CLASS:10TH

SUBJECT:Mathematics

SESSION 2024 - 2025

Holyfaith presentation school.
Rawalpora Srinagar

UNIT2ND

Chapter3

PairofLinearEquations in Two Variables

Exercise3.1

1. Formthepair oflinearequationsinthefollowingproblems,andfindtheirsolutionsgraphically.

(i) 10 students ofClassXtook partinaMathematics quiz.Ifthenumberof girlsis 4morethanthe number of boys, find the number of boys and girls who took part in the quiz.

(ii) 5pencilsand7penstogethercost50, whereas7pencilsand5penstogethercost46.Findthe cost of one pencil and that of one pen.

Solution:

(i) Letthere arex numberofgirlsandynumberof boys.Asperthegiven question,thealgebraic expression can be represented as follows.

x+y=10 x–

y = 4

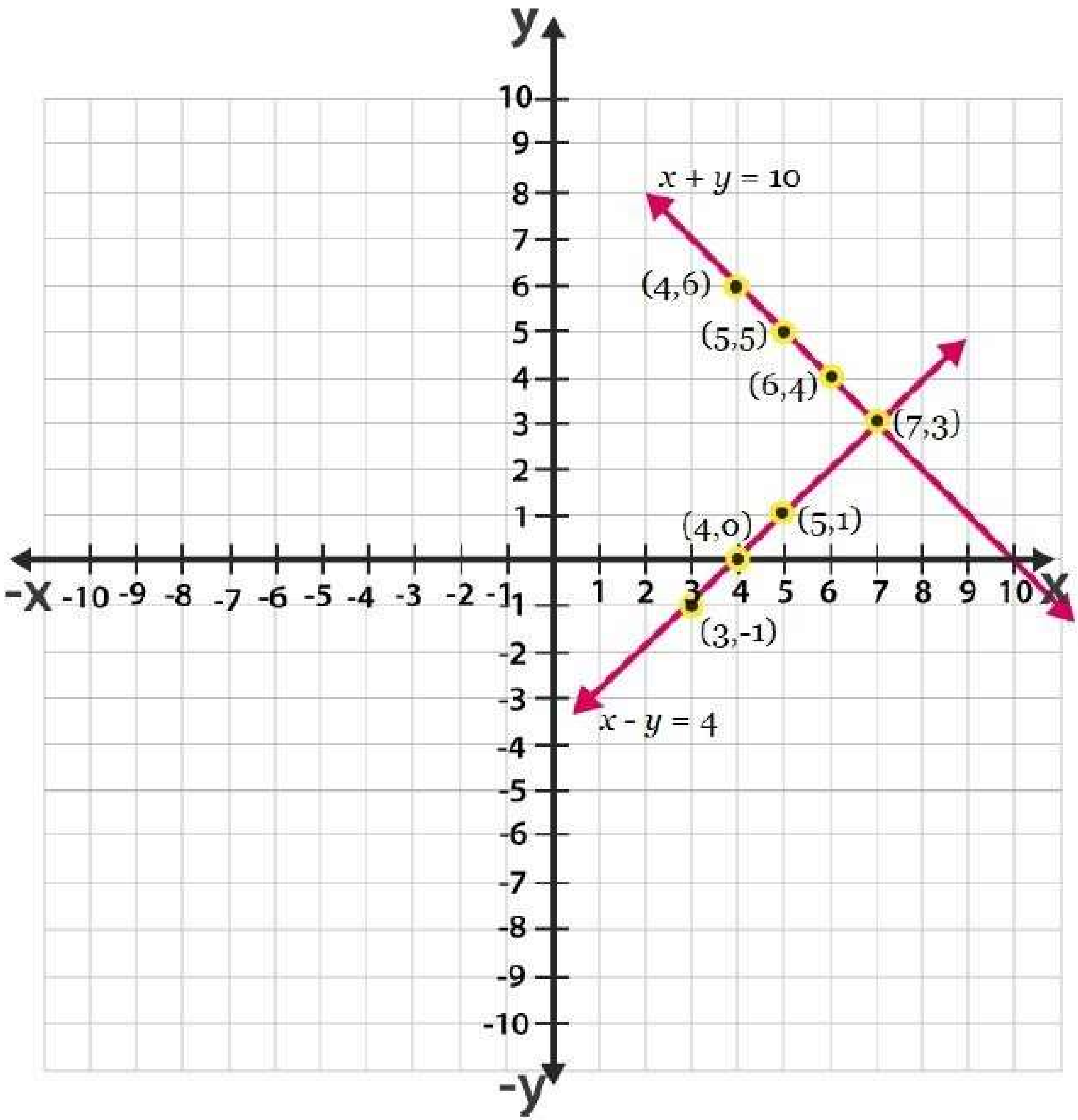
Now,forx+y=10orx=10−y,thesolutionsare;

x	5	4	6
y	5	6	4

Forx–y=4orx=4 +y,thesolutionsare;

x	4	5	3
y	0	1	-1

Thegraphicalrepresentationisasfollows;



Fromthegraph, itcanbeseenthatthegivenlinescrosseachotheratpoint(7,3). Therefore,there are 7 girls and 3 boys in the class.

(ii) Let1pencilcostsRs.xand1 pencostsRs.y.

Accordingtothequestion,thealgebraicexpressioncabberepresentedas; 5x +

7y = 50

7x+5y=46

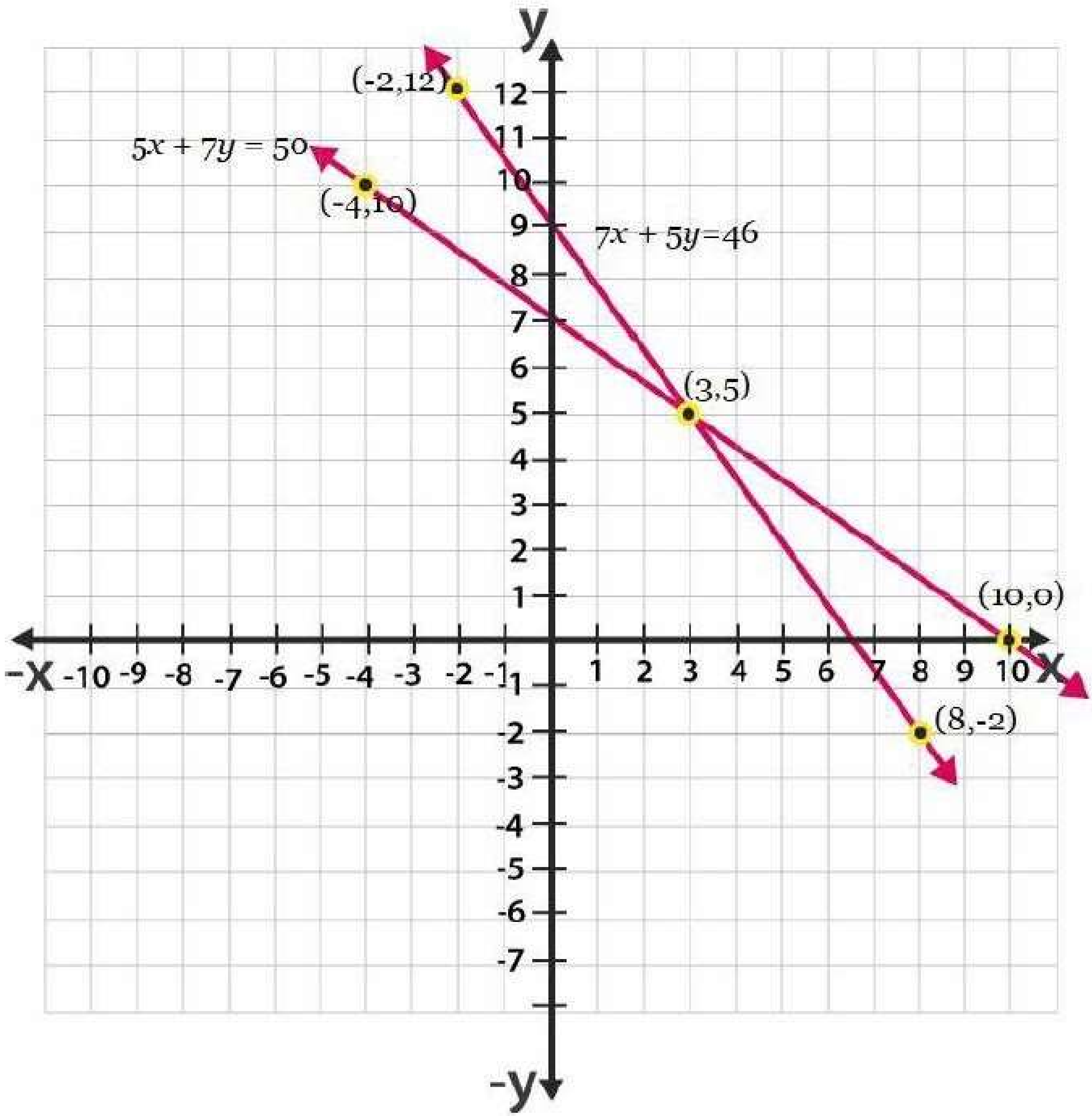
For,5x+7y=50orx=(50-7y)/5,thesolutionsare;

x	3	10	-4
y	5	0	10

For7x+5y=46 orx =(46-5y)/7,thesolutionsare;

x	8	3	-2
y	-2	5	12

Hence,thegraphicalrepresentationisasfollows;



Fromthegraph,itiscan beseenthatthe givenlines crosseachotheratpoint (3, 5). So, the

cost of a pencil is 3/- and cost of a pen is 5/-.

2. Oncomparingtheratiosa1/a2, b1/b2,c1/c2 findoutwhether the linesrepresentingthe following pairs of linear equations intersect at a point, are parallel or coincident:

(i) 5x–4y+8 =0

$7x+6y–9=0$

(ii) $9x+3y+12=0$

$18x + 6y + 24 = 0$

(iii) $6x–3y+10=0$ $2x$

$– y + 9 = 0$

Solutions:

(i) Givenexpressions;

$5x-4y+8=0$

$7x+6y-9=0$

Comparingtheseequationswitha1x+b1y+c1=0 And

$a_2x+b_2y+c_2 = 0$

Weget,

$a_1=5, b_1=-4, c_1= 8$

$a_2=7, b_2=6, c_2=-9$

$(a_1/a_2) = 5/7$

$(b_1/b_2)=-4/6=-2/3$

$(c_1/c_2)=8/-9$

Since, $(a_1/a_2)\neq (b_1/b_2)$

So,thepairsofequationsgiveninthe questionhaveauniquesolutionand thelinescross eachother at exactly one point.

(ii) Givenexpressions;

$9x+3y+12=0$

$18x+6y+24=0$

Comparingtheseequationswitha1x+b1y+c1=0 And

$a_2x+b_2y+c_2 = 0$

Weget,

$a_1=9, b_1=3, c_1=12$

$a_2=18, b_2=6, c_2=24$

$(a_1/a_2) = 9/18 = 1/2$

$(b_1/b_2) = 3/6 = 1/2$

$(c_1/c_2)=12/24=1/2$

Since $(a_1/a_2)=(b_1/b_2)=(c_1/c_2)$

So,thepairsofequationsgiveninthe questionhaveinfinitepossiblesolutions andthelines are coincident.

(iii) GivenExpressions;

$6x–3y+10=0$

$2x–y+9=0$

Comparingtheseequationswith $a_1x+b_1y+c_1=0$ And

$a_2x+b_2y+c_2 = 0$

Weget,

$a_1=6, b_1=-3, c_1= 10$

$a_2=2, b_2=-1, c_2= 9$

$(a_1/a_2) = 6/2 = 3/1$

$(b_1/b_2)=-3/-1=3/1$

$(c_1/c_2)=10/9$

Since $(a_1/a_2)=(b_1/b_2)\neq (c_1/c_2)$

So,thepairsofequationsgiveninthe questionareparalleltoeach otherandthelinesneverintersect each other at any point and there is no possible solution for the given pair of equations.

3. Oncomparingtheratio,(a_1/a_2) ,(b_1/b_2), (c_1/c_2)findoutwhetherthe followingpairoflinear equations are consistent, or inconsistent.

(i) $3x+2y=5;2x–3y=7$

(ii) $2x – 3y = 8$; $4x – 6y = 9$

(iii)$(3/2)x+(5/3)y=7;9x–10y=14$

(iv) $5x–3y=11;–10x+6y=–22$

(v)(4/3)x+2y =8;2x +3y =12

Solutions:

(i) Given:3x+2y=5or3x+2y-5=0 and 2x

– 3y = 7 or 2x – 3y -7 = 0

Comparingtheseequationswitha1x+b1y+c1=0 And

a2x+b2y+c2 = 0

Weget,

a1=3,b1=2,c1=-5

a2=2,b2=-3,c2= -7

(a1/a2) = 3/2

(b1/b2) = 2/-3

(c1/c2)=-5/-7=5/7

Since,(a1/a2)≠ (b1/b2)

So,thegiven equationsintersect each otheratonepoint andthey have onlyonepossiblesolution.The equations are consistent.

(ii) Given2x–3y=8and4x –6y =9

Therefore,

a1=2,b1=-3,c1= -8

a2=4,b2=-6,c2= -9

(a1/a2) = 2/4 = 1/2

(b1/b2)=-3/-6=1/2

(c1/c2)=-8/-9=8/9

Since,(a1/a2) =(b1/b2) ≠(c1/c2)

So,theequations are parallel toeachotherandtheyhavenopossiblesolution.Hence,the equationsare inconsistent.

(iii)Given(3/2)x+(5/3)y=7and9x –10y=14

Therefore,

$a1=3/2, b1=5/3, c1=-7$

$a2=9$, $b2 =-10$, $c2= -14$

$(a1/a2) = 3/(2×9) = 1/6$

$(b1/b2)= 5/(3×-10)= -1/6$

$(c1/c2) = -7/-14 = 1/2$

Since, $(a1/a2)\neq (b1/b2)$

So,theequations areintersectingeachother atonepointandtheyhaveonlyonepossiblesolution. Hence, the equations are consistent.

(iv) Given, 5x– 3y=11and –10x +6y=–22

Therefore,

$a1=5, b1=-3, c1= -11$

$a2 = -10$, $b2 = 6$, $c2 = 22$

$(a1/a2)=5/(-10)=-5/10=-1/2$

$(b1/b2) = -3/6 = -1/2$

$(c1/c2)=-11/22=-1/2$

Since $(a1/a2)=(b1/b2)=(c1/c2)$

Theselinearequationsarecoincidentlines andhaveinfinitenumberof possiblesolutions. Hence,the equations are consistent.

(v) Given,(4/3)x+2y=8and2x+3y=12

$a1 = 4/3$, $b1= 2$, $c1 = -8a2$

$= 2$, $b2 = 3$, $c2 = -12$

$(a1/a2)=4/(3×2)=4/6=2/3$

$(b1/b2)=2/3$

$(c1/c2)=-8/-12=2/3$

Since $(a1/a2)=(b1/b2)=(c1/c2)$

Theselinearequationsarecoincidentlines andhaveinfinitenumberof possible solutions. Hence,the equations are consistent.

4. Whichofthefollowingpairsoflinearequationsareconsistent/inconsistent?Ifconsistent, obtain the solution graphically:

(i) x +y= 5,2x+2y=10

(ii) x –y= 8,3x–3y=16

(iii) 2x+y–6=0,4x–2y–4=0

(iv) 2x–2y–2=0,4x–4y–5=0

Solutions:

(i) Given,x+y=5and2x+2y=10

(a1/a2)=1/2

(b1/b2)=1/2

(c1/c2)=1/2

Since(a1/a2)=(b1/b2)=(c1/c2)

∴Theequationsarecoincidentandtheyhaveinfinitenumberofpossiblesolutions. So, the equations are consistent.

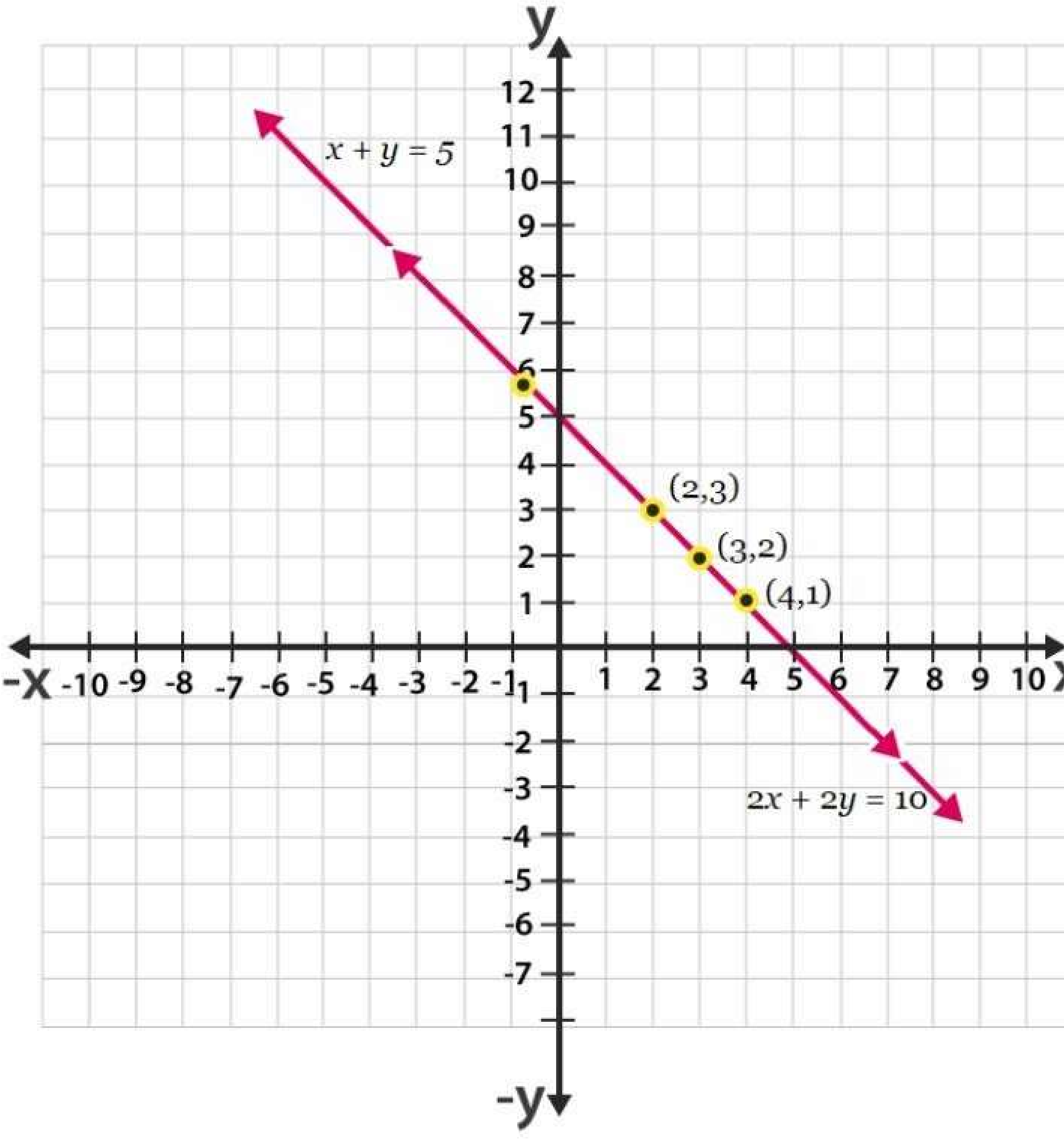
For,x+y= 5orx=5–y

x	4	3	2
y	1	2	3

For2x+2y=10orx=(10-2y)/2

x	4	3	2
y	1	2	3

So,theequationsarerepresentedingraphsasfollows:



Fromthefigure,wecansee,thatthelinesareoverlappingeachother. Therefore,

the equations have infinite possible solutions.

(ii) Given,x–y=8and 3x–3y= 16

$(a1/a2) = 1/3$

$(b1/b2)=-1/-3=1/3$

$(c1/c2)=8/16=1/2$

Since, $(a1/a2)=(b1/b2)\neq(c1/c2)$

Theequationsareparalleltoeachotherandhaveno solutions.Hence, thepairof linearequationsis inconsistent.

(iii) Given,2x+y–6=0 and4x–2y–4= 0

$(a1/a2) = 2/4 = ½$

$(b1/b2) = 1/-2$

$(c1/c2)=-6/-4=3/2$

Since, $(a1/a2)\neq (b1/b2)$

Thegivenlinearequations areintersectingeachotherat onepointand have onlyonesolution.Hence, the pair of linear equations is consistent.

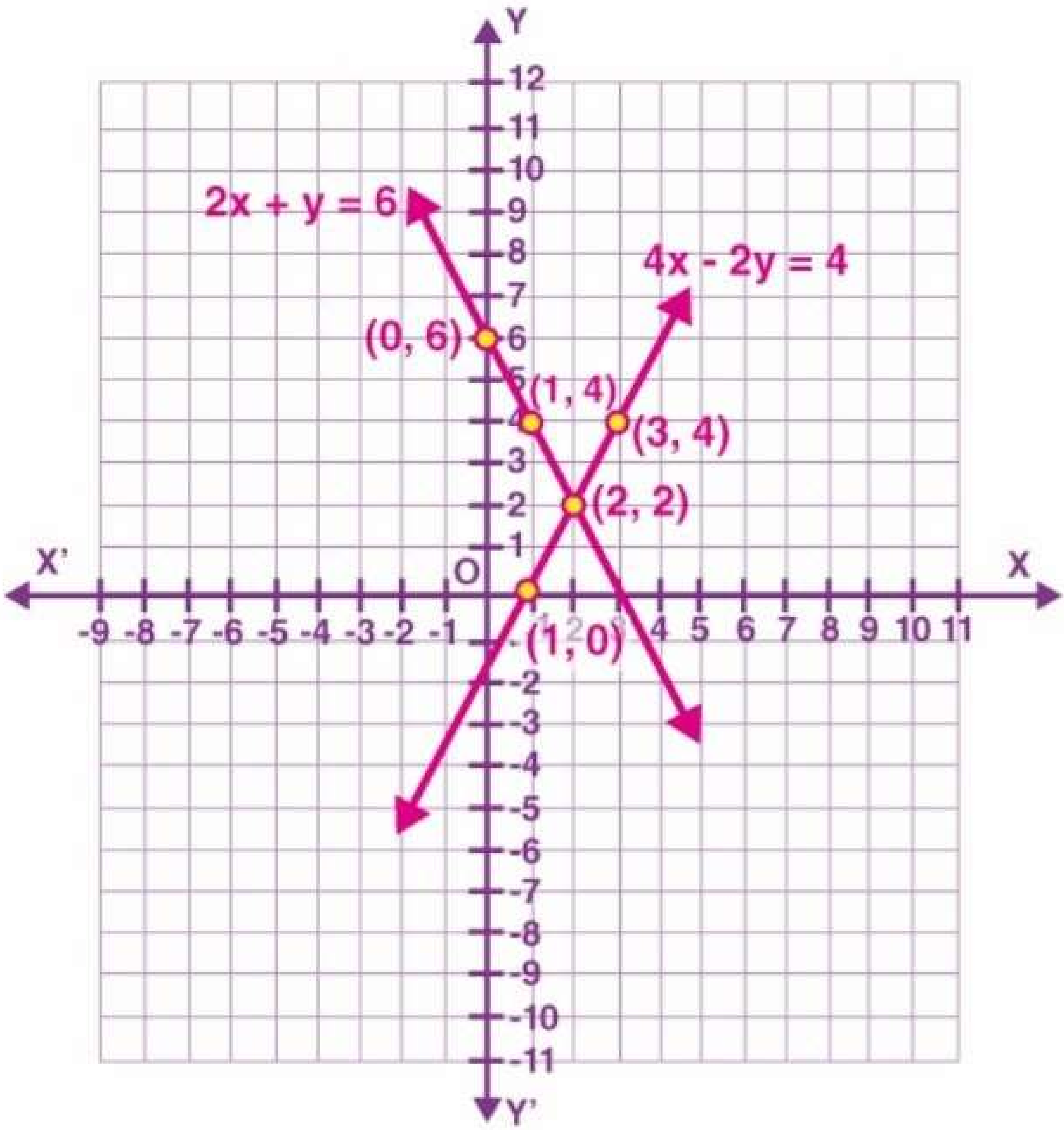
Now,for**2x+y–6=0ory=6–2x**

x	0	1	2
y	6	4	2

Andfor**4x–2y–4=0ory=(4x-4)/2**

x	1	2	3
y	0	2	4

So,theequationsarerepresentedingraphsasfollows:



Fromthegraph,itcanbeseenthattheselinesareintersectingeachotheratonlyonepoint,(2,2).

(iv) Given,2x–2y–2= 0and4x–4y–5= 0

(a1/a2) = 2/4 = ½

(b1/b2)=-2/-4=1/2

(c1/c2)=2/5

Since,a1/a2 =b1/b2≠c1/c2

Thus,theselinearequationshaveparallelandhavenopossiblesolutions.Hence, thepairof linear equations are inconsistent.

5. Halfthe perimeterof arectangulargarden,whoselengthis 4mmorethanitswidth, is36m. Find the dimensions of the garden.

Solutions: Letusconsider.

Thewidthofthegardenisxandlengthisy.

Now,accordingtothequestion,wecanexpress thegivenconditionas; y – x

= 4

and

y+x=36

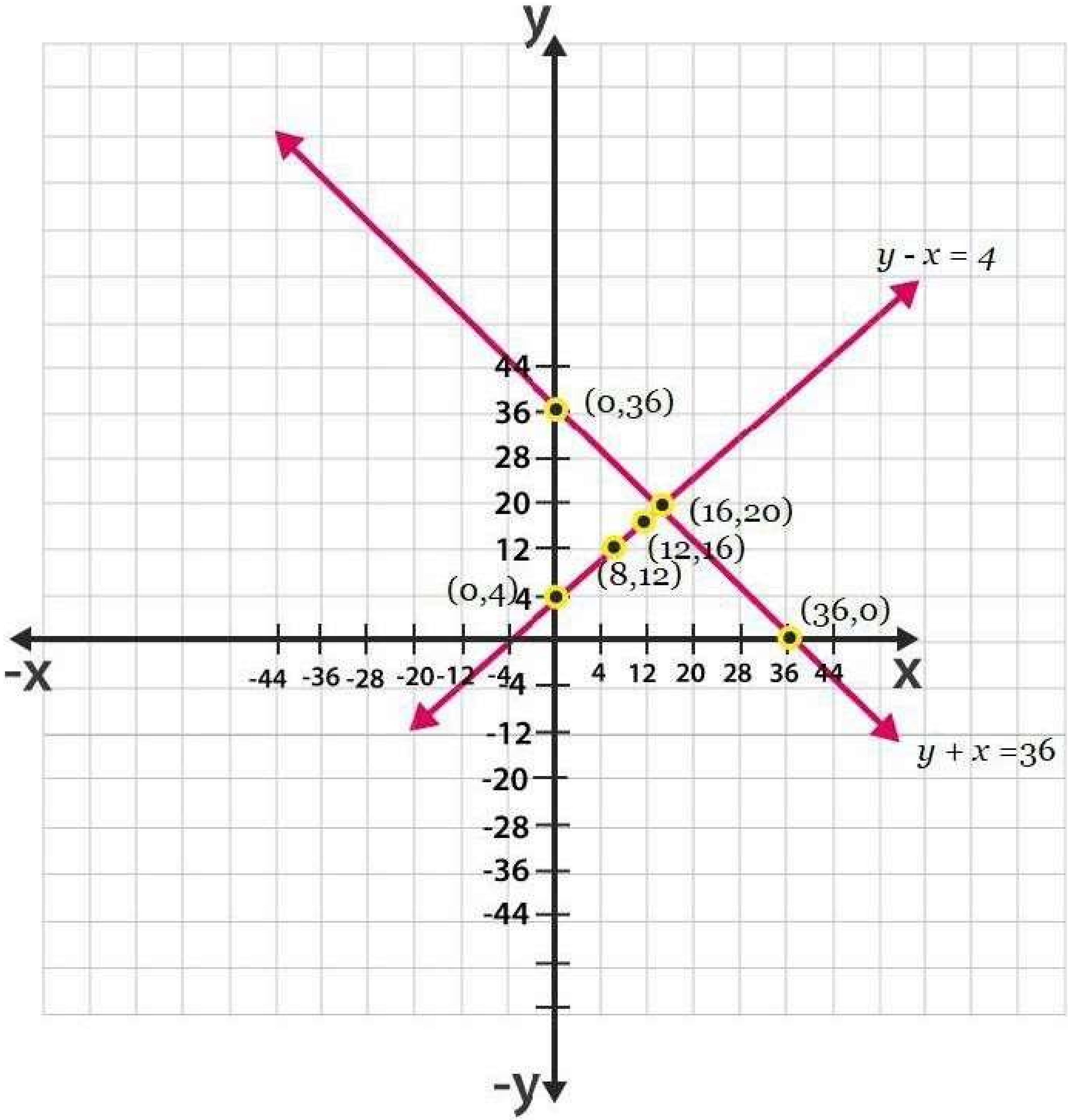
Now,takingy–x=4ory=x+ 4

x	0	8	12
y	4	12	16

Fory+x= 36,y=36–x

x	0	36	16
y	36	0	20

Thegraphical representationofboththeequationisas follows;



Fromthegraph youcansee, thelines intersectseachotherata point(16, 20).Hence,thewidthofthe garden is 16 and length is 20.

6. Giventhelinear equation 2x+ 3y– 8=0,writeanother linear equationintwovariablessuchthat the geometrical representation of the pair so formed is:

(i) Intersectinglines

(ii) Parallellines

(iii) Coincidentlines

Solutions:

(i) Giventhelinear equation2x+3y–8=0.

Tofind anotherlinearequationintwovariables suchthat thegeometricalrepresentation ofthepairso formed is intersecting lines, it should satisfy below condition;

$(a1/a2)\neq(b1/b2)$

Thus,anotherequation could be2x– 7y+9=0,suchthat; (a1/a2)

= 2/2 = 1 and (b1/b2) = 3/-7

Clearly,youcanseeanotherequationsatisfiesthecondition.

(ii) Given thelinear equation2x+3y–8=0.

Tofind anotherlinearequationintwovariables suchthat thegeometricalrepresentation ofthepairso formed is parallel lines, it should satisfy below condition;

$(a1/a2)=(b1/b2)\neq (c1/c2)$

Thus,anotherequationcouldbe6x +9y+9=0,suchthat; (a1/a2) =

2/6 = 1/3

(b1/b2)=3/9=1/3

(c1/c2) = -8/9

Clearly,youcanseeanotherequationsatisfiesthecondition.

(iii) Given thelinear equation2x+3y–8=0.

Tofind anotherlinearequationintwovariables suchthat thegeometricalrepresentation ofthepairso formed is coincident lines, it should satisfy below condition;

(a1/a2)=(b1/b2)=(c1/c2)

Thus,anotherequation could be4x +6y – 16= 0,such that;

(a1/a2)=2/4=1/2,(b1/b2)=3/6=1/2,(c1/c2)= -8/-16= 1/2

Clearly,youcanseeanotherequationsatisfiesthecondition.

7. Draw thegraphsoftheequationsx –y+ 1=0and 3x+2y–12=0. Determine the coordinates of the vertices of the triangle formed by these lines and the x-axis, and shade the triangular region.

Solution: Given,theequationsforgraphsarex–y+1=0and 3x+2y–12= 0.

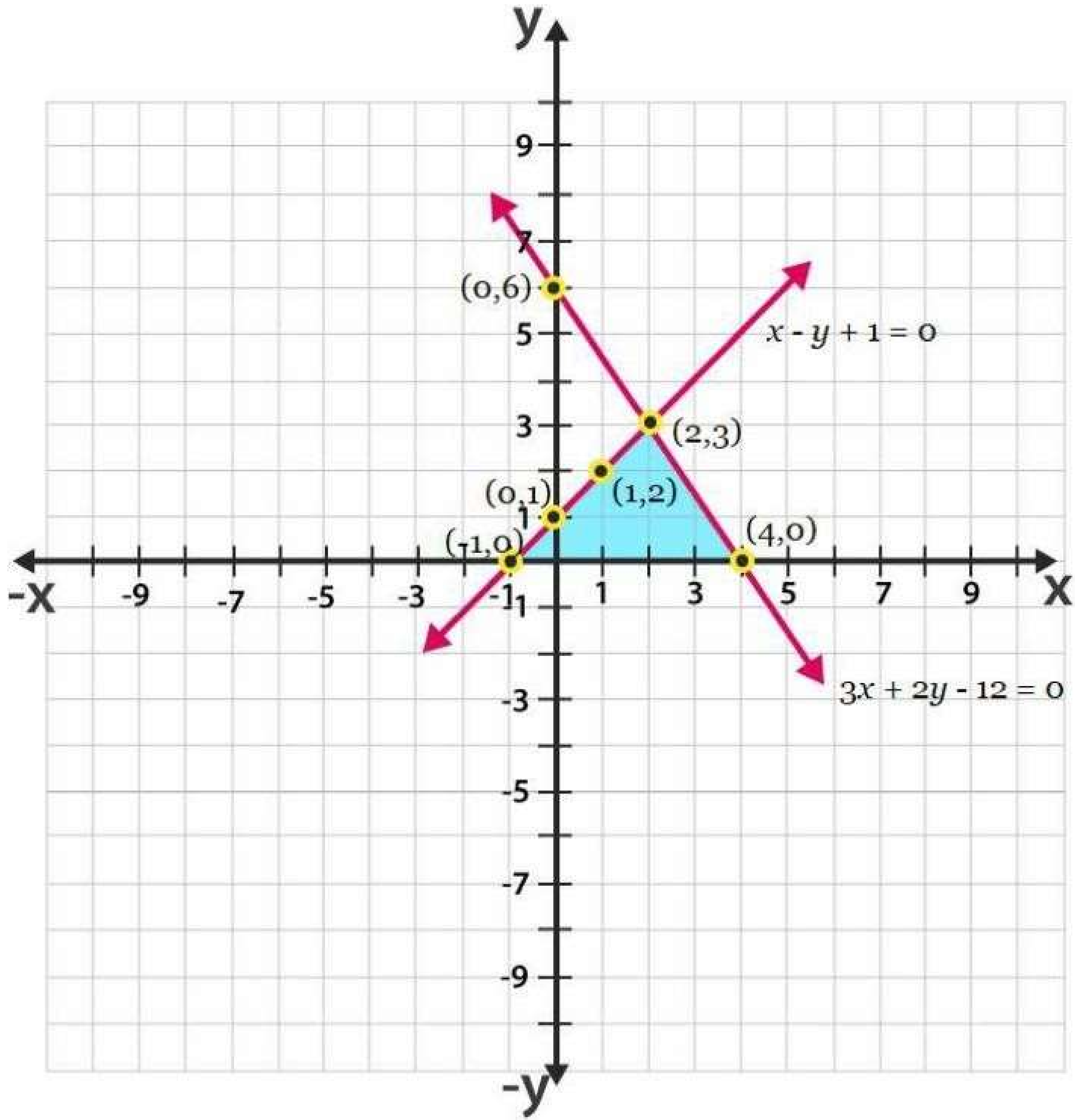
For,x–y+1=0orx=-1+y

x	0	1	2
y	1	2	3

For,3x+2y–12=0 orx=(12-2y)/3

x	4	2	0
y	0	3	6

Hence,thegraphicalrepresentationoftheseequationsisasfollows;



From thefigure,it canbeseenthat theselinesareintersectingeach otheratpoint(2,3) andx-axisat (−1, 0) and (4, 0). Therefore, the vertices of the triangle are (2, 3), (−1, 0), and (4, 0).

Exercise3.2

1. Solvethefollowingpairoflinearequations bythesubstitutionmethod

(i) x+y=14 x

– y = 4

(ii) s–t= 3

(s/3)+(t/2)=6

(iii) 3x–y=3

9x – 3y = 9

(iv) 0.2x+0.3y=1.3

0.4x + 0.5y = 2.3

(v) √2 x+√3y=0

√3x-√8y=0

(vi) (3x/2)–(5y/3)=-2

(x/3)+(y/2)=(13/6)

Solutions:

(i) Given,

x+y=14andx–y =4arethetwoequations. From

1_{st}equation, we get,

x=14–y

Now,substitutethevalueofxinsecondequationtoget, (14 –

y) – y = 4

14–2y=4

2y=10

Ory=5

Bythevalueofy,wecannowfindtheexactvalueofx;

∵x=14–y

∴x=14–5

Orx=9

Hence,x=9andy=5.

(ii) Given,

s–t=3 and (s/3)+(t/2)=6arethetwoequations. From

1stequation, we get,

s = 3 + t ________________(1)

Now,substitutethevalueofsinsecondequationtoget, (3+t)/3 +

(t/2) = 6

⇒(2(3+t)+3t)/6=6

⇒(6+2t+3t)/6=6

⇒(6+5t)=36

⇒5t=30

⇒t=6

Now,substitutethevalueoftinequation(1) s =

3 + 6 = 9

Therefore,s=9andt=6.

(iii) Given,

3x–y=3and9x–3y =9arethetwo equations. From

1stequation, we get,

x=(3+y)/3

Now,substitutethevalueof xinthegivensecondequationtoget, 9(3+y)/3 –

3y = 9

⇒9+3y-3y=9

⇒9=9

Therefore,yhasinfinitevaluesandsince,x=(3+y)/3,soxalsohasinfinitevalues.

(iv) Given,

0.2x+0.3y=1.3and0.4x+0.5y=2.3arethe twoequations. From

1st equation, we get,

x = (1.3-0.3y)/0.2 __________________(1)

Now,substitutethevalueof xinthegivensecondequationtoget,

0.4(1.3-0.3y)/0.2 + 0.5y = 2.3

⇒2(1.3–0.3y)+0.5y=2.3

⇒2.6–0.6y +0.5y=2.3

⇒2.6–0.1y=2.3

⇒0.1y=0.3

⇒y=3

Now,substitutethevalueofyinequation(1),weget, x =

(1.3-0.3(3))/0.2 = (1.3-0.9)/0.2 = 0.4/0.2 = 2

Therefore,x=2andy=3.

(v) Given,

√2x+√3y =0and√3x –√8y =0 are

the two equations.

From1st equation,weget,

x = – (√3/√2)y ___________________(1)

Puttingthevalueofxinthegivensecondequationtoget,

√3(-√3/√2)y–√8y=0⇒(-3/√2)y-√8y =0

⇒y=0

Now,substitutethevalueofyinequation(1),weget, x = 0

Therefore,x=0andy=0.

(vi) Given,

(3x/2)-(5y/3)=-2and(x/3)+(y/2)=13/6arethetwoequations. From

1$_{st}$equation, we get,

(3/2)x =-2 +(5y/3)

⇒x=2(-6+5y)/9=(-12+10y)/9 (1)

Puttingthevalueof xinthegivensecondequationtoget, ((-

12+10y)/9)/3 + y/2 = 13/6

⇒y/2=13/6–((-12+10y)/27)+y/2=13/6

$$\frac{\frac{-12+10y}{9}}{3}+\frac{y}{2}=\frac{13}{6} \Rightarrow \frac{-12+10y}{27}+\frac{y}{2}=\frac{13}{6}$$

$$\Rightarrow \frac{y}{2}=\frac{13}{6}-\frac{-12+10y}{27} \Rightarrow \frac{y}{2}=\frac{117}{54}-\frac{-24+20y}{54}$$

$$\Rightarrow \frac{y}{2}=\frac{117+24-20y}{54}$$

$$\Rightarrow y = 3$$

Now,substitutethevalueofyinequation(1),weget, (3x/2) –

5(3)/3 = -2

⇒(3x/2)–5=-2

⇒x=2

Therefore,x=2andy=3.

2. Solve2x+ 3y= 11and2x–4y=–24andhencefindthevalueof'm'for whichy=mx+3.

Solution:

2x+3y=11 (I)

2x–4y= -24 (II)

Fromequation(II),weget

x=(11-3y)/2 (III)

Substitutingthevalueofxinequation(II),weget

2(11-3y)/2 – 4y = 24

11–3y–4y= -24

-7y=-35

$y=5$ (IV)

Puttingthevalueofyinequation(III),weget x =

$(11-3\times5)/2 = -4/2 = -2$

Hence,$x=-2$,$y=5$ Also,

$y=mx+3$

$5=-2m+3$

$-2m=2$

$m=-1$

Thereforethevalueofmis-1.

3. Formthepair oflinearequationsforthefollowingproblemsandfindtheirsolutionby substitution method.

(i) Thedifference betweentwonumbersis26andonenumberisthree timestheother.Findthem.

Solution:

Letthetwonumbers bexandyrespectively,suchthaty >x.

According to the question,

$y=3x$ (1)

$y–x=26$ (2)

Substitutingthevalueof(1)into(2),weget 3x –

x = 26

$x=13$ (3)

Substituting(3)in(1),wegety =39

Hence,thenumbersare13and39.

(ii) Thelargeroftwosupplementaryanglesexceedsthesmallerby18 degrees.Findthem.

Solution:

Letthelargeranglebyx$_o$andsmaller anglebey$_o$.

Weknowthatthesumoftwosupplementarypairof anglesisalways180$_o$.

According to the question,

$x + y = 180^\circ$ (1)

$x – y = 18^\circ$ (2)

From(1),wegetx=$180^\circ – y$ (3)

Substituting(3)in(2),weget 180°

$– y – y = 18^\circ$

$162^\circ = 2y$

$y = 81^\circ$ (4)

Usingthevalueofyin(3),weget x =

$180^\circ – 81^\circ$

$= 99^\circ$

Hence,theanglesare99°and81°.

(iii) Thecoachofa cricketteambuys 7batsand6ballsforRs3800. Later,she buys3bats and5 ballsforRs.1750.Findthecostofeachbatandeachball.

Solution:

Letthecostabatbexandcostof aballbey. According

to the question,

$7x+6y=3800$ (I)

$3x+5y=1750$ (II)

From(I),weget

$y=(3800-7x)/6$ (III)

Substituting(III)in(II).weget,

$3x+5(3800-7x)/6 =1750$

$\Rightarrow 3x+9500/3–35x/6=1750$

$\Rightarrow 3x-35x/6=1750–9500/3$

$\Rightarrow (18x-35x)/6=(5250–9500)/3$

$\Rightarrow -17x/6=-4250/3$

⇒-17x=-8500

x=500 (IV)

Substitutingthevalueofxin(III),weget y =

(3800-7 ×500)/6 = 300/6 = 50

Hence, the costofabatis Rs500andcostofaballis Rs50.

(iv) The taxi charges in a city consist of a fixed charge together with the charge for the distance covered.For adistanceof10 km,the chargepaidisRs105and for ajourneyof15 km,the charge paidisRs155.Whatarethefixedchargesand the charge per km?How muchdoesa personhave to pay for travelling a distance of 25 km?

Solution:

Letthe fixedcharge beRsxandperkmchargebeRsy. According

to the question,

x+10y=105 (1)

x+15y=155 (2)

From(1),wegetx=105– 10y (3)

Substitutingthevalueofxin(2),weget 105

– 10y + 15y = 155

5y=50

y=10 (4)

Puttingthevalueofyin(3),weget x =

105 – 10 × 10 = 5

Hence,fixedchargeisRs5andperkmcharge=Rs10 Charge

for 25 km = x + 25y = 5 + 250 = Rs 255

(v) Afractionbecomes 9/11, if2isaddedtoboththe numeratorandthe denominator.If, 3is added to both the numerator and the denominator it becomes 5/6. Find the fraction.

Solution:

Let the fraction be x/y.

Accordingtothequestion,

(x+2)/(y+2)=9/11

11x+22=9y+18

11x–9y=-4 (1)

(x+3)/(y+3)=5/6

6x+18=5y+15

6x–5y= -3 (2)

From(1),weget x=(-4+9y)/11 (3)

Substitutingthevalueofxin(2),weget 6(-

4+9y)/11 -5y = -3

-24+54y–55y=-33

-y=-9

y=9 (4)

Substitutingthevalueofyin(3),weget x =

(-4+9×9)/11 = 7

Hencethefractionis7/9.

(vi) Five yearshence,theageofJacob willbethreetimesthat of his son.Fiveyearsago,Jacob’s age was seven times that of his son. What are their present ages?

Solutions:

LettheageofJacobandhissonbexandyrespectively. According

to the question,

(x+5) =3(y+5)

x–3y=10 (1)

(x–5)=7(y–5)

x–7y=-30 (2)

From(1),wegetx=3y+10 (3)

Substitutingthevalueofxin(2),weget 3y +

10 – 7y = -30

$-4y=-40$

$y=10$ (4)

Substitutingthevalueofyin(3),weget x =

3 x 10 + 10 = 40

Hence,thepresentageofJacob'sand hissonis40yearsand10yearsrespectively.

Exercise3.3

1. Solvethefollowingpairoflinearequationsbytheeliminationmethodandthesubstitution method:

(i) x+y=5and2x–3y=4

(ii) 3x+4y=10and2x–2y=2

(iii) 3x–5y–4=0and9x=2y+7

(iv) x/2+2y/3=-1andx-y/3=3

Solutions:

(i) x + y = 5 and 2x – 3y =

4Bythemethodofelimination.

$x+y=5$ (i)

$2x-3y=4$ (ii)

Whentheequation(i)ismultipliedby2,weget 2x +

2y = 10 (iii)

Whentheequation(ii)issubtractedfrom(iii)weget, 5y =

6

$y=6/5$ (iv)

Substitutingthevalueofyineq.(i)weget,

$x=5-6/5 = 19/5$

∴x=19/5,y=6/5

Bythemethodofsubstitution.

Fromtheequation(i),weget:

x=5–y… (v)

Whenthevalueisputinequation(ii)weget, 2(5

– y) – 3y = 4

-5y=-6

y=6/5

Whenthevaluesaresubstitutedinequation(v),weget: x

=5− 6/5 = 19/5

∴x=19/5,y=6/5

(ii) 3x +4y=10and2x –2y =2 By

the method of elimination.

3x+4y=10 (i)

2x–2y=2 (ii)

Whentheequation(i)and(ii)ismultipliedby2,weget: 4x –

4y = 4 (iii)

WhentheEquation(i)and(iii)areadded,weget: 7x =

14

x=2 (iv)

Substitutingequation(iv)in(i)weget, 6 +

4y = 10

4y=4

y=1

Hence,x=2 andy= 1

BythemethodofSubstitution

Fromequation(ii)weget,

$x=1+y$ (v)

Substitutingequation(v)inequation(i)weget, 3(1 +

y) + 4y = 10

$7y=7$

$y=1$

Wheny=1issubstitutedinequation(v)weget, A = 1

+ 1 = 2

Therefore,A=2 andB=1

(iii) 3x–5y–4 =0and9x=2y+7 By

the method of elimination:

$3x-5y-4=0$ (i)

$9x=2y+7$

$9x-2y-7=0$ (ii)

Whentheequation(i)and(iii)ismultipliedweget, 9x –

15y – 12 = 0 (iii)

Whentheequation(iii)issubtractedfromequation(ii)weget, 13y =

-5

$y=-5/13$ (iv)

Whenequation(iv)issubstitutedinequation(i)weget, 3x

+25/13 −4=0

$3x=27/13$

$x=9/13$

∴x=9/13andy=-5/13

BythemethodofSubstitution:

Fromtheequation(i)weget,

$x=(5y+4)/3$ (v)

Puttingthevalue(v)inequation(ii)weget, 9(5y+4)/3

−2y −7=0

13y = -5

y=-5/13

Substitutingthisvalueinequation(v)weget, x =

(5(-5/13)+4)/3

x=9/13

∴x=9/13**,y=-5/13**

(iv) x/2+2y/3=-1andx-y/3=3 By

the method of Elimination.

3x+4y=-6 (i)

x-y/3=3

3x–y=9 (ii)

Whentheequation(ii)issubtractedfromequation(i)weget, 5y = -

15

y=-3 (iii)

Whentheequation(iii)issubstitutedin(i)weget, 3x –

12 = -6

3x=6

x=2

Hence, x=2 ,y=-3

BythemethodofSubstitution:

Fromtheequation(ii)we get,

x=(y+9)/3 (v)

Puttingthevalueobtainedfromequation(v)inequation(i)weget, 3(y+9)/3

+4y =−6

5y=-15

y=-3

Wheny=-3issubstitutedinequation(v)weget, x = (-

3+9)/3 = 2

Therefore,x=2andy=-3

2. Formthepair oflinearequationsinthefollowingproblems,andfindtheirsolutions(ifthey exist) by the elimination method:

(i) Ifweadd1tothe numeratorandsubtract 1fromthedenominator,afractionreduces to1.It becomes if we only add 1 to the denominator. What is the fraction?

Solution:

Letthe fractionbea/b

Accordingtothegiveninformation,

(a+1)/(b-1) = 1

=>a–b=-2 (i)

a/(b+1)=1/2

=>2a-b=1 (ii)

Whenequation(i)issubtractedfromequation(ii)weget, a =

3 (iii)

Whena=3issubstitutedinequation(i)weget, 3 – b

= -2

-b=-5

b=5

Hence,thefractionis3/5.

(ii) Fiveyearsago,NuriwasthriceasoldasSonu. Tenyearslater,Nuriwillbe twiceas old as Sonu. How old are Nuri and Sonu?

Solution:

Letusassume, presentageofNuriisx

AndpresentageofSonuisy.

Accordingtothegivencondition,wecanwriteas; x –

5 = 3(y – 5)

x–3y=-10 (1)

Now,

x+10=2(y+10)

x–2y=10 (2)

Subtracteq.1from 2,toget,

y=20 (3)

Substitutingthevalueofyineq.1,weget, x –

3.20 = -10

x–60= -10

x=50

Therefore,

Age of Nuri is 50 years

AgeofSonuis20years.

(iii) The sumof thedigits ofatwo-digitnumberis9.Also,nine timesthisnumberis twicethe number obtained by reversing the order of the digits. Find the number.

Solution:

Lettheunitdigit andtensdigit ofanumberbexandyrespectively. Then,

Number (n) = 10B + A

Nafterreversingorderofthe digits=10A+B

Accordingtothegiveninformation,A+B= 9 (i)

9(10B+A)=2(10A+B)

88B–11A=0

-A+8B=0 (ii)

Addingthe equations(i)and (ii)weget,

9B=9

B=1 (3)

SubstitutingthisvalueofB,intheequation(i)wegetA= 8

Hencethenumber(N)is10B+A=10x 1+8=18

(iv) Meenawenttoabanktowithdraw Rs.2000.Sheaskedthe cashiertogiveherRs.50and Rs.100 notesonly. Meenagot25notesinall.Findhow manynotesofRs.50 andRs.100she received.

Solution:

Letthe numberofRs.50 notesbeAand thenumberofRs.100notesbe B

According to the given information,

A+B =25 (i)

50A+100B=2000 (ii)

Whenequation (i)ismultipliedwith(ii)weget,

50A+50B= 1250 (iii)

Subtractingtheequation(iii)fromtheequation(ii)weget, 50B

= 750

B=15

Substitutingintheequation(i)weget, A

= 10

Hence,Meenahas10notesofRs.50and15notesofRs.100.

(v) Alendinglibraryhasafixed chargefor the firstthreedaysandanadditionalcharge for each day thereafter. Saritha paid Rs.27 for a book kept for seven days, while Susy paid Rs.21 for the book she kept for five days. Find the fixed charge and the charge for each extra day.

Solution:

Letthe fixedcharge forthefirst threedays beRs.Aandthechargeforeachday extra beRs.B. According to the

information given,

A +4B =27 (i)

A +2B =21 (ii)

Whenequation(ii)issubtractedfromequation(i)weget, 2B =

6

B=3 (iii)

SubstitutingB=3inequation(i)weget, A +

12 = 27

A=15

Hence,thefixedchargeisRs.15

And the Charge per day is Rs.3

QuadraticEquation

Exercise4.1

1. Checkwhetherthefollowingarequadraticequations:

(i) $(x+1)_2= 2(x-3)$

(ii) $x_2-2x=(-2)(3-x)$

(iii) $(x-2)(x+1)=(x-1)(x+3)$

(iv) $(x-3)(2x+1)=x(x+5)$

(v) $(2x-1)(x-3)=(x+5)(x-1)$

(vi) $x_2+3x+1=(x-2)_2$

(vii) $(x+2)_3=2x(x_2-1)$

(viii) x_3-4x_2-

$x+1=(x-2)_3$Solutions:

(i) Given,

$(x+1)_2=2(x-3)$

Byusingtheformulafor $(a+b)_2=a_2+2ab+b_2$

$\Rightarrow x_2+2x+1=2x-6$

$\Rightarrow x_2+7=0$

The above equation is in the form of $ax^2 + bx + c = 0$

Therefore,thegivenequationisaquadraticequation.

(ii) Given, $x^2 – 2x = (–2)(3 – x)$

$\Rightarrow x^2 – 2x = -6 + 2x$

$\Rightarrow x^2 – 4x + 6 = 0$

The above equation is in the form of $ax^2 + bx + c = 0$

Therefore,thegivenequationisaquadraticequation.

(iii) Given, $(x – 2)(x + 1) = (x – 1)(x + 3)$ By

multiplication,

$\Rightarrow x^2 – x – 2 = x^2 + 2x – 3$

$\Rightarrow 3x – 1 = 0$

The above equation is not in the form of $ax^2 + bx + c = 0$

Therefore,thegivenequationisnotaquadraticequation.

2. Representthefollowingsituationsintheformof quadraticequations:

(i) Thearea ofarectangularplotis 528m^2.The lengthofthe plot (inmetres)is onemorethan twice its breadth. We need to find the length and breadth of the plot.

(ii) Theproductoftwoconsecutivepositiveintegersis306.We needtofindtheintegers.

(iii) Rohan'smother is26years older thanhim.The productoftheirages(inyears)3yearsfrom now will be 360. We would like to find Rohan's present age.

(iv) Atraintravelsa distanceof480 km ata uniform speed. Ifthespeedhad been8km/hless, then it would have taken

Solutions:

(i) Letusconsider,

Thebreadthoftherectangularplot=xm

Thus, the length of the plot = (2x + 1) m

As we know,

Areaofrectangle=length×breadth=528m^2

Puttingthevalueof thelengthandbreadthof theplotintheformula,we get, (2x +

1) × x = 528

$\Rightarrow 2x^2+x=528$

$\Rightarrow 2x^2+x-528=0$

Therefore,thelengthand breadthof the plotsatisfythequadratic equation,$2x^2+x-528=0$,whichisthe required representation of the problem mathematically.

(ii) Letus consider,

Thefirstintegernumber=x

Thus,thenext consecutivepositiveintegerwillbe=x+1 Product of

two consecutive integers = x × (x +1) = 306

$\Rightarrow x^2+x=306$

$\Rightarrow x^2+x-306=0$

Therefore,thetwointegersx andx+1satisfythequadraticequation,$x^2+x-306=0$,whichisthe required representation of the problem mathematically.

(iii) Letusconsider,

Age of Rohan's =

xyearsTherefore,asperthegivenquest

ion, Rohan's mother's age = x + 26

After3years,

AgeofRohan's=x+3

AgeofRohan'smotherwill be=x+26+3=x+29

Theproduct oftheiragesafter3yearswillbe equalto360,suchthat (x +

3)(x + 29) = 360

$\Rightarrow x^2+29x+3x+87=360$

$\Rightarrow x^2+32x+87-360=0$

$\Rightarrow x^2+32x-273=0$

Therefore,the ageofRohanand hismothersatisfiesthe quadraticequation,x_2+32x–273=0,whichis the required representation of the problem mathematically.

(iv) Letusconsider,

Thespeedofthetrain=$x$km/h And

Timetakento travel 480km=480/xkm/hr

Aspersecondcondition,thespeedoftrain=(x–8)km/h

Alsogiven,the trainwilltake3hourstocoverthesame distance. Therefore,

time taken to travel 480 km = (480/x)+3 km/h

Asweknow,

Speed×Time=Distance

Therefore,

(x–8)(480/x)+3=480

⇒480+3x –3840/x–24=480

⇒3x–3840/x=24

⇒x_2–8x–1280=0

Therefore,thespeed ofthetrain satisfiesthequadratic equation,x_2–8x–1280 =0,whichisthe required representation of the problem mathematically.

Exercise4.2

1. Findtherootsofthe followingquadraticequations byfactorisation:

(i) x_2–3x–10=0
(ii) 2x_2+x–6=0
(iii) √2x_2+ 7x+5√2=0
(iv) 2x_2–x+1/8=0
(v) 100x_2–20x +1=0

Solutions:

(i) Given,x_2–3x–10=0

Taking L.H.S.,

$\Rightarrow x_2 – 5x + 2x – 10$

$\Rightarrow x(x – 5) + 2(x – 5)$

$\Rightarrow (x – 5)(x + 2)$

The roots of this equation, $x_2 – 3x – 10 = 0$ are the values of x for which $(x – 5)(x + 2) = 0$ Therefore, $x – 5 = 0$

or $x + 2 = 0$

$\Rightarrow x = 5$ or $x = -2$

(ii) Given, $2x_2 + x – 6 = 0$

Taking L.H.S.,

$\Rightarrow 2x_2 + 4x – 3x – 6$

$\Rightarrow 2x(x + 2) – 3(x + 2)$

$\Rightarrow (x + 2)(2x – 3)$

The roots of this equation, $2x_2 + x – 6 = 0$ are the values of x for which $(xx + 2)(2x – 3) = 0$ Therefore, $x + 2 =$

0 or $2x – 3 = 0$

$\Rightarrow x = -2$ or $x = 3/2$

(iii) $\sqrt{2}x_2 + 7x + 5\sqrt{2} = 0$

Taking L.H.S.,

$\Rightarrow \sqrt{2}x_2 + 5x + 2x + 5\sqrt{2}$

$\Rightarrow x(\sqrt{2}x + 5) + \sqrt{2}(\sqrt{2}x + 5) = (\sqrt{2}x + 5)(x + \sqrt{2})$

The roots of this equation, $\sqrt{2}x_2 + 7x + 5\sqrt{2} = 0$ are the values of x for which $(\sqrt{2}x + 5)(x + \sqrt{2}) = 0$ Therefore, $\sqrt{2}x + 5$

$= 0$ or $x + \sqrt{2} = 0$

$\Rightarrow x = -5/\sqrt{2}$ or $x = -\sqrt{2}$

(iv) $2x_2 – x + 1/8 = 0$

Taking L.H.S.,

$= 1/8(16x_2 – 8x + 1)$

$= 1/8\ (16x_2 – 4x - 4x + 1)$

$= 1/8(4x(4x – 1) - 1(4x – 1))$

$=1/8\ (4x-1)^2$

The roots of this equation, $2x^2 - x + 1/8 = 0$, are the values of x for which $(4x-1)^2 = 0$ Therefore,

$(4x - 1) = 0$ or $(4x - 1) = 0$

$\Rightarrow x = 1/4$ or $x = 1/4$

(v) Given, $100x^2 - 20x + 1 = 0$

Taking L.H.S.,

$= 100x^2 - 10x - 10x + 1$

$= 10x(10x-1) - 1(10x-1)$

$= (10x-1)^2$

The roots of this equation, $100x^2 - 20x + 1 = 0$, are the values of x for which $(10x-1)^2 = 0$

$\therefore (10x-1) = 0$ or $(10x-1) = 0$

$\Rightarrow x = 1/10$ or $x = 1/10$

2. Solve the problems given in Example 1.

Represent the following situations mathematically:

(i) John and Jivanti together have 45 marbles. Both of them lost 5 marbles each, and the product of the number of marbles they now have is 124. We would like to find out how many marbles they had to start with.

(ii) A cottage industry produces a certain number of toys in a day. The cost of production of each toy (in rupees) was found to be 55 minus the number of toys produced in a day. On a particular day, the total cost of production was Rs. 750. We would like to find out the number of toys produced on that day.

Solutions:

(i) Let us say the number of marbles John has =

x Therefore, the number of marble Jivanti has $= 45 - x$ After

losing 5 marbles each,

Number of marbles John has $= x - 5$

Number of marble Jivanti has $= 45 - x - 5 = 40 - x$

Given that the product of their marbles is 124.

$\therefore (x-5)(40-x)=124$

$\Rightarrow x^2-45x+324=0$

$\Rightarrow x^2-36x-9x+324=0$

$\Rightarrow x(x-36)-9(x-36)=0$

$\Rightarrow (x-36)(x-9)=0$

Thus,wecansay,

$x-36=0$ or $x-9=0$

$\Rightarrow x=36$ or $x=9$

Therefore,

IfJohn'smarbles=36

Then,Jivanti'smarbles=45–36=9 And if

John's marbles = 9

Then,Jivanti'smarbles=45– 9=36

(ii) Letussaythenumberoftoysproducedin adayis x.

Therefore,costof production ofeachtoy =Rs(55 – x)

Giventhetotalcostofproductionofthetoys=Rs750

$\therefore x(55-x)=750$

$\Rightarrow x^2-55x+750=0$

$\Rightarrow x^2-25x-30x+750=0$

$\Rightarrow x(x-25)-30(x-25)=0$

$\Rightarrow (x-25)(x-30)=0$

Thus,either x-25=0or x–30=0

$\Rightarrow x=25$ or $x=30$

Hence,the numberoftoysproducedin adaywillbeeither25or30.

3. Findtwonumberswhose sum is 27and productis 182.

Solution:

Letussaythefirstnumberisx,andthesecondnumberis27 –x.

Therefore,theproductoftwonumbers

x(27 – x) = 182

⇒x^2–27x–182=0

⇒x^2–13x–14x+182=0

⇒x(x–13)-14(x–13)=0

⇒(x–13)(x-14)=0

Thus,either,x=-13= 0orx–14=0

⇒x=13orx=14

Therefore,iffirstnumber=13,thensecondnumber=27 –13=14 And if

first number = 14, then second number = 27 – 14 = 13 Hence, the

numbers are 13 and 14.

4. Findtwoconsecutivepositiveintegers,thesumofwhosesquaresis365.

Solution:

Letussay thetwoconsecutivepositiveintegersare*x*and*x*+1. Therefore, as

per the given questions,

$x^2+(x+1)^2=365$

⇒$x^2+x^2+1+2x=365$

⇒$2x^2+2x-364=0$

⇒$x^2+x-182=0$

⇒$x^2+14x-13x-182=0$

⇒$x(x+14)-13(x+14)=0$

⇒$(x+14)(x-13)=0$

Thus,either,$x+14=0$ or$x-13=0$,

⇒$x=-14$ or$x=13$

Sincetheintegersarepositive,*x*canbe13only.

$\therefore x+1=13+1=14$

Therefore,twoconsecutivepositiveintegers willbe13and14.

5. Thealtitudeofaright triangleis 7cm less thanits base. Ifthehypotenuseis13cm,findthe other two sides.

Solution:

Letussaythebaseoftherighttriangleisxcm. Given,

the altitude of right triangle = (x – 7)cm From

Pythagoras' theorem, we know,

$Base^2+Altitude^2=Hypotenuse^2$

$\therefore x^2+(x-7)^2=13^2$

$\Rightarrow x^2+x^2+49-14x=169$

$\Rightarrow 2x^2-14x-120=0$

$\Rightarrow x^2-7x-60=0$

$\Rightarrow x^2-12x+5x-60=0$

$\Rightarrow x(x-12)+5(x-12)=0$

$\Rightarrow (x-12)(x+5)=0$

Thus,either$x-12=0$or$x+5=0$,

$\Rightarrow x=12$or$x=-5$

Sincesidescannotbe negative,xcanonlybe12.

Therefore,the baseofthe giventriangleis 12cm, and thealtitude ofthistriangle willbe(12 –7)cm=5 cm.

6. A cottage industry produces a certain number of pottery articles in a day. It was observed on a particular day that the cost of production of each article (in rupees) was 3 more than twice the numberofarticlesproducedonthatday.Ifthe totalcostofproductiononthatdaywasRs.90,find the number of articles produced and the cost of each article.

Solution:

Let us say the number of articles produced is x.

Therefore,costof productionofeacharticle=Rs$(2x+3)$

GiventhetotalcostofproductionisRs.90

$\therefore x(2x+3)=90$

$\Rightarrow 2x^2+3x-90=0$

$\Rightarrow 2x^2+15x-12x-90=0$

$\Rightarrow x(2x+15)-6(2x+15)=0$

$\Rightarrow (2x+15)(x-6)=0$

Thus,either$2x+15=0$or$x-6=0$

$\Rightarrow x=-15/2$or$x=6$

Asthenumberofarticles producedcanonlybea positive integer,xcan onlybe 6. Hence,

the number of articles produced = 6

Costofeacharticle=2× 6+ 3=Rs15

Exercise4.3

1. Findthe natureoftheroots ofthe followingquadraticequations.Ifthe realrootsexist, find them.
(i) $2x^2-3x+5=0$
(ii) $3x^2-4\sqrt{3}x+4=0$
(iii) $2x^2-6x+3=0$

Solutions:

(i) Given,

$2x^2-3x+5=0$

Comparingtheequationwith$ax^2+bx+c=0$,we get

$a=2$,$b=-3$and$c=5$

Weknow,Discriminant=b^2-4ac

$=(-3)^2-4(2)(5)=9-40$

$=-31$

Asyoucansee,$b^2-4ac<0$

Therefore,no realrootispossibleforthe givenequation,$2x^2-3x+5=0$

(ii) $3x_2–4\sqrt{3}x+4=0$

Comparingtheequationwith$ax_2+ bx+c=0$,we get

$a=3,b=-4\sqrt{3}$and$c=4$

Weknow,Discriminant=$b_2–4ac$

$=(-4\sqrt{3})_2–4(3)(4)$

$=48–48= 0$

As$b_2–4ac=0$,

Realrootsexistforthegivenequation,andtheyareequaltoeachother. Hence, the

roots will be $–b/2a$ and $–b/2a$.

$–b/2a=-(-4\sqrt{3})/2×3=4\sqrt{3}/6=2\sqrt{3}/3=2/\sqrt{3}$ Therefore,

the roots are $2/\sqrt{3}$ and $2/\sqrt{3}$.

(iii) $2x_2–6x+3=0$

Comparingtheequationwith$ax_2+ bx+c=0$,we get

$a=2,b=-6,c=3$

Asweknow,Discriminant=$b_2–4ac$

$=(-6)_2–4(2)(3)$

$=36–24= 12$

As$b_2–4ac>0$,

Therefore,therearedistinct realrootsthatexistforthisequation, $2x_2– 6x+3=0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$=(-(-6)\pm\sqrt{(-6_2-4(2)(3))})/2(2)$

$=(6\pm2\sqrt{3})/4$

$=(3\pm\sqrt{3})/2$

Therefore,the roots forthegivenequationare$(3+\sqrt{3})/2$and$(3-\sqrt{3})/2$

2. Findthevaluesof$k$for eachof thefollowing quadraticequationssothat theyhavetwoequal roots.
(i) $2x^2+kx+3=0$
(ii) $kx(x–2)+6=0$

Solutions:

(i) $2x^2+kx+3=0$

Comparingthegivenequationwith$ax^2+bx+c= 0$,weget,

$a=2$,$b=k$and$c=3$

Asweknow,Discriminant=$b^2–4ac$

$=(k)^2–4(2)(3)$

$=k^2–24$

Forequalroots,weknow,

Discriminant=0

$k^2–24=0$

$k^2=24$

$k=\pm\sqrt{24}=\pm2\sqrt{6}$

(ii) $kx(x–2)+6=0$

or$kx^2– 2kx+6 =0$

Comparingthegivenequationwith$ax^2+bx+c= 0$,weget

$a=k$,$b=–2k$and$c=6$

Weknow,Discriminant=$b^2–4ac$

$=(–2k)^2–4(k)(6)$

$=4k^2–24k$

Forequalroots,weknow,

$b^2– 4ac = 0$

$4k^2–24k=0$

$4k(k–6)=0$

Either$4k$= 0or$k$=6 =0

k=0or$k$=6

However,if$k$ =0,thentheequation willnothavethe terms‘x_2‘and‘x‘. Therefore, if

this equation has two equal roots, k should be 6 only.

3. Isit possibletodesignarectangular mango grove whoselengthistwiceits breadthandthe area is 800 m_2? If so, find its length and breadth.

Solution:

Letthe breadthofthe mangogrovebe l.

Thelengthofthemangogrovewillbe$2l$.

Area of the mango grove = $(2l)$ (l)= $2l_2$

$2l_2$=800

l_2=800/2=400

l_2–400=0

Comparingthegivenequationwithax_2+bx+c= 0,weget

a=1,b=0,c=400

Asweknow,Discriminant=b_2–4ac

=>$(0)_2$– 4×(1) ×(– 400) =1600

Here,b_2–4ac >0

Thus,theequationwillhaverealroots.Andhence, thedesiredrectangularmango grovecanbe designed.

l =±20

Asweknow,thevalueoflengthcannotbenegative.

Therefore, the breadth of the mango grove = 20 m

Length of mango grove = 2 × 20 = 40 m

4. Isthefollowingsituationpossible?Ifso,determine theirpresent ages.Thesumofthe agesof two friends is 20 years. Four years ago, the product of their age in years was 48.

Solution:

Let’ssaytheageofonefriendisxyears.

Then,theageoftheotherfriendwillbe(20–x)years. Four

years ago,

AgeofFirstfriend= (x–4)years

AgeofSecond friend= (20–x –4)=(16–x)years As per

the given question, we can write,

$(x – 4) (16 – x) = 48$

$16x–x^2–64+4x=48$ –

$x^2+ 20x – 112 = 0$

$x^2–20x+112=0$

Comparingtheequationwith$ax^2+ bx+c=0$,we get

$a=1$,$b=-20$and$c=112$

Discriminant = $b^2– 4ac$

$=(-20)^2–4×112$

$=400–448 =-48$

$b^2–4ac<0$

Therefore,therewillbenorealsolutionpossiblefortheequations.Hence,theconditiondoesn’t exist.

5. Isitpossibletodesignarectangular parkof perimeter80andanarea of400m2?Ifso,findits length and breadth.

Solution:

Letthelengthandbreadthoftheparkbe$l$andb.

Perimeter oftherectangularpark= $2(l + b) =80$ *So,*

$l + b = 40$

Or,$b= 40–l$

Areaoftherectangularpark=$l×b= l(40–l)=40l–l^2=400$

$l^2– 40l + 400 = 0$, which is a quadratic equation.

Comparingtheequationwith $ax^2+bx+c =0$,weget

$a=1, b=-40, c=400$

Since,Discriminant $=b^2-4ac$

$=(-40)^2-4\times400$

$=1600-1600=0$

Thus,$b^2-4ac=0$

Therefore,thisequationhasequalrealroots.Hence,the situationispossible. The root

of the equation,

$l=-b/2a$

$l=-(-40)/2(1)=40/2=20$

Therefore,thelength oftherectangularpark,l=20m

Andthebreadthofthepark,b=40–l=40–20=20m.

Chapter6–Triangles

Exercise6.1

1. Fillintheblanksusingcorrectwordgivenin thebrackets:-

(i) All circles are __________.(congruent,similar)

Answer:Similar

(ii) All squares are __________.(similar,congruent)

Answer:Similar

(iii) All ___________trianglesaresimilar.(isosceles,equilateral)

Answer:Equilateral

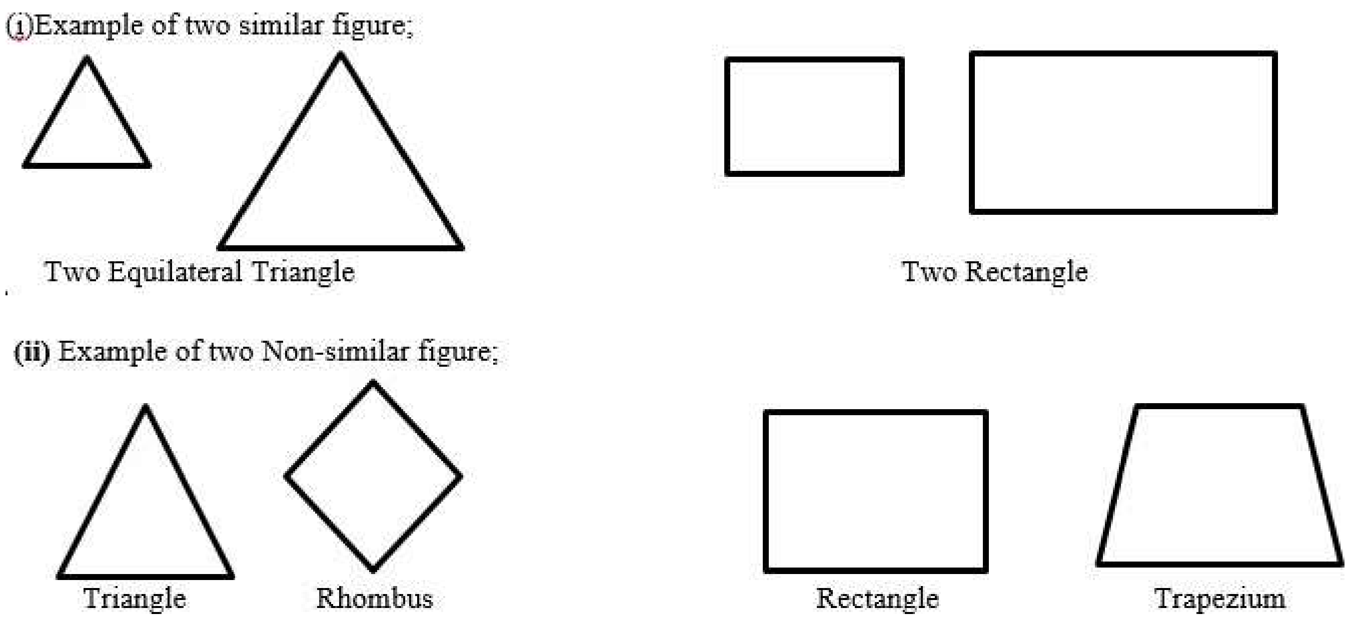
(iv) Twopolygonsofthesamenumberofsides aresimilar,if(a)theircorrespondinganglesare ___________and(b)their correspondingsides are_____________.(equal,proportional)

Answer:(a)Equal

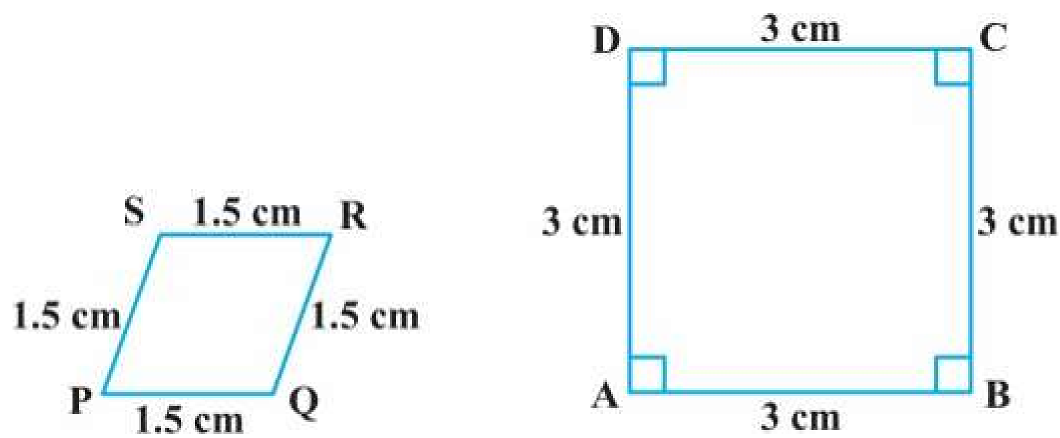
(b)Proportional

2. Give twodifferentexamplesofpairof
(i) Similarfigures
(ii) Non-similarfigures

Solution:



3. Statewhetherthefollowingquadrilateralsaresimilarornot:

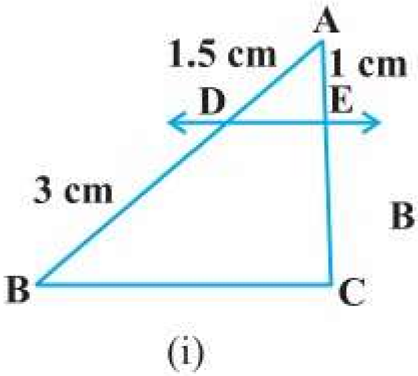


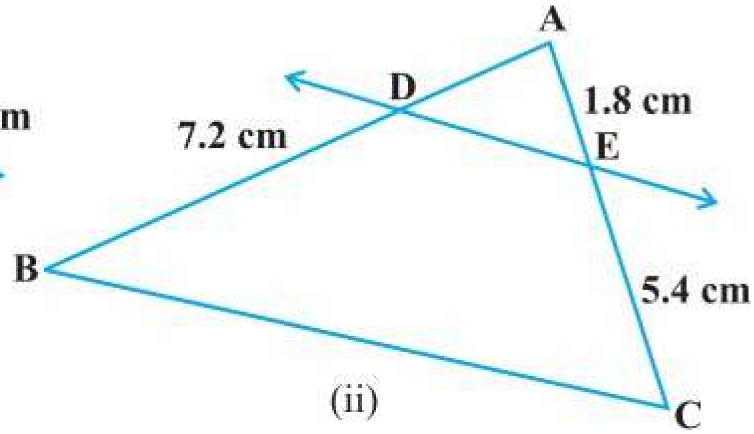
Solution:

Fromthegiventwofigures, wecanseetheircorrespondingangles are differentor unequal. Therefore, they are not similar.

Exercise6.2Page:128

1. Infigure.(i) and (ii),DE ||BC.FindECin(i) andADin(ii).





Solution:

(i) Given,in△ABC,DE∥BC

∴ AD/DB =AE/EC[UsingBasicproportionalitytheorem]

⇒1.5/3=1/EC

⇒EC=3/1.5

EC=3×10/15=2cm

Hence,EC=2cm.

(ii) Given,in△ABC,DE∥BC

∴ AD/DB =AE/EC[UsingBasicproportionalitytheorem]

⇒AD/7.2=1.8/5.4

⇒AD=1.8×7.2/5.4=(18/10)×(72/10)×(10/54)=24/10

⇒AD=2.4

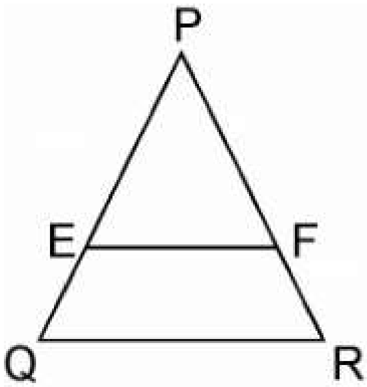
Hence,AD=2.4cm.

2. Eand Fare points on the sidesPQandPR,respectively ofaΔPQR.For eachofthefollowing cases, state whether EF || QR.
(i) PE=3.9cm,EQ=3cm,PF=3.6cmandFR=2.4cm

(ii) PE=4cm,QE =4.5cm,PF=8cmandRF=9cm
(iii) PQ=1.28 cm,PR=2.56 cm,PE=0.18 cmandPF=0.63 cm

Solution:

Given,inΔPQR,EandFaretwopointsonsidePQandPR,respectively.See thefigurebelow;



(i) Given,PE=3.9cm, EQ=3cm,PF=3.6cmand FR=2,4cm

Therefore, by using Basic proportionality theorem, we get,PE/EQ

= 3.9/3 = 39/30 = 13/10 = 1.3

AndPF/FR=3.6/2.4=36/24 =3/2=1.5

So, we get, PE/EQ ≠ PF/FR

Hence,EFisnotparalleltoQR.

(ii) Given,PE=4 cm,QE=4.5cm,PF=8cmandRF =9cm

Therefore, by using Basic proportionality theorem, we get,

PE/QE = 4/4.5 = 40/45 = 8/9

And,PF/RF=8/9 So,

we get here,

PE/QE = PF/RF

Hence,EFisparalleltoQR.

(iii) Given,PQ= 1.28cm,PR=2.56cm,PE=0.18 cmandPF=0.36cm From

the figure,

EQ = PQ – PE = 1.28 – 0.18 = 1.10 cm

And,FR=PR–PF=2.56–0.36=2.20cm

So,PE/EQ=0.18/1.10=18/110=9/55 **(i)**

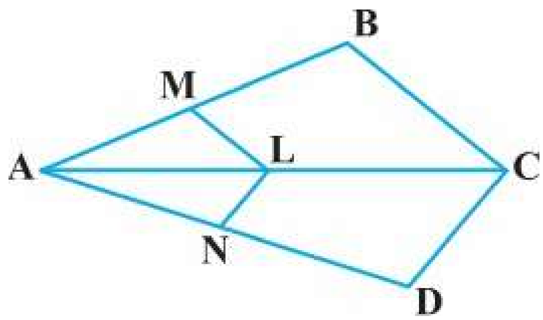
And,PE/FR=0.36/2.20=36/220=9/55 **(ii)**

So,wegethere,

PE/EQ =PF/FR

Hence,EFisparalleltoQR.

3. Inthe figure,ifLM||CBandLN|| CD,prove thatAM/AB=AN/AD



Solution:

Inthegivenfigure,wecansee,LM ||CB,

Byusingbasicproportionalitytheorem,weget,

AM/AB = AL/AC **(i)**

Similarly,given,LN||CDandusingbasic proportionalitytheorem,

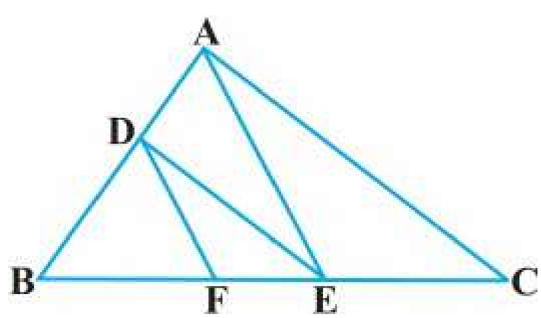
∴AN/AD=AL/AC **(ii)**

Fromequation**(i)**and**(ii)**,weget,

AM/AB = AN/AD

Hence,proved.

4. Inthefigure,DE||ACandDF||AE.ProvethatBF/FE=BE/EC



Solution:

InΔABC,givenas,DE ||AC

Thus,by usingBasicProportionalityTheorem,weget,

∴BD/DA=BE/EC **(i)**

InΔBAE,givenas,DF||AE

Thus,by usingBasicProportionalityTheorem,weget,

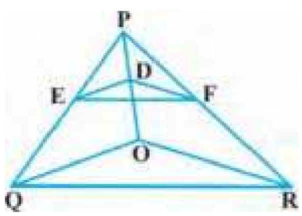
∴BD/DA=BF/FE **(ii)**

Fromequation**(i)**and**(ii)**,weget

BE/EC = BF/FE

Hence,proved.

5. Inthefigure,DE||OQandDF||OR,showthatEF||QR.



Solution:

Given,

InΔPQO,DE ||OQ

SobyusingBasicProportionalityTheorem,

PD/DO = PE/EQ **(i)**

Againgiven,inΔPOR,DF|| OR,

SobyusingBasicProportionalityTheorem,

PD/DO = PF/FR **(ii)**

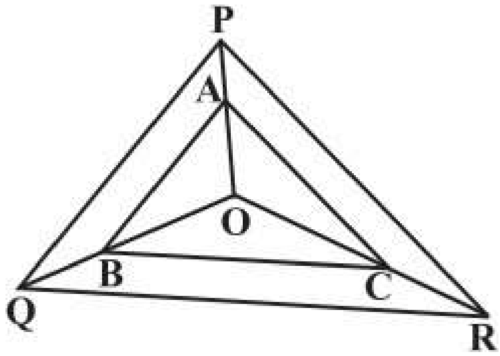
Fromequation**(i)**and**(ii)**,weget,

PE/EQ = PF/FR

Therefore,byconverseofBasicProportionalityTheorem, EF

|| QR, in ΔPQR.

6. Inthe figure,A,BandCare points onOP,OQ and ORrespectively suchthatAB||PQandAC|| PR. Show that BC || QR.



Solution:

Givenhere,

In∆OPQ,AB||PQ

ByusingBasicProportionalityTheorem,

OA/AP = OB/BQ **(i)**

Alsogiven,

In∆OPR,AC||PR

ByusingBasicProportionalityTheorem

∴OA/AP=OC/CR (ii)

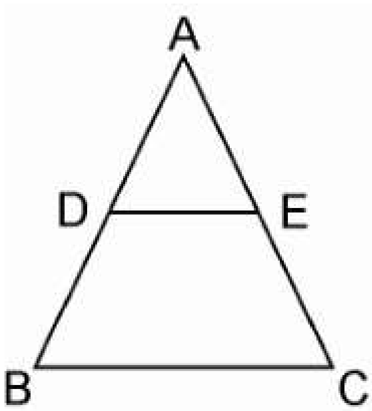
Fromequation**(i)**and**(ii)**,weget,

OB/BQ = OC/CR

Therefore,byconverseofBasicProportionalityTheorem, In

∆OQR, BC || QR.

7. UsingBasic proportionalitytheorem,provethat alinedrawnthroughthemid-points of one side ofa triangleparalleltoanother sidebisectsthethird side. (Recall thatyou have provedit inClass IX).



Solution:

Given,in∆ABC,DisthemidpointofABsuchthatAD=DB.

AlineparalleltoBCintersectsACatEasshowninabovefigure suchthatDE|| BC. We have

to prove that E is the mid point of AC.

Since,Disthemid-pointofAB.

∴AD=DB

⇒AD/DB=1 **(i)**

In∆ABC,DE ||BC,

ByusingBasicProportionalityTheorem,

Therefore, AD/DB = AE/EC

Fromequation(i),wecanwrite,

⇒1=AE/EC

∴AE=EC

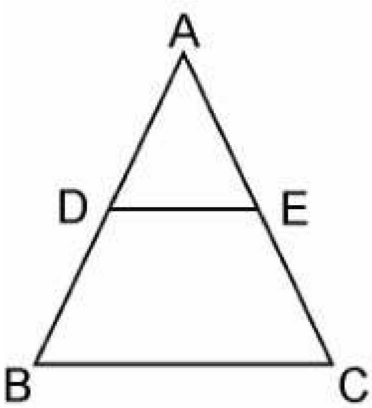
Hence,proved,EisthemidpointofAC.

8. UsingConverse ofbasic proportionalitytheorem,provethatthelinejoining themid-pointsof anytwosides of atriangleisparalleltothethirdside. (Recall thatyouhave doneit inClassIX).

Solution:

Given,in∆ABC,DandEarethe midpointsofABandAC,respectively, suchthat, AD=BD

and AE=EC.



Wehavetoprovethat:DE||BC.

Since, D is the midpoint of AB

∴AD=DB

⇒AD/BD=1 **(i)**

Alsogiven,Eisthemid-pointofAC.

∴AE=EC

⇒AE/EC=1

Fromequation**(i)**and**(ii)**,weget,

AD/BD = AE/EC

ByconverseofBasicProportionalityTheorem, DE

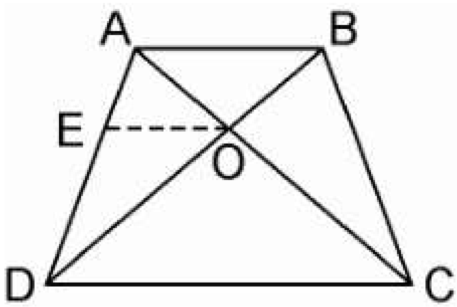
|| BC

Hence,proved.

9. ABCDisatrapeziuminwhichAB|| DCandits diagonals intersecteachotherat thepointO. Show that AO/BO = CO/DO.

Solution:

Given,ABCDisatrapeziumwhereAB||DC anddiagonalsACandBDintersecteach otheratO.



Wehavetoprove,AO/BO=CO/DO

FromthepointO, drawa lineEOtouchingADatE,insuchawaythat, EO || DC

|| AB

InΔADC,wehaveOE ||DC

Therefore,byusingBasicProportionalityTheorem

AE/ED = AO/CO **(i)**

Now,InΔABD,OE||AB

Therefore,byusingBasicProportionalityTheorem

DE/EA = DO/BO **(ii)**

Fromequation**(i)**and**(ii)**,weget,

AO/CO = BO/DO

⇒AO/BO=CO/DO

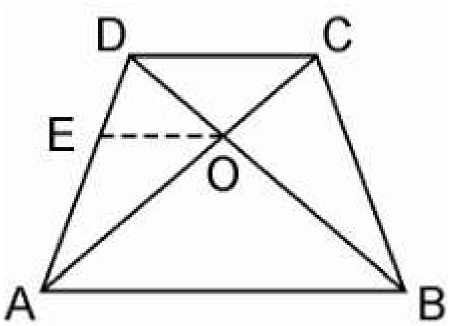
Hence,proved.

10. ThediagonalsofaquadrilateralABCDintersecteachother atthepointOsuch that AO/BO = CO/DO. Show that ABCD is a trapezium.

Solution:

Given,QuadrilateralABCDwhereACandBDintersecteachotheratO suchthat, AO/BO =

CO/DO.



Wehavetoprovehere,ABCDisatrapezium

From thepointO, drawa lineEOtouchingADatE,insuchaway that, EO ||

DC || AB

In∆DAB,EO||AB

Therefore,byusingBasicProportionalityTheorem

DE/EA = DO/OB (i)

Also, given,

AO/BO=CO/DO

⇒AO/CO=BO/DO

⇒CO/AO=DO/BO

⇒DO/OB=CO/AO **(ii)**

Fromequation**(i)**and**(ii)**,weget

DE/EA = CO/AO

Therefore,byusingconverseofBasicProportionalityTheorem, EO ||

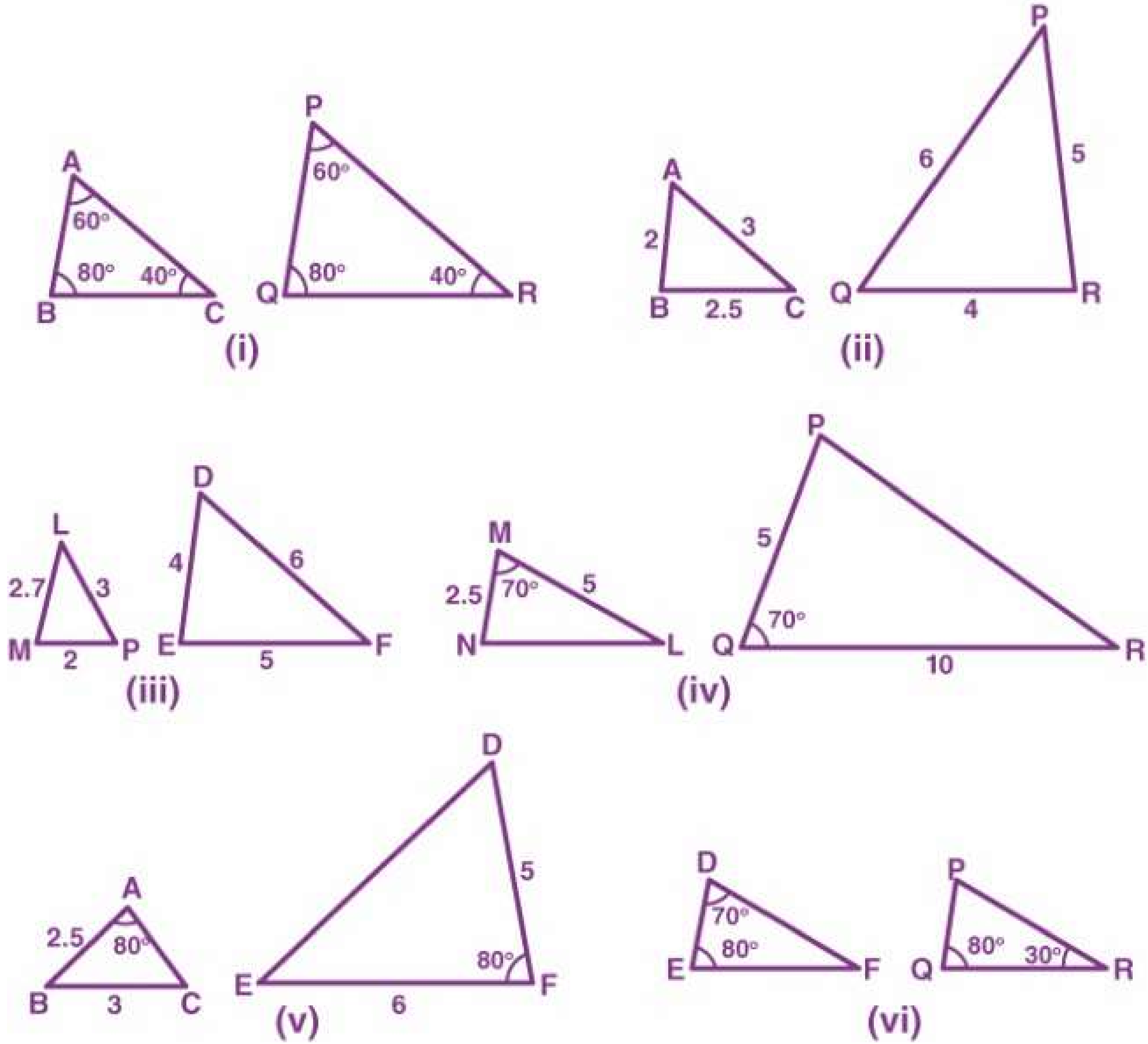
DC also EO || AB

⇒AB||DC.

Hence,quadrilateralABCDisatrapeziumwithAB||CD.

Exercise6.3Page:138

1. Statewhichpairsof trianglesinthefigure aresimilar.Writethe similaritycriterion usedby you for answering the question and also write the pairs of similar triangles in the symbolic form:



Solution:

(i) Given,inΔABCandΔPQR,

$\angle A=\angle P=60°$

$\angle B=\angle Q=80°$

$\angle C=\angle R=40°$

Therefore,byAAAsimilaritycriterion,

∴ΔABC~ΔPQR

(ii) Given,inΔABCandΔPQR,

AB/QR = 2/4 = 1/2,

BC/RP=2.5/5=1/2,

CA/PA = 3/6 = 1/2

BySSS similaritycriterion,

ΔABC~ΔQRP

(iii) Given,inΔLMPandΔDEF,

LM=2.7,MP=2,LP=3,EF=5,DE=4,DF=6 MP/DE =

2/4 = 1/2

PL/DF = 3/6 = 1/2

LM/EF=2.7/5=27/50

Here,MP/DE =PL/DF≠LM/EF

Therefore,ΔLMPandΔDEFarenotsimilar.

(iv) InΔMNLandΔQPR,itisgiven,

MN/QP = ML/QR = 1/2

∠M=∠Q=70°

Therefore,bySASsimilaritycriterion

∴ΔMNL~ΔQPR

(v) InΔABCandΔDEF,giventhat,

AB=2.5,BC=3,∠A=80°,EF=6,DF=5,∠F=80° Here , AB/DF = 2.5/5 = 1/2

And,BC/EF=3/6 =1/2

⇒∠B≠∠F

Hence,ΔABCandΔDEFare notsimilar.

(vi) InΔDEF,bysumofanglesoftriangles,weknowthat,

∠D+∠E+∠F=180°

⇒70°+80°+∠F=180°

⇒∠F=180°–70°–80°

⇒∠F=30°

Similarly,InΔPQR,

∠P+∠Q+∠R=180(SumofanglesofΔ)

⇒∠P+80°+30°=180°

⇒∠P=180°–80°-30°

⇒∠P=70°

Now,comparingboththetriangles,ΔDEFandΔPQR,wehave

∠D=∠P=70°

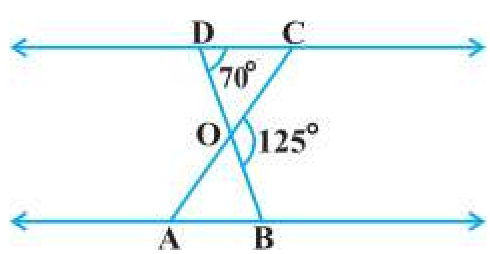
∠F=∠Q=80°

∠F=∠R=30°

Therefore,byAAAsimilaritycriterion,

Hence, ΔDEF ~ ΔPQR

2. Infigure6.35,ΔODC~ΔOBA,∠BOC=125°and∠CDO=70°.Find∠DOC,∠DCOand∠OAB.



Solution:

Aswecanseefromthefigure,DOBisastraightline. Therefore,

∠DOC + ∠ COB = 180°

⇒∠DOC=180°–125°(Given,∠BOC=125°)

=55°

InΔDOC,sumof themeasuresoftheanglesofatriangleis180º

Therefore,∠DCO+∠CDO+∠DOC=180°

⇒∠DCO+70º+55º=180°(Given,∠CDO=70°)

⇒∠DCO=55°

Itisgiventhat,ΔODC~ΔOBA,

Therefore, ΔODC ~ ΔOBA.

Hence,correspondinganglesareequalinsimilartriangles

∠OAB=∠OCD

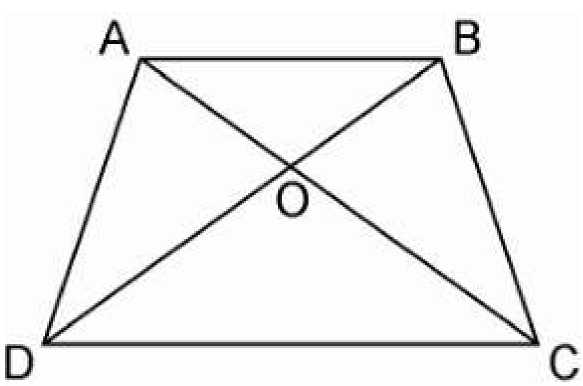
⇒∠OAB=55°

∠OAB=∠OCD

⇒∠OAB=55°

3. DiagonalsACandBDof atrapeziumABCDwithAB||DCintersecteachotheratthe pointO. Using a similarity criterion for two triangles, show that AO/OC = OB/OD

Solution:



InΔDOCandΔBOA,

AB||CD,thusalternateinteriorangleswillbe equal,

∴∠CDO=∠ABO

Similarly,

∠DCO=∠BAO

Also,forthe twotrianglesΔDOCandΔBOA,verticallyoppositeangleswillbeequal;

∴∠DOC=∠BOA

Hence,byAAAsimilaritycriterion,

ΔDOC ~ ΔBOA

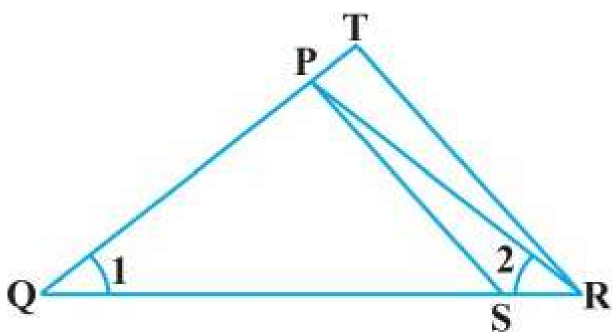
Thus,thecorrespondingsidesareproportional.

DO/BO = OC/OA

⇒OA/OC=OB/OD

Hence,proved.

4. Inthefig.6.36,QR/QS=QT/PRand∠1=∠2.ShowthatΔPQS~ΔTQR.



Solution:

InΔPQR,

∠PQR=∠PRQ

∴PQ=PR **(i)**

Given,

QR/QS=QT/PRUsingequation**(i)**,weget

QR/QS = QT/QP **(ii)**

InΔPQSandΔTQR,byequation(ii),

QR/QS = QT/QP

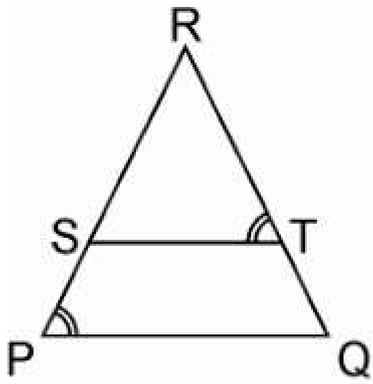
∠Q=∠Q

∴ΔPQS~ΔTQR[BySASsimilaritycriterion]

5. SandTarepointonsidesPRandQRofΔPQRsuchthat∠P=∠RTS.ShowthatΔRPQ~ΔRTS. Solution:

Given,Sand TarepointonsidesPRandQR ofΔPQR And $\angle$P

= $\angle$RTS.



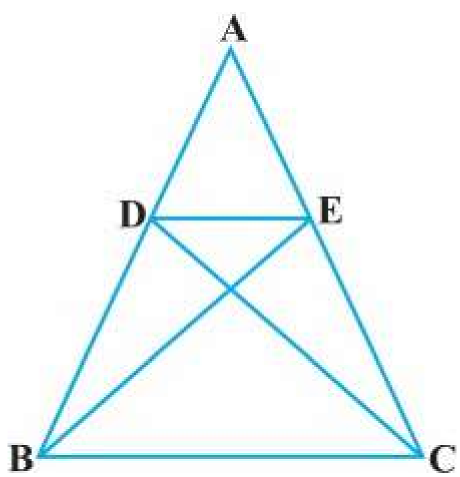
InΔRPQandΔRTS,

$\angle$RTS=$\angle$QPS(Given)

$\angle$R=$\angle$R(Commonangle)

$\therefore\Delta$RPQ~ΔRTS(AAsimilaritycriterion)

6. Inthe figure,ifΔABE≅ΔACD,show thatΔADE~ΔABC.



Solution:

Given,ΔABE≅ΔACD.

$\therefore$AB=AC[ByCPCT] **(i)**

And,AD=AE [ByCPCT] **(ii)**

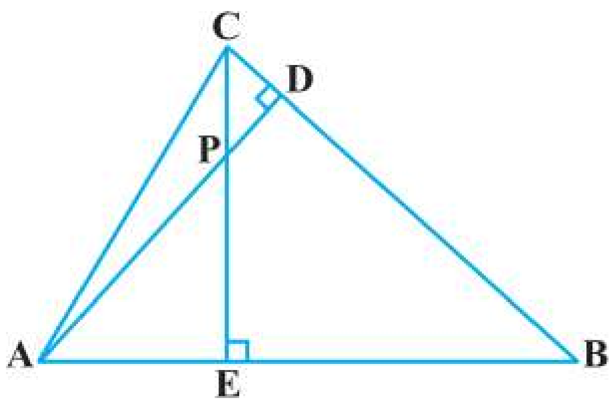
InΔADEandΔABC,dividingeq.(ii)byeq(i),

AD/AB =AE/AC

∠A=∠A[Commonangle]

∴ΔADE~ΔABC[SASsimilaritycriterion]

7. Inthefigure,altitudesADandCEofΔABCintersecteachother atthepointP.Showthat:



(i) ΔAEP~ΔCDP
(ii) ΔABD ~ΔCBE
(iii) ΔAEP~ΔADB
(iv) ΔPDC ~ΔBEC

Solution:

Given,altitudesADandCEof ΔABCintersecteachotheratthepointP.

(i) InΔAEP andΔCDP,

∠AEP=∠CDP(90°each)

∠APE=∠CPD(Verticallyoppositeangles)

Hence, by AA similarity criterion,

ΔAEP~ΔCDP

(ii) InΔABDandΔCBE,

∠ADB=∠CEB(90°each)

∠ABD=∠CBE(Common Angles)

Hence,byAAsimilaritycriterion,

ΔABD~ΔCBE

(iii) InΔAEPandΔADB,

∠AEP=∠ADB(90°each)

∠PAE=∠DAB(CommonAngles)

Hence,byAAsimilaritycriterion,

ΔAEP~ΔADB

(iv) InΔPDCandΔBEC,

∠PDC=∠BEC(90°each)

∠PCD=∠BCE(Commonangles)

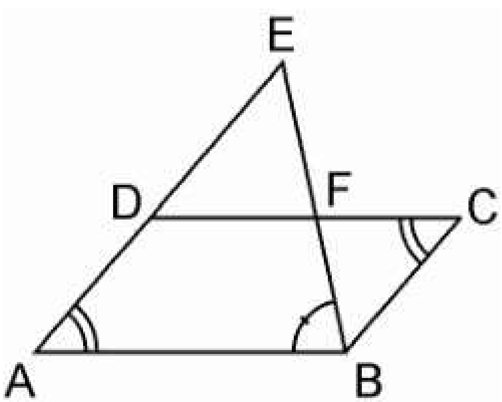
Hence,byAAsimilaritycriterion,

ΔPDC ~ ΔBEC

8. EisapointonthesideAD producedofaparallelogramABCDandBEintersects CDatF.Show that ΔABE ~ ΔCFB.

Solution:

Given,Eisapointontheside ADproducedof aparallelogramABCDandBEintersectsCDat F. Consider the figure below,



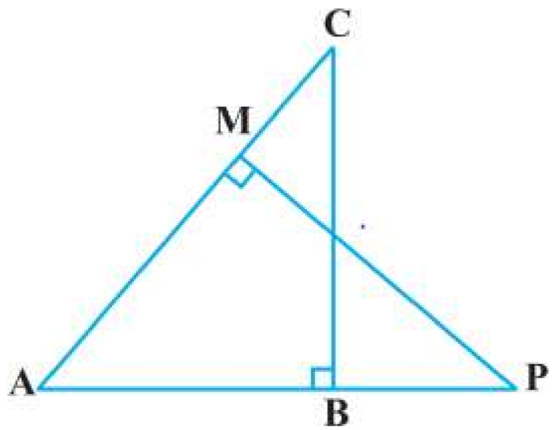
InΔABEandΔCFB,

∠A=∠C(Oppositeanglesofaparallelogram)

∠AEB=∠CBF(AlternateinterioranglesasAE||BC)

∴ΔABE~ΔCFB(AAsimilaritycriterion)

9. Inthe figure,ABCand AMParetworighttriangles, rightangledatBandM,respectively,prove that:



(i) ΔABC ~ΔAMP

(ii) CA/PA=BC/MP

Solution:

Given,ABCandAMParetworighttriangles, rightangledatBandM,respectively.

(i) InΔABCandΔAMP,wehave,

∠CAB=∠MAP(commonangles)

∠ABC=∠AMP=90°(each90°)

∴ΔABC~ΔAMP(AAsimilaritycriterion)

(ii) As,ΔABC~ΔAMP(AA similaritycriterion)

Iftwotrianglesaresimilarthenthecorrespondingsides arealwaysequal, Hence,

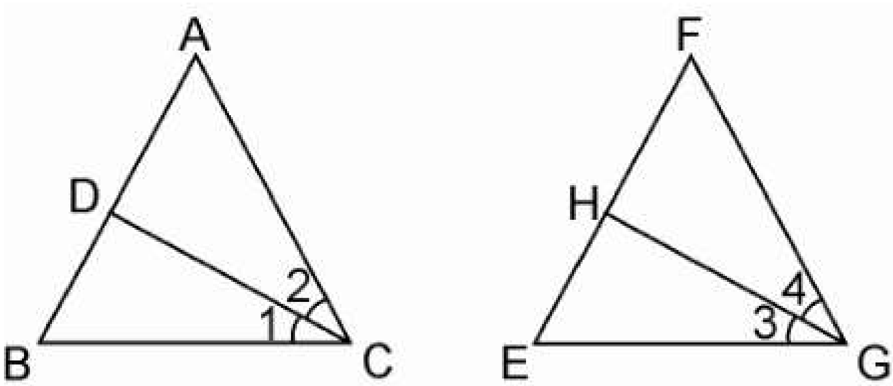
CA/PA = BC/MP

10. CDandGHarerespectivelythebisectorsof∠ACBand∠EGFsuchthatDandHlieonsides AB and FE of ΔABC and ΔEFG respectively. If ΔABC ~ ΔFEG, Show that:

(i) CD/GH=AC/FG
(ii) ΔDCB~ΔHGE
(iii) ΔDCA ~ΔHGF

Solution:

Given,CDandGHarerespectivelythebisectorsof∠ACBand∠EGFsuchthatDandHlieonsidesAB and FE of ΔABC and ΔEFG, respectively.



(i) Fromthegivencondition,

ΔABC ~ ΔFEG.

∴∠A=∠F,∠B=∠E,and∠ACB=∠FGE Since,

∠ACB = ∠FGE

∴∠ACD=∠FGH(Anglebisector)

And,∠DCB=∠HGE(Anglebisector) In

ΔACD and ΔFGH,

∠A=∠F

∠ACD=∠FGH

∴ΔACD~ΔFGH(AAsimilaritycriterion)

⇒CD/GH=AC/FG

(ii) InΔDCBandΔHGE,

∠DCB=∠HGE (Alreadyproved)

∠B=∠E(Alreadyproved)

∴ΔDCB~ΔHGE(AAsimilaritycriterion)

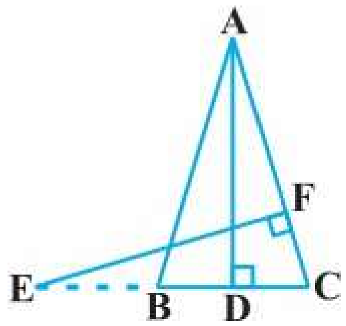
(iii) InΔDCAandΔHGF,

∠ACD=∠FGH(Alreadyproved)

∠A=∠F(Alreadyproved)

∴ΔDCA~ΔHGF(AAsimilaritycriterion)

11. Inthefollowingfigure,Eisa pointonsideCBproducedofan isoscelestriangleABCwithAB =AC.IfAD⊥BCandEF⊥AC,provethatΔABD~ΔECF.



Solution:

Given,ABCisanisoscelestriangle.

∴AB=AC

⇒∠ABD=∠ECF

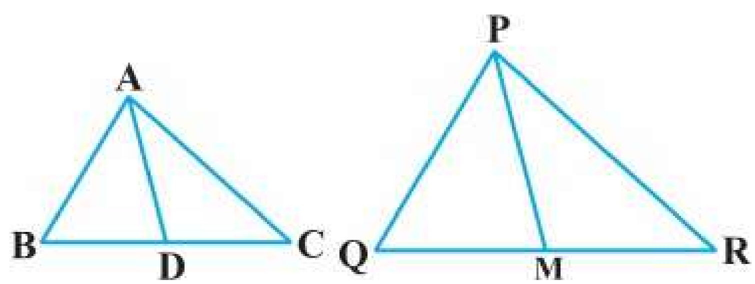
InΔABDandΔECF,

∠ADB=∠EFC(Each90°)

∠BAD= ∠CEF(Already proved)

∴ΔABD~ΔECF(usingAAsimilaritycriterion)

12. Sides ABandBCandmedianADofatriangleABCarerespectively proportional tosidesPQ and QR and median PM of ΔPQR (see Fig 6.41). Show that ΔABC ~ ΔPQR.



Solution:

Given,ΔABCandΔPQR,AB,BCandmedianADofΔABCareproportionaltosidesPQ,QR and median PM of ΔPQR

i.e.AB/PQ=BC/QR=AD/PM

Wehavetoprove:ΔABC~ΔPQR

Asweknowhere,

AB/PQ=BC/QR=AD/PM

$$\frac{AB}{PQ} = \frac{\frac{1}{2}BC}{\frac{1}{2}QR} = \frac{AD}{PM} \text{..................................} (1)$$

⇒AB/PQ=BC/QR=AD/PM(DisthemidpointofBC.Misthemidpointof QR)

⇒ΔABD~ΔPQM[SSSsimilaritycriterion]

∴∠ABD=∠PQM[Correspondinganglesoftwosimilartrianglesareequal]

⇒ ∠ABC =

∠PQRInΔABCandΔ
PQR

AB/PQ =BC/QR **(i)**

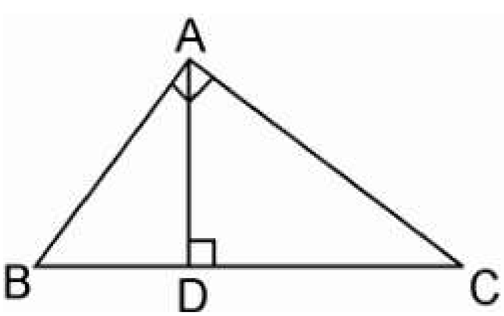
∠ABC=∠PQR **(ii)**

From equation **(i)** and **(ii)**, we get,

ΔABC~ΔPQR[SASsimilaritycriterion]

13. DisapointonthesideBCofatriangleABCsuchthat ∠ADC=∠BAC.ShowthatCA$_2$=CB.CD Solution:

Given,DisapointonthesideBCofatriangleABCsuchthat∠ADC=∠BAC.



InΔADCandΔBAC,

∠ADC =∠BAC (Alreadygiven)

∠ACD=∠BCA(Commonangles)

∴ΔADC~ΔBAC(AAsimilaritycriterion)

Weknowthatcorrespondingsides ofsimilartrianglesareinproportion.

∴CA/CB=CD/CA

⇒CA^2=CB.CD.

Hence,proved.

14. Sides ABandACandmedianADofatriangleABCare respectively proportional tosidesPQ and PR and median PM of another triangle PQR. Show that ΔABC ~ ΔPQR.

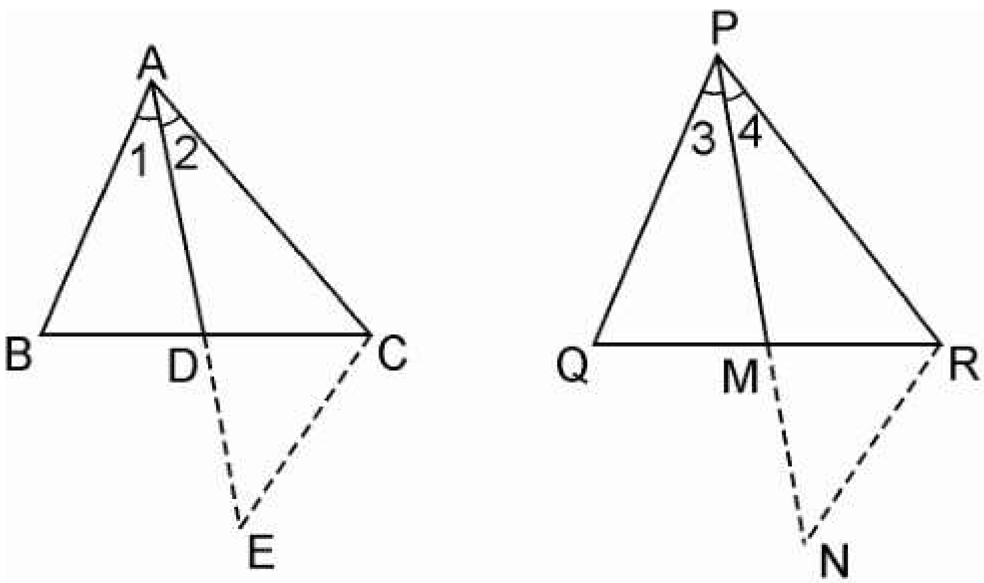
Solution:

Given:Two trianglesΔABCandΔPQRinwhichADand PMaremedianssuchthat; AB/PQ

= AC/PR = AD/PM

Wehavetoprove,ΔABC~ΔPQR

Letusconstruct first:ProduceADtoEso thatAD=DE.JoinCE,SimilarlyproducePMtoNsuchthatPM =MN,alsoJoinRN.



InΔABDandΔCDE,wehave AD

= DE[By Construction.]

BD=DC[Since,APisthemedian]

and,∠ADB= ∠CDE[Verticallyoppositeangles]

∴ΔABD≅ΔCDE[SAScriterionofcongruence]

⇒AB=CE[ByCPCT] **(i)**

Also, in ΔPQM and ΔMNR,

PM=MN[ByConstruction.]

QM=MR[Since,PMisthemedian]

and,∠PMQ= ∠NMR[Verticallyoppositeangles]

∴ΔPQM=ΔMNR[SAScriterionofcongruence]

⇒PQ=RN[CPCT] **(ii)**

Now,AB/PQ=AC/PR=AD/PM

Fromequation**(i)**and**(ii)**,

⇒CE/RN=AC/PR=AD/PM

⇒CE/RN=AC/PR=2AD/2PM

⇒CE/RN=AC/PR=AE/PN[Since2AD=AEand2PM=PN]

∴ΔACE~ΔPRN[SSSsimilaritycriterion]

Therefore, ∠2 = ∠4

Similarly,∠1=∠3

∴∠1+∠2=∠3+∠4

⇒∠A=∠P **(iii)**

Now,inΔABCandΔPQR,wehave

AB/PQ = AC/PR (Already given)

From equation (iii),

∠A=∠P

∴ΔABC~ΔPQR[SASsimilaritycriterion]

15. Averticalpoleofalength6mcastsashadow 4mlongonthegroundandatthe sametime a tower casts a shadow 28 m long. Find the height of the tower.

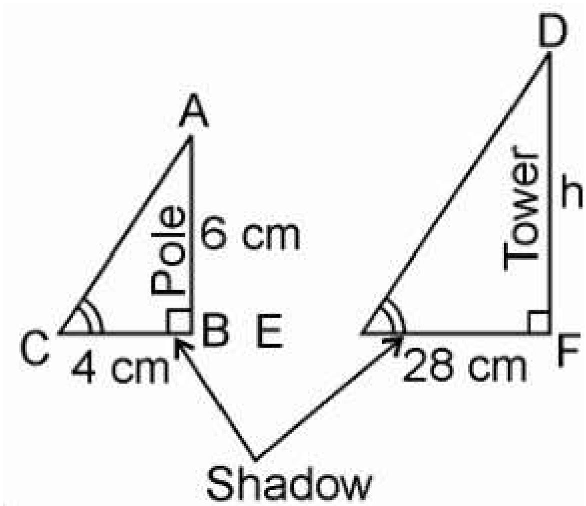
Solution:

Given,Lengthoftheverticalpole=6m Shadow

of the pole = 4 m

LetHeightoftower=*h*m

Lengthofshadowofthetower=28m



InΔABCandΔDEF,

∠C=∠E(angularelevationofsum)

∠B=∠F=90°

∴ΔABC~ΔDEF(AAsimilaritycriterion)

∴AB/DF=BC/EF(Iftwotrianglesaresimilarcorrespondingsidesareproportional)

∴6/h=4/28

⇒h=(6×28)/4

⇒h=6×7

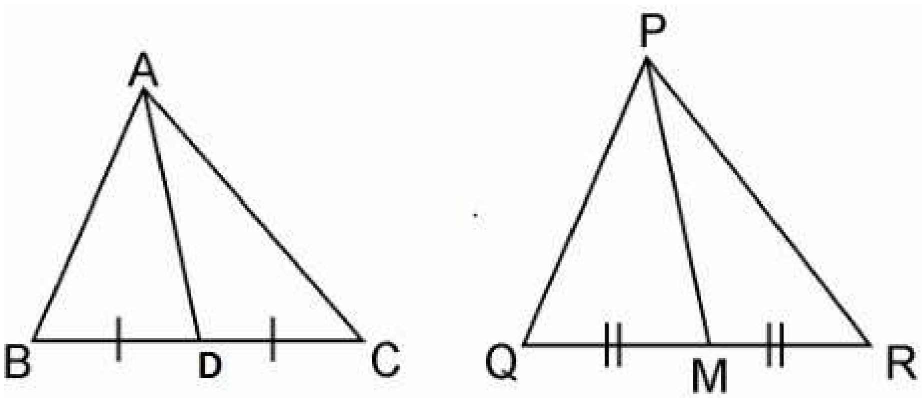
⇒h=42m

Hence,the heightofthetoweris42m.

16. IfADandPM aremediansoftrianglesABCandPQR,respectively where ΔABC~ΔPQRprove that AB/PQ = AD/PM.

Solution:

Given,ΔABC~ΔPQR



Weknowthatthecorrespondingsides ofsimilartriangles areinproportion.

∴AB/PQ=AC/PR=BC/QR **(i)**

Also,∠A=∠P,∠B=∠Q,∠C=∠R **(ii)**

SinceADandPMaremedians,theywilldividetheiroppositesides.

∴BD=BC/2andQM=QR/2 **(iii)**

From equations **(i)** and **(iii)**, we getAB/PQ

= BD/QM **(iv)**

InΔABDandΔPQM,

Fromequation(ii),wehave

∠B=∠Q

Fromequation**(iv),wehave,**

AB/PQ =BD/QM

∴ΔABD~ΔPQM(SASsimilaritycriterion)

⇒AB/PQ=BD/QM=AD/PM