$

We know that $(x + a)(x + b) = x^2 + (a + b)x + ab$

Here $x = 100, a = 3, b = 7$

$(100 + 3)(100 + 7) = (100)^2 + 10 \times 100 + 3 \times 7$

$= 10000 + 1000 + 21$

$= 11021$

(ii) $95 \times 96 = (100 - 5)(100 - 4)$

We know that $(x + a)(x + b) = x^2 + (a + b)x + ab$

Here $x = 100, a = -5, b = -4$ $95 \times 96 = (100 - 5)(100 - 4)$

$= (100)^2 + [(-5 + (-4))]100 + (-5)(-4)$

$= 10000 - 900 + 20$

$= 9120.$

(iii) $104 \times 96 = (100 + 4)(100 - 4)$

We know that $(x + a)(x - a) = x^2 - a^2$

Here $x = 100, a = 4$

$(100 + 4)(100 - 4) = (100)^2 - (4)^2$

$= 10000 - 16$

$= 9984$

3. Factorise the following using appropriate identities

(i) $9x^2 + 6xy + y^2 = (3x)^2 + 2 \cdot 3x \cdot y + (y)^2$

We know that $x^2 + 2xy + y^2 = (x + y)^2$

$9x^2 + 6xy + y^2 = (3x^2) + 2 \cdot 3x \cdot y + (y)^2$

$= (3x + y)^2$

$= (3x + y)(3x + y)$

(ii) $4y^2 - 4y + 1 = (2y)^2 - 2 \cdot 2y(1) + 1$

We know that $x^2 - 2xy + y^2 = (x - y)^2$

$(2y)^2 - 2 \cdot 2y)(1) + 1 = (2y - 1)^2$

$= (2y - 1)(2y - 1)$

(iii) $x^2 - y^2/100$

We know that $(x + a)(x - a) = x^2 - a^2$

$= x^2 - (y/10)^2$

$=(x+y/10)(x-y/10)$

4. Expand each of the following, using suitable Identities

(i) $(x + 2y + 4z)^2$

We know that $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$(x + 2y + 4z)^2 = x^2 + (2y)^2 + (4z)^2 + 2 \cdot x \cdot 2y + 2 \cdot 2y \cdot 4z + 2 \cdot x \cdot 4z$

$= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8xz$

(ii) $(2x - y + z)^2$

We know that $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$(2x - y + z)^2 = (2x)^2 + (-y)^2 + (z)^2 + 2 \cdot 2x(-y) + 2(-y)(z) + 2 \cdot 2x \cdot z$

$= 4x^2 + y^2 + z^2 - 4xy - 2yz + 4xz$

(iii) $(-2x + 3y + 2z)^2$

We know that $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$(-2x + 3y + 2z)^2 = (-2x)^2 + (3y)^2 + (2z)^2 + 2 \cdot (-2x) \cdot 3y + 2 \cdot 3y \cdot 2z + 2 \cdot (-2x) \cdot 2z$

$= 4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8xz$

(iv) $(3a - 7b - c)^2$

We know that $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$(3a - 7b - c)^2 = (3a)^2 + (-7b)^2 + (-c)^2 + 2 \cdot 3a \cdot (-7b) + 2 \cdot (-7b)(-c) + 2.(3a).(-c)$

$= 9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ac.$

(v) $(-2x + 5y - 3z)^2$

We know that $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$(-2x + 5y - 3z)^2 = (-2x)^2 + (5y)^2 + (-3z)^2 + 2 \cdot (-2x)(5y) + 2 \cdot (5y)(-3z) + 2 \cdot (-2x)(-3z)$

$= 4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12xz$ {part vi same as i,ii,iii,iv and v}

5. Factorise

Sol.

i $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

$= (2x)^2 + (3y)^2 + (-4z)^2 + 2 \cdot 2x \cdot 3y + 2 \cdot (3y)(-4z) + 2 \cdot (2x)(-4z)$

We know that $x^2 + y^2 + z^2 + 2xy + 2yz + 2xz = (x + y + z)^2$

$= (2x)^2 + (3y)^2 + (-4z)^2 + 2 \cdot 2x \cdot 3y + 2 \cdot (3y)(-4z) + 2 \cdot (2x)(-4z)$

$= (2x + 3y - 4z)^2$

$= (2x + 3y - 4z)(2x + 3y - 4z)$

ii $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

We know that $x^2 + y^2 + z^2 + 2xy + 2yz + 2xz = (x + y + z)^2$

$(-\sqrt{2}x)^2 + y^2 + (2\sqrt{2}z)^2 + 2(-\sqrt{2}x)(y) + 2(y)(2\sqrt{2}z) + 2(2\sqrt{2}z)(-\sqrt{2}x)$

$= (-\sqrt{2}x + y + 2\sqrt{2}z)^2$

$= (-\sqrt{2}x + y + 2\sqrt{2}z)(-\sqrt{2}x + y + 2\sqrt{2}z)$

6. Write the following cubes in expanded form

Sol. i) We know that $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

Given polynomial is $(2x + 1)^3$ $a = 2x, b = 1$

$(2x + 1)^3 = (2x)^3 + (1)^3 + 3 \cdot (2x) \cdot (1)(2x + 1)$

$= 8x^3 + 1 + 6x(2x + 1)$

$= 8x^3 + 1 + 12x^2 + 6x$

$= 8x^3 + 12x^2 + 6x + 1$

ii) We know that $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

$(2a - 3b)^3 = (2a)^3 - (3b)^3 - 3(2a)(3b)(2a - 3b)$

$= 8a^3 - 27b^3 - 18ab(2a - 3b)$

$= 8a^3 - 27b^3 - 36a^2b + 54ab^2$

$= 8a^3 - 36a^2b + 54ab^2 - 27b^3$

Rest parts are same as part (i)(ii)

7. Evaluate the following using suitable identities

Solution: i. $(99)^3 = (100 - 1)^3$

We know that $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$ a

$= 100, b = 1$

$(99)^3 = (100 - 1)^3 = (100)^3 - (1)^3 - 3(100)(1)(100\text{-}1)$
$= (100)^3 - (1)^3 - 3(100)(1)(99)$

$= 1000000 - 1 - 29,700$

$= 9\ 70\ 299.$

ii. $(102)^3 = (100 + 2)^3$

We know that $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

$a = 100, b = 2$

$(102)^3 = (100 + 2)^3 = (100)^3 + (2)^3 + 3 \cdot 100 \cdot 2(100 + 2)$

$= 1000000 + 8 + 600 \times 102$

$= 1000008 + 61{,}200$

$= 1061208$

iii. $(998)^3 = (1000 - 2)^3$

We know that $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

Here $a = 1000, b = 2$

$(998)^3 = (1000 - 2)^3 = (1000)^3 - 8 - 3(1000)(2)(200\text{-}2)$

$=(1000)^3 - 8 - 3(1000)(2)(998)$

$= 1000000000 - 8 - 6000 \times 998$

$= 994011992$

8. Factorise each of the following

Solution:

i. $8a^3 + b^3 + 12a^2b + 6ab^2$

$= (2a)^3 + (b)^3 + 3 \cdot (2a)(b)(2a + b)$

We know that $a^3 + b^3 + 3ab(a + b) = (a + b)^3$

$(2a)^3 + b^3 + 3(2a) \cdot b(2a + b) = (2a + b)^3$

$= (2a + b)(2a + b)(2a + b)$

ii. $8a^3 - b^3 - 12a^2b + 6ab^2$

We know that $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

$8a^3 - b^3 - 12a^2b + 6ab^2 = (2a)^3 - (b)^3 - 3 \cdot (2a)(b)(2a - b)$

$= (2a - b)^3$

$= (2a - b)(2a - b)(2a - b)$

iii. $27 - 125a^3 - 135a + 225a^2$

we know that $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

$27 - 125a^3 - 135a + 225a^2 = -(125a^3 - 27 - 225a^2 + 135a)$

$= [\,(3)^3 - (5a)^3 - 3 \cdot (3)(5a)(3\text{-}5a)]$

$= [3 - 5a]^3$

$= (3 - 5a)^3$

$= (3 - 5a)(3 - 5a)(3 - 5a)$

iv. $64a^3 - 27b^3 - 144a^2b + 108ab^2$

we know that $a^3 - b^3 - 3ab(a - b) = (a - b)^3$

$64a^3 - 27b^3 - 144a^2b + 108ab^2$

$= (4a)^3 - (3b)^3 - 3 \cdot (4a) \cdot (3b)(4a - 3b)$

$= (4a - 3b)^3$

$= (4a - 3b)(4a - 3b)(4a - 3b)$

v. $27p^3 - -\frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

we know that $a^3 - b^3 - 3ab(a - b) = (a - b)^3$

$= (3p)^3 - (-\frac{1}{6})^3 - \frac{9}{2}p^2 + \frac{1}{4}p$

$= (3p)^3 - (-\frac{1}{6})^3 - 3(3p)(\frac{1}{6})\ [3p - \frac{1}{6}]$

$= (3p - \frac{1}{6})^3$

$= (3p - \frac{1}{6})(3p - \frac{1}{6})(3p - \frac{1}{6})$

9. Verify

I. $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

ii. $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

Solution:

i $x^3+y^3=(x+y)(x^2-xy+y^2)$

Take R.H.S $(x + y)(x^2 - xy + y^2)$

= $x^3- x^2 y + x y^2+ x^2 y - x y^2 + y^3$

= x^3+y^3

Therefore R.H.S =L.H.S

Hence verified.

ii. $x^3-y^3=(x-y)(x^2+xy+y^2)$

Take R.H.S $(x - y)(x^2 + xy + y^2)$

= $x^3+ x^2 y + x y^2- x^2 y - x y^2 - y^3$

= x^3-y^3

Therefore R.H.S =L.H.S

Hence verified.

10. Factorise each of the following

Sol. I. $27y^3 + 125z^3 = (3y)^3+ (5z)^3$

We know that $x^3 + y^3 = (x + y)(x^2 + y^2 - xy)$

$(3y)^3+ (5z)^3= (3y + 5z)\{(3y)^2+ (5z)^2-(3y)(5z)\}=27y^3 + 125z^3 = (3y + 5z)(9y^2 + 25z^2 - 15yz)$

II. $64m^3 - 343n^3 = (4m)^3-(7n)^3$

We know that $(x)^3-(y)^3= (x - y)(x^2 + xy + y^2)$

$(4m)^3-(7n)^3= (4m - 7n)\{(4m)^2+ (4m)7n +(7n)^2\}=(4m - 7n)(16m^2+ 28mn +49n)^2\}$

11. Factorise$27x^2 + y^3 + z^3 - 9xyz$

Solution: $27x^2 + y^3 + z^3 - 9xyz$

$= (3x)^3+ (y)^3+ (z)^3- 3(3x)(y)(z)$

We know that

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - xz)$

Now, $27x^3 + y^3 + z^3 - 9xyz = (3x)^3+ (y)^3+ (z)^3- 3(3x)(y)(z)$

$= (3x + y + z)((3x)^2+ (y)^2+ (z)^2- 3xy - yz - 3xz)$

$=(3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3xz)$

12 Verify that $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (x - z)^2]$

Solution: We know that

$$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - xz)$$

$$= (x + y + z)\ \frac{1}{2}\ (2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2xz)$$

$$(x + y + z)\ \frac{1}{2}\ (x^2 + y^2 - 2xy + y^2 + z^2 - 2yz + x^2 + z^2 - 2xz)$$

We know that $a^2 + b^2 - 2ab = (a - b)^2$

$\frac{1}{2}\ (x + y + z)((x + y)^2 + (y - z)^2 + (x - z)^2)$

$x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}\ (x + y + z)((x - y)^2 + (y - z)^2 + (x - z)^2)$

Hence verified

13. If $x + y + z = 0$, show that $x^3 + y^3 + z^3 = 3xyz$

Solution: We know that,

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - xz)$

Given that $x + y + z = 0$

$x^3 + y^3 + z^3 - 3xyz = (0)(x^2 + y^2 + z^2 - xy - yz - xz)$

$\Rightarrow\ x^3 + y^3 + z^3 - 3xyz = 0$

$\Rightarrow\ x^3 + y^3 + z^3 = 3xyz$

Hence proved

14 Without actually calculating the cubes, find the value of each of the following

Solution: I) . $(-12)^3 + (7)^3 + (5)^3$

Let $x = -12,\ y = 7,\ z = 5$

now $x + y + z = -12 + 7 + 5 = 0$

we know if $x + y + z = 0$

then, $x^3 + y^3 + z^3 = 3xyz$

Therefore, $(-12)^3 + (7)^3 + (5)^3 = 3(12)(7)(5)$

$= -1260$

ii. $(28)^3 + (-15)^3 + (-13)^3$

Let $x = -28$, $y = -15$, $z = -13$ = $x + y + z = -28 - 15 - 13 = 0$

We know if $x + y + z = 0$

Then , $x^3 + y^3 + z^3 = 3xyz$

Therefore, $(28)^3 + (-15)^3 + (-13)^3 = 3(28)(-15)(-13)$

$= 16380$

15 Give possible expressions for the length and breadth of each of the following rectangles, in which the areas are given

Solution: i. Given area = $25a^2 - 35a + 12$

$= 25a^2 - 15a - 20a + 12$

$= 5a(5a - 3) - 4(5a - 3)$

$= (5a - 3)(5a - 4)$

We know that area = length × breadth

So possible expression for length = $5a - 3$

possible expression for breadth = $5a - 4$.

ii. Given area = $35^2 + 13y^2 - 12$

$= 35y^2 + 28y - 15y - 12$

$= 7y(5y + 4) - 3(5y + 4)$

$= (5y + 4)(7y - 3)$

We know that area = length × breadth

So possible expression for length $= 5y + 4$

possible expression for breadth $= 7y - 3$.

16. What are the possible expressions for the dimension of the cuboids whose volume are given below?

(i) Volume= $3x^2 - 12x$

(ii) Volume= $12ky^2 + 8ky - 20k$

Solution:

(i) Given volume $= 3x^2 - 12x$

$= 3(x^2 - 4x)$

$= 3x(x-4)$

We know that Volume of cuboid = length × breadth × height

Possible expression for length of cuboid $= 3$

Possible expression for breadth $= x$

Possible expression for height $= x - 4$.

(ii) Given Volume $= 12ky^2 + 8ky - 20k$

$= 4k(3y^2 + 2y - 5)$

$= 4k(3y^2 + 5y - 3y - 5)$

$= 4k(y(3y+5) - 1(3y+5)$

$= 4k(3y+5)(y-1)$

Possible value of length of cuboid $= 4k$

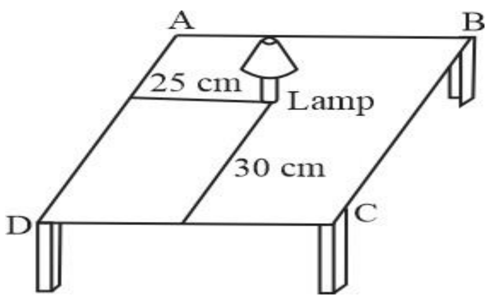
Possible expression for breadth $= 3y + 5$

Possible expression for breadth $= y - 1$.

Chapter 4
Coordinate Geomerty
Exercise: 4.1

1. How will you describe the position of a table lamp on study table to another person?

Solution:



Since, there are two dimensions of the surface of a table (length and breadth), so two references will be required to accurately describe position of the object on the table.

As a reference we can take DC and DA as a pair of perpendicular edges with common point D.

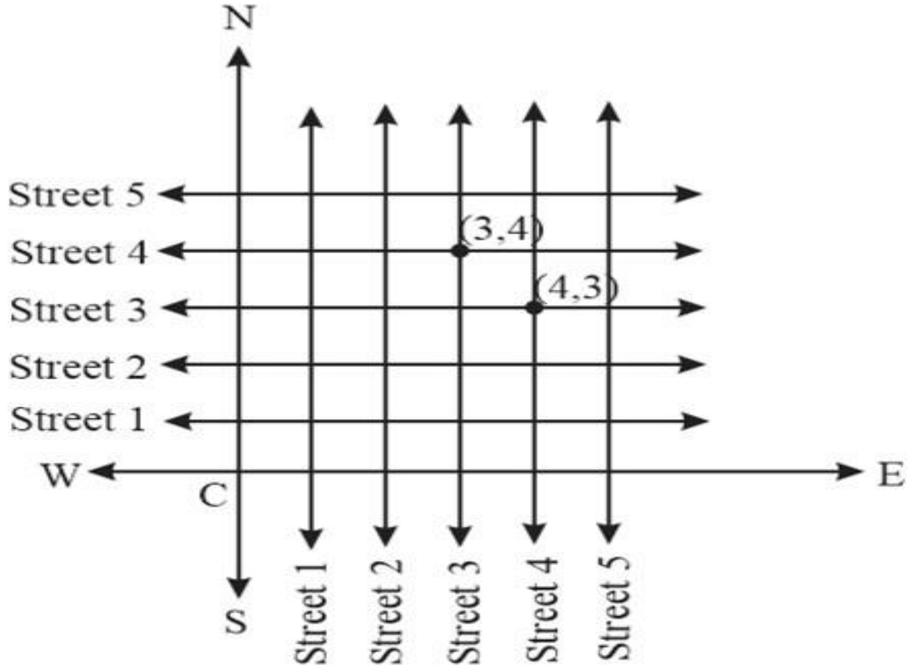
Now, any point on the table can be uniquely determined by its distances from both the edges DC and DA. In the shown figure, the lamp is at a distance of 25 cm from AD and 30 cm from DC.

2.(Street Plan): A city has two main roads which cross each other at the centre of the city. These two roads are along the North-South direction and East-West Direction. All the other streets of the city run parallel to these roads and are200 m apart. There are 5 streets in each direction. Using 1 cm=200 m, draw a model of the city on your notebook. Represent the roads/streets by single lines.There are many cross-streets in your model. A particular cross-street is made by two streets, one running in the North-South direction and another in the East-West direction. Each cross street is referred to in the following manner: If the 2nd street running in the North-South direction and 5th in the East-West direction meet at some crossing, then we will call this cross-street (2,5). Using this convention, find:

(i) How many cross - streets can be referred to as (4,3).

(ii) How many cross - streets can be referred to as (3,4).

Solution:



(i) The given cross street is marked in the figure and it can be observed that there is only one street referred as (4,3).

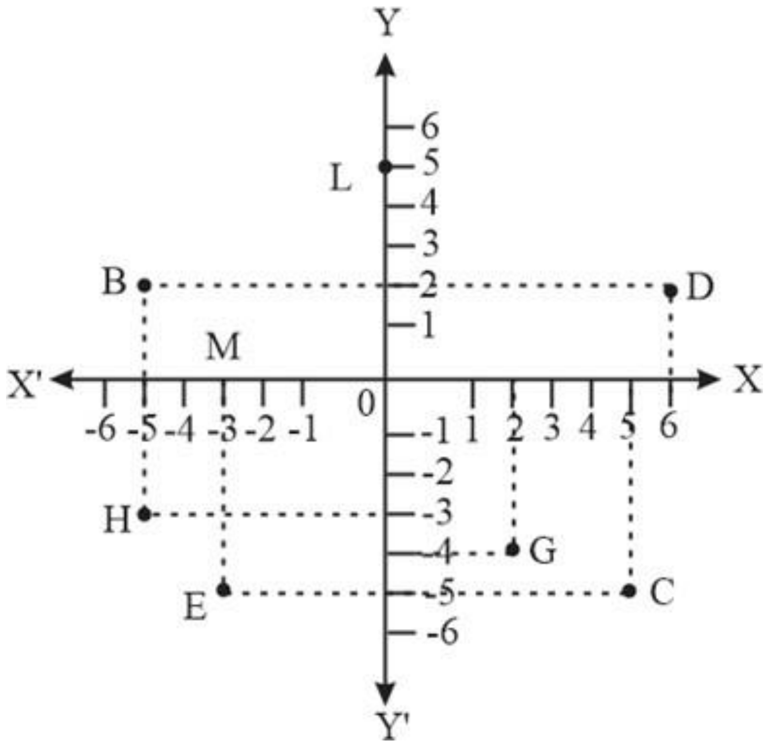
(ii) The given cross street is marked in the figure and it can be observed that there is only one street referred as (3,4).

Exercise: 4.2

1. Write the answer of each of the following questions:

(i) What is the name of horizontal and the vertical lines drawn to determine the position of any point in the Cartesian plane?

(ii) What is the name of each part of the plane formed by these two lines?
(iii) Write the name of point where these two lines intersect.
Solution:
(i) The Horizontal and vertical lines are called X axis and Y axis respectively.
(ii) The part of plane formed by the two lines are called quadrants.
(iii) The point where the two lines intersect is called origin.
2. See the given figure, and write the following:
(i) The coordinates of B.
(ii) The coordinates of C.
(iii) The point identified by the coordinates (−3,−5).
(iv) The point identified by the coordinates (2,−4).
(v) The abscissa of the point D.
(vi) The ordinate of the point H.
(vii) The coordinates of the point L.
(viii) The coordinates of the point M



Solution:
(i) The coordinates of point B is (−5,2)
(ii) The coordinates of point C is (5,−5)
(iii) The point identified by the coordinates (−3,−5) is point E.
(iv) The point identified by the coordinates (2,−4) is point G.
(v) Abscissa of point D is 6
(vi) Ordinate of point H is −3
(vii) The coordinates of point L is (0,5)
(viii) The coordinates of point M is (−3,0)