Holyfaith presentation school.
Rawalpora Srinagar

CLASS:10TH

SUBJECT:Mathematics SESSION

2024 – 2025

Term 2nd

IntroductiontoTrignometry

Exercise8.1

1. In△ABC,right-angledatB,AB=24cm,BC=7cm.Determine:

(i) sinA,cosA

(ii) sinC,cosC

Solution:

In a given triangle ABC, right angled at B = ∠B = 90°

Given: AB = 24 cm and BC = 7 cm

AccordingtothePythagorasTheorem,

Inaright-angled triangle,thesquaresof thehypotenusesideisequaltothesumof the squares of the other two sides.

By applying Pythagoras theorem, we get

$AC^2=AB^2+BC^2$

$AC^2=(24)^2+7^2$

$AC^2 = (576+49)$

$AC^2 = 625cm^2 AC$

$= \sqrt{625} = 25$

Therefore,AC=25cm

(i) TofindSin(A),Cos(A)

We know that sine (or) Sin function is the equal to the ratio of length of the opposite side to the hypotenuse side. So it becomes

Sin(A)=Oppositeside/Hypotenuse=BC/AC=7/25

Cosine or Cos function is equal to the ratio of the length of the adjacent side to the hypotenuse side and it becomes,

Cos(A)=Adjacentside/Hypotenuse=AB/AC=24/25

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(ii) TofindSin(C),Cos(C)

Sin (C) = AB/AC = 24/25

Cos (C) = BC/AC = 7/25

2. InFig.8.13,findtanP–cotR

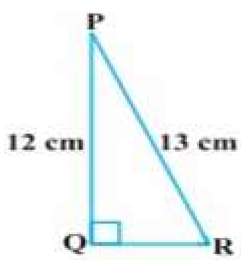


Fig. 8.13

Solution:

In the given triangle PQR, the given triangle is right angled at Q and the given measures are:

PR = 13cm,

PQ = 12cm

Since the given triangle is right angled triangle, to find the side QR, apply the Pythagorean theorem

AccordingtoPythagoreantheorem,

Inaright-angled triangle,thesquaresof thehypotenusesideisequaltothesumof the squares of the other two sides.

$PR^2=QR^2+PQ^2$

Substitute the values of PR and PQ

$13^2 = QR^2+12^2$

$169=QR^2+144$

Therefore,$QR^2=169-144$

$QR^2=25$

QR=√25=5

Therefore,thesideQR=5cm To

find tan P – cot R:

According to the trigonometric ratio, the tangent function is equal to the ratio of thelengthoftheoppositesidetotheadjacentsides,thevalueoftan(P) becomes

tan(P)**=**Oppositeside/Adjacentside= QR/PQ=5/12

Since cot function is the reciprocal of the tan function, the ratio of cot function becomes,

Cot (R) = Adjacent side/Opposite side = QR/PQ = 5/12

Therefore,

tan (P) – cot (R) = 5/12 – 5/12 = 0

Therefore,tan(P) – cot(R) = 0

3. IfsinA=3/4,calculatecosAandtanA.

Solution:

Let us assume a right angled triangle ABC, right angled at B

Given: Sin A = 3/4

We know that, Sin function is the equal to the ratio of length of the opposite side to the hypotenuse side.

Therefore,SinA=Oppositeside/Hypotenuse=3/4

Let BC be 3k and AC will be 4k

wherekisapositiverealnumber.

AccordingtothePythagorastheorem,thesquaresofthehypotenusesideis equalto the sum of the squares of the other two sides of a right angle triangle and we get,

$AC^2=AB^2+BC^2$

Substitute the value of AC and BC

$(4k)^2=AB^2+ (3k)^2$

$16k^2-9k^2=AB^2$

$AB^2=7k^2$

Therefore,AB =$\sqrt{7}k$

Now, we have to find the value of cos A and tan A

We know that,

Cos(A)=Adjacentside/Hypotenuse

Substitute the value of AB and AC and cancel the constant k in both numerator and denominator, we get

AB/AC = $\sqrt{7}k/4k = \sqrt{7}/4$

Therefore,cos(A)=$\sqrt{7}/4$

tan(A) =Oppositeside/Adjacent side

Substitute the Value of BC and AB and cancel the constant k in both numerator and denominator, we get,

BC/AB = $3k/\sqrt{7}k = 3/\sqrt{7}$

Therefore, tan A = $3/\sqrt{7}$

4. Given15cotA=8,findsinAandsecA.

Solution:

LetusassumearightangledtriangleABC,rightangledatB Given: 15

cot A = 8

So,CotA=8/15

We know that, cot function is the equal to the ratio of length of the adjacent side to the opposite side.

Therefore, cot A = Adjacent side/Opposite side = AB/BC = 8/15

LetAB be 8kand BC will be 15k

Where,kisapositiverealnumber.

AccordingtothePythagorastheorem,thesquaresofthehypotenusesideis equalto the sum of the squares of the other two sides of a right angle triangle and we get,

$AC^2=AB^2+BC^2$

Substitute the value of AB and BC

$AC^2= (8k)^2 + (15k)^2$

$AC^2= 64k^2 + 225k^2$

$AC^2= 289k^2$

Therefore,AC=17k

Now, we have to find the value of sin A and sec A

We know that,

Sin(A)=Oppositeside/Hypotenuse

Substitute the value of BC and AC and cancel the constant k in both numerator and denominator, we get

SinA=BC/AC=15k/17k=15/17

Therefore,sinA=15/17

Since secant orsec function is the reciprocal of the cos function which is equal to the ratio of the length of the hypotenuse side to the adjacent side.

Sec(A)=Hypotenuse/Adjacentside

Substitute the Value of BC and AB and cancel the constant k in both numerator and denominator, we get,

AC/AB=17k/8k=17/8

Thereforesec(A)=17/8

5. Givensecθ=13/12Calculateallothertrigonometricratios

Solution:

We know that sec function is the reciprocal of the cos function which is equal to the ratio of the length of the hypotenuse side to the adjacent side

LetusassumearightangledtriangleABC,rightangledatB

secθ=13/12=Hypotenuse/Adjacentside=AC/AB

Let AC be 13k and AB will be 12k

Where,kisapositiverealnumber.

AccordingtothePythagorastheorem,thesquaresofthehypotenusesideis equalto the sum of the squares of the other two sides of a right angle triangle and we get,

$AC^2=AB^2+BC^2$

Substitute the value of AB and AC

$(13k)^2= (12k)^2 + BC^2$

$169k^2=144k^2+BC^2$

$169k^2= 144k^2 + BC^2$

$BC^2 = 169k^2 - 144k^2$

$BC^2= 25k^2$

Therefore,BC=5k

Now, substitute the corresponding values in all other trigonometric ratios

So,

Sin θ = Opposite Side/Hypotenuse = BC/AC = 5/13

Cos θ = Adjacent Side/Hypotenuse = AB/AC = 12/13

tan θ = Opposite Side/Adjacent Side = BC/AB = 5/12

Cosec θ = Hypotenuse/Opposite Side = AC/BC = 13/5

cotθ =AdjacentSide/OppositeSide= AB/BC=12/5

6. If∠Aand∠Bareacuteanglessuchthatcos A=cos B,thenshowthat∠A=∠B.

Solution:

LetusassumethetriangleABCinwhichCD⊥AB

Give that the angles A and B are acute angles, such that

Cos (A) = cos (B)

As per the angles taken, the cos ratio is written as

AD/AC = BD/BC

Now, interchange the terms, we get

AD/BD = AC/BC

Let takeaconstantvalue AD/BD

= AC/BC = k

Nowconsidertheequationas

AD=k BD…(1)

AC=kBC…(2)

ByapplyingPythagorastheoremin△CADand△CBDweget,

$CD^2=BC^2-BD^2$...(3)

$CD^2=AC^2-AD^2$....(4)

From theequations(3) and (4) weget,

$AC^2-AD^2 = BC^2-BD^2$

Nowsubstitute the equations(1) and (2) in (3) and (4)

$K^2(BC^2-BD^2)=(BC^2-BD^2)$ $k^2=1$

Putting this value in equation, we obtain

AC = BC

∠A=∠B(Anglesoppositetoequalsideareequal-isoscelestriangle)

7. Ifcotθ=7/8,evaluate:

(i) (1 +sinθ)(1–sinθ)/(1+cosθ)(1-cosθ)

(ii) $\cot^2\theta$

Solution:

Letusassumea△ABCinwhich∠B=90°and∠C=θ

Given:

cotθ=BC/AB=7/8

Let BC = 7k and AB = 8k, where k is a positive real number

According to Pythagoras theorem in △ABC we get.

$AC^2 = AB^2+BC^2$

$AC^2=(8k)^2+(7k)^2$

$AC^2 = 64k^2+49k^2$

$AC^2 = 113k^2$

$AC=\sqrt{113}k$

Accordingtothesineandcosfunctionratios,itiswrittenas

sin θ = AB/AC = Opposite Side/Hypotenuse = 8k/√113 k = 8/√113 and

cosθ=AdjacentSide/Hypotenuse=BC/AC=7k/√113k=7/√113 Now apply

the values of sin function and cos function:

$$(i)\ \frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}=\frac{1-\sin^2\theta}{1-\cos^2\theta}$$

$$=\frac{1-\left(\frac{8}{\sqrt{113}}\right)^2}{1-\left(\frac{7}{\sqrt{113}}\right)^2}=\frac{49}{64}$$

$$(ii)\ \cot^2\theta=\left(\frac{7}{8}\right)^2=\frac{49}{64}$$

8. If 3cotA= 4,checkwhether(1-tan$_2$A)/(1+tan$_2$A)= cos$_2$A–sin$_2$A ornot.

Solution:

Let△ABCinwhich∠B=90°

We know that, cot function is the reciprocal of tan function and it is written

ascot(A) = AB/BC = 4/3

Let AB = 4k an BC =3k, where k is a positive real number.

According to the Pythagorean theorem,

$AC_2=AB_2+BC_2$

$AC_2=(4k)_2+(3k)_2$

$AC_2=16k_2+9k_2$

$AC_2=25k_2AC=5k$

Now, apply the values corresponding to the ratios

tan(A) = BC/AB = 3/4

sin(A)=BC/AC=3/5

cos(A)=AB/AC=4/5

Nowcomparethelefthandside(LHS)withrighthandside(RHS)

$$\text{L.H.S.} = \frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2} = \frac{1-\frac{9}{16}}{1+\frac{9}{16}} = \frac{7}{25}$$

$$\text{R.H.S.} = \cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

Since,boththeLHSandRHS=7/25

R.H.S.=L.H.S.

Hence,**$(1-\tan^2 A)/(1+\tan^2 A)=\cos^2 A-\sin^2 A$**isproved

9. IntriangleABC,right-angledatB,iftanA=1/√3findthevalueof:

(i) sinAcosC+cosAsinC

(ii) cosAcosC–sinAsinC

Solution:

LetΔABCinwhich∠B=90°

tanA=BC/AB=1/√3

LetBC=1kandAB=√3k,

Where kisthe positive real number of the problem

By Pythagoras theorem in ΔABC we get:

$AC^2=AB^2+BC^2$

$AC^2=(\sqrt{3}k)^2+(k)^2$

$AC^2=3k^2+k^2$

$AC^2=4k^2$

AC=2k

Now find the values of cos A, Sin A

Sin A = BC/AC = 1/2

CosA=AB/AC=√3/2

Thenfindthevaluesofcos C andsinC Sin C

= AB/AC = √3/2

CosC=BC/AC=1/2

Now,substitutethevaluesinthegivenproblem

(i) sin A cosC+cosAsinC=(1/2)×(1/2)+√3/2 ×√3/2 = 1/4 +3/4 = 1

(ii) cosAcosC− sinA sinC =(√3/2)(1/2)− (1/2) (√3/2) =0

10. In∆PQR,right-angledatQ,PR+QR=25cmandPQ=5cm.Determinethe values of sin P, cos Pand tan P

Solution:

In a given triangle PQR, right angled at Q, the following measures are

PQ = 5 cm

PR+QR=25cm

Nowletusassume,QR=x PR

= 25-QR

PR=25-x

According to the Pythagorean Theorem,

$PR^2 = PQ^2 + QR^2$

SubstitutethevalueofPRasx

$(25- x)^2 = 5^2 + x^2$

$25^2+x^2-50x=25 +x^2$

$625+x^2-50x-25-x^2=0$

$-50x = -600$

$x= -600/-50$

$x = 12 = QR$

Now,findthevalueofPR PR

= 25- QR

SubstitutethevalueofQR PR

= 25-12

PR=13

Now,substitutethevaluetothegivenproblem

(1) sinp=OppositeSide/Hypotenuse=QR/PR=12/13

(2) Cosp=Adjacent Side/Hypotenuse=PQ/PR=5/13

(3) tanp=OppositeSide/Adjacentside=QR/PQ= 12/5

11. Statewhetherthefollowingaretrueorfalse.Justifyyouranswer.

(i) ThevalueoftanAisalwayslessthan1.

(ii) secA=12/5forsomevalueofangleA.

(iii) cosAistheabbreviationusedforthecosecantofangleA.

(iv) cotAistheproductofcotandA.

(v) sinθ=4/3forsomeangleθ.

Solution:

(i) Thevalueoftan Aisalwayslessthan1.

Answer: **False**

Proof: In ΔMNC in which $\angle N=90^\circ$, MN=3,NC=4andMC=5

ValueoftanM=4/3whichisgreaterthan1.

The triangle can be formed with sides equal to 3, 4 and hypotenuse = 5 as it will follow the Pythagoras theorem.

$MC^2=MN^2+NC^2$

$5^2=3^2+4^2$

25=9+16

25=25

(ii) sec A = 12/5 for some value of angle A

Answer: **True**

Justification:LetaΔMNCinwhich∠N=90º,

MC=12k and MB=5k, where k is a positive real number.

By Pythagoras theorem we get,

$MC^2=MN^2+NC^2$

$(12k)^2=(5k)^2+NC^2$

$NC^2+25k^2=144k^2$

$NC^2=119k^2$

Sucha triangleis possibleasitwillfollowthe Pythagorastheorem.

(iii) cosAistheabbreviationusedforthecosecantofangleA.

Answer: **False**

Justification:AbbreviationusedforcosecantofangleMiscosec M. cosMisthe abbreviation used for cosine of angle M.

(iv) cotAistheproductofcotandA.

Answer: **False**

Justification: cotMisnottheproductofcotandM.Itisthecotangentof∠M.

(v) sinθ=4/3forsomeangleθ.

Answer: **False**

Justification: sinθ=Opposite/Hypotenuse

Weknowthatinarightangledtriangle,Hypotenuseisthelongestside.

∴sinθwillalwayslessthan1anditcanneverbe4/3foranyvalueofθ.

Exercise8.2

1. Evaluatethefollowing:

(i) sin60°cos30°+sin30°cos60°

(ii) 2tan$_2$45°+cos$_2$30°–sin$_2$60

(iii) $$\frac{\cos 45°}{\sec 30° + \operatorname{cosec} 30°}$$

(iv) $$\frac{\sin 30° + \tan 45° - \operatorname{cosec} 60°}{\sec 30° + \cos 60° + \cot 45°}$$

(v) $$\frac{5\cos^2 60° + 4\sec^2 30° - \tan^2 45°}{\sin^2 30° + \cos^2 30°}$$

Solution:

(i) sin60° cos30°+ sin30°cos 60°

First, find the values of the given trigonometric ratios

sin 30° = 1/2

cos30°=√3/2

sin 60°= 3/2

$cos60° = 1/2$

Now,substitutethevaluesinthegivenproblem

$sin60°cos30° + sin30°cos60° = \sqrt{3}/2 \times \sqrt{3}/2 + (1/2) \times (1/2) = 3/4 + 1/4 = 4/4$
$= 1$

(ii) $2tan^2 45° + cos^2 30° - sin^2 60$

Weknowthat,thevaluesofthetrigonometricratiosare:

$sin\ 60° = \sqrt{3}/2$

$cos\ 30° = \sqrt{3}/2$

$tan\ 45° = 1$

Substitutethevaluesinthegivenproblem

$2tan^2 45° + cos^2 30° - sin^2 60 = 2(1)^2 + (\sqrt{3}/2)^2 - (\sqrt{3}/2)^2$

$2tan^2 45° + cos^2 30° - sin^2 60 = 2 + 0$

$2tan^2 45° + cos^2 30° - sin^2 60 = 2$

(iii) $cos45°/(sec30° + cosec30°)$

We know that,

$cos\ 45° = 1/\sqrt{2}$

$sec\ 30° = 2/\sqrt{3}$

$cosec\ 30° = 2$

Substitutethevalues,weget

$$\frac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ} = \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}}+2} = \frac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}}$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2+2\sqrt{3}}$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2(1+\sqrt{3})} = \frac{\sqrt{3}}{2\sqrt{2}(1+\sqrt{3})} = \frac{\sqrt{3}}{2\sqrt{2}(\sqrt{3}+1)}$$

Now, rationalize the terms we get,

$$= \frac{\sqrt{3}}{2\sqrt{2}(\sqrt{3}+1)} \times \frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{3-\sqrt{3}}{2\sqrt{2}\,(3-1)} = \frac{3-\sqrt{3}}{2\sqrt{2}\,(2)}$$

Now,multiplybooththenumeratoranddenominatorby√2,weget

$$= \frac{3-\sqrt{3}}{2\sqrt{2}\,(2)} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}-\sqrt{3}\sqrt{2}}{8} = \frac{3\sqrt{2}-\sqrt{6}}{8}$$

Therefore,cos45°/(sec30°+cosec30°)=(3√2−√6)/8

(iv) $\frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$

We know

that,sin30°=1/2

tan 45° =

1cosec60°=2/√3

sec 30° = 2/√3

cos60°=1/2 cot

45° = 1

Substitutethevaluesinthegivenproblem,weget

$$\frac{\sin 30°+\tan 45°-\operatorname{cosec} 60°}{\sec 30°+\cos 60°+\cot 45°}=\frac{\frac{1}{2}+1-\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}+\frac{1}{2}+1}=\frac{\frac{\sqrt{3}+2\sqrt{3}-4}{2\sqrt{3}}}{\frac{4+\sqrt{3}+2\sqrt{3}}{2\sqrt{3}}}$$

Now, cancel the term 2√3, in numerator and denominator, we get

$$=\frac{\sqrt{3}+2\sqrt{3}-4}{4+\sqrt{3}+2\sqrt{3}}=\frac{3\sqrt{3}-4}{3\sqrt{3}+4}$$

Now, rationalize the terms

$$=\frac{3\sqrt{3}-4}{3\sqrt{3}+4}\times\frac{3\sqrt{3}-4}{3\sqrt{3}-4}$$

$$=\frac{27-12\sqrt{3}-12\sqrt{3}+16}{27-12\sqrt{3}+12\sqrt{3}+16}=\frac{27-24\sqrt{3}+16}{11}=\frac{43-24\sqrt{3}}{11}$$

Therefore,

$$\frac{\sin 30°+\tan 45°-\operatorname{cosec} 60°}{\sec 30°+\cos 60°+\cot 45°}=\frac{43-24\sqrt{3}}{11}$$

(v) $\frac{5\cos^2 60°+4\sec^2 30°-\tan^2 45°}{\sin^2 30°+\cos^2 30°}$

We know that,

cos 60° = 1/2

sec30° = 2/√3

tan 45° = 1

sin 30°= 1/2

cos30°=√3/2

Now, substitute the values in the given problem, we get

$(5\cos^2 60° + 4\sec^2 30° – \tan^2 45°)/(\sin^2 30° + \cos^2 30°)$

$=5(1/2)^2+4(2/\sqrt{3})^2-1^2/(1/2)^2+(\sqrt{3}/2)^2$

=(5/4+16/3-1)/(1/4+3/4)

=(15+64-12)/12/(4/4)

=67/12

2. Choosethecorrectoptionandjustifyyourchoice:

(i) $2\tan 30°/1+\tan^2 30°$=

(A)sin60° **(B)cos60°** **(C)tan60°** **(D)sin30°**

(ii) $1-\tan^2 45°/1+\tan^2 45°=$

(A)tan90° **(B)1** **(C)sin45°** **(D)0**

(iii) sin2A=2sinAistruewhenA=

(A)0° **(B)30°** **(C)45°** **(D)60°**

(iv) $2\tan 30°/1-\tan^2 30°=$

(A)cos60° **(B)sin60°** **(C)tan 60°** **(D)sin30°**

Solution:

(i) (A)iscorrect.

Substitutetheoftan30°inthegivenequation

tan30°=1/√3

$2\tan 30°/1+\tan^2 30°=2(1/\sqrt{3})/1+(1/\sqrt{3})^2$

=(2/√3)/(1+1/3)=(2/√3)/(4/3)

=6/4√3=√3/2=sin60°

Theobtainedsolutionisequivalenttothetrigonometricratiosin60°

(ii) (D)iscorrect.

Substitute the of tan 45° in the given equation

tan 45° = 1

$1-\tan^2 45°/1+\tan^2 45°=(1-1^2)/(1+1^2)$

=0/2=0

Thesolutionoftheaboveequationis0.

(iii) (A)iscorrect.

Tofind thevalueofA, substitutethedegree given in theoptionsone byone sin 2A

= 2 sin A is true when A = 0°

Assin2A= sin 0°= 0

$2 \sin A = 2 \sin 0° = 2 \times 0 = 0$

or,

Apply the sin 2A formula, to find the degree value

$\sin 2A = 2\sin A \cos A$

$\Rightarrow 2\sin A\cos A = 2 \sin A$

$\Rightarrow 2\cos A = 2 \Rightarrow \cos A = 1$

Now,we havetocheck,togetthesolutionas1,which degreevaluehastobeapplied. When 0

degree is applied to cos value, i.e., cos 0 =1

Therefore,$\Rightarrow A = 0°$

(iv) (C)iscorrect.

Substitutetheoftan30°inthegivenequation

$\tan 30° = 1/\sqrt{3}$

$2\tan 30°/1-\tan^2 30° = 2(1/\sqrt{3})/1-(1/\sqrt{3})^2$

$=(2/\sqrt{3})/(1-1/3) = (2/\sqrt{3})/(2/3) = \sqrt{3} = \tan 60°$

Thevalueofthegivenequationisequivalenttotan60°.

3. If $\tan(A+B)=\sqrt{3}$ and $\tan(A–B)=1/\sqrt{3}$, $0°<A+B\leq 90°$; $A>B$, find A and B.

Solution:

$\tan (A + B) = \sqrt{3}$

Since $\sqrt{3} = \tan 60°$

Nowsubstitutethedegreevalue

$\Rightarrow \tan(A+B) = \tan 60°$

$(A+ B) = 60°$…(i)

Theaboveequationisassumedasequation(i) tan

(A – B) = 1/√3

Since1/√3=tan30°

Nowsubstitutethedegreevalue

⇒tan(A−B)=tan30°

(A−B)=30°…equation(ii)

Nowadd the equation (i) and (ii),we get A

+ B + A – B = 60° + 30°

Cancel the termsB

2A = 90°

A=45°

Now, substitute the value of A in equation (i) to find the value of B

45° + B = 60°

B=60°−45°

B=15°

ThereforeA=45°andB=15°

4. Statewhetherthefollowingaretrueorfalse.Justifyyouranswer.

(i) sin(A+B)=sinA+sinB.

(ii) Thevalueofsinθincreasesasθincreases.

(iii) Thevalueofcosθincreasesasθincreases.

(iv) sinθ=cosθforallvaluesofθ.

(v) cotAisnotdefinedforA=0°.

Solution:

(i) False.

Justification:

LetustakeA=30°andB=60°,then

Substitutethevaluesinthesin(A+B)formula,weget sin (A

+ B) = sin (30° + 60°) = sin 90° = 1 and,

sinA+sinB=sin30°+sin60°

=1/2+√3/2=1+√3/2

Sincethevaluesobtainedarenotequal,thesolutionisfalse.

(ii) True.

Justification:

According tothevaluesobtained aspertheunit circle,thevaluesofsinare: sin 0°

= 0

sin 30° = 1/2

sin 45° = 1/√2

sin 60° = √3/2

sin 90° = 1

Thusthevalueofsinθincreasesasθincreases.Hence,thestatementistrue

(iii) False.

Accordingtothevaluesobtainedaspertheunitcircle,thevaluesofcosare: cos 0° =

1

cos 30° = √3/2

cos45°=1/√2

cos60°=1/2

$\cos 90° = 0$

Thus,thevalueofcosθdecreasesasθincreases.So,thestatementgivenaboveis false.

(iv) False

sinθ=cosθ,whena righttriangle has2anglesof (π/4). Therefore,theabove statementisfalse.

(v) True.

Since cot function is the reciprocal of the tan function, it is also written as:

cot A = cos A/sin A

NowsubstituteA=0°

cot 0°= cos0°/sin 0°= 1/0 = undefined. Hence,

it is true

Exercise8.3

1. ExpressthetrigonometricratiossinA,secAandtanAintermsofcotA.

Solution:

To convert the given trigonometric ratios in terms of cot functions, use trigonometric formulas

We know that,

$\text{cosec}^2 A - \cot^2 A = 1$

$\text{cosec}^2 A = 1 + \cot^2 A$

Since cosec function is the inverse of sin function,it is written as

$1/\sin^2 A = 1 + \cot^2 A$

Now,rearrangetheterms,itbecomes

$\sin^2 A=1/(1+\cot^2 A)$

Now,takesquarerootsonbothsides,weget

$\sin A=\pm 1/(\sqrt{(1+\cot^2 A})$

The above equation defines the sin function in terms of cot function

Now, to express sec function in terms of cot function, use this formula

$\sin^2 A = 1/ (1+\cot^2 A)$

Now, represent the sin function as cos function

$1 - \cos^2 A = 1/ (1+\cot^2 A)$

Rearrange the terms,

$\cos^2 A = 1 - 1/(1+\cot^2 A)$

$\Rightarrow \cos^2 A=(1-1+\cot^2 A)/(1+\cot^2 A)$

Sincesecfunctionistheinverseofcosfunction,

$\Rightarrow 1/\sec^2 A=\cot^2 A/(1+\cot^2 A)$

Takethereciprocaland squarerootsonbothsides,weget

$\Rightarrow \sec A=\pm\sqrt{(1+\cot^2 A)}/\cot A$

Now, to express tan function in terms of cot function

$\tan A = \sin A/\cos A$ and $\cot A = \cos A/\sin A$

Since cot function is the inverse of tan function, it is rewritten as

$\tan A = 1/\cot A$

2. Writealltheothertrigonometricratiosof∠AintermsofsecA.

Solution:

CosAfunctionintermsofsecA:

secA=1/cosA

⇒cosA=1/secA

secAfunction in termsof secA:

$\cos^2 A + \sin^2 A = 1$

Rearrange the terms

$\sin^2 A = 1 - \cos^2 A$

$\sin^2 A = 1 - (1/\sec^2 A)$

$\sin^2 A=(\sec^2 A-1)/\sec^2 A$

$\sin A=\pm\sqrt{(\sec^2 A-1)}/\sec A$

cosecAfunctionintermsofsecA: sin

A = 1/cosec A

⇒cosecA=1/sinA

$\mathrm{cosec} A=\pm\sec A/\sqrt{(\sec^2 A-1)}$

Now, tan A function in terms of sec A:

$\sec^2 A - \tan^2 A = 1$

Rearrangetheterms

$\Rightarrow\tan^2 A=\sec^2 A-1$

$\tan A=\sqrt{(\sec^2 A-1)}$

cot A function in terms of sec A:

tan A = 1/cot A

⇒cotA=1/tanA

$\cot A=\pm 1/\sqrt{(\sec^2 A-1)}$

3. Evaluate:

(i) $(\sin^2 63° + \sin^2 27°)/(\cos^2 17° + \cos^2 73°)$

(ii) $\sin 25° \cos 65° + \cos 25° \sin 65°$

Solution:

(i) $(\sin^2 63° + \sin^2 27°)/(\cos^2 17° + \cos^2 73°)$

To simplify this, convert some of the sin functions into cos functions and cos function into sin function and it becomes,

$=[\sin^2(90°-27°)+\sin^2 27°]/[\cos^2(90°-73°)+\cos^2 73°)]$

$=(\cos^2 27°+\sin^2 27°)/(\sin^2 27°+\cos^2 73°)$

$= 1/1 = 1$ (since $\sin^2 A + \cos^2 A = 1$)

Therefore, $(\sin^2 63°+\sin^2 27°)/(\cos^2 17° +\cos^2 73°)=1$

(ii) $\sin 25° \cos 65°+\cos 25° \sin 65°$

To simplify this, convert some of the sin functions into cos functions and cos function into sin function and it becomes,

$=\sin(90°-25°)\cos 65°+\cos(90°-65°)\sin 65°$

$=\cos 65° \cos 65° +\sin 65° \sin 65°$

$=\cos^2 65°+\sin^2 65°=1$ (since $\sin^2 A+\cos^2 A=1$) Therefore, sin

$25° \cos 65° + \cos 25° \sin 65° = 1$

4. Choosethecorrectoption.Justifyyourchoice.

(i) $9\sec^2 A - 9\tan^2 A =$

(A)1 (B)9 (C)8 (D)0

(ii) $(1+\tan\theta+\sec\theta)(1+\cot\theta-\text{cosec}\theta)$

(A)0 (B)1 (C)2 (D)–1

(iii) $(\sec A+\tan A)(1-\sin A)=$

(A)secA (B)sinA (C)cosecA (D)cosA

(iv) $1+\tan^2 A/1+\cot^2 A=$

(A)$\sec^2 A$ (B)-1 (C)$\cot^2 A$ (D)$\tan^2 A$

Solution:

(i) (B)iscorrect.

Justification:

Take9outside,anditbecomes 9

$\sec^2 A - 9\tan^2 A$

$=9(\sec^2 A - \tan^2 A)$

$=9\times 1=9$ $(\because \sec^2 A - \tan^2 A = 1)$

Therefore, $9\sec^2 A - 9\tan^2 A = 9$

(ii) (C)iscorrect

Justification:

$(1+\tan\theta+\sec\theta)(1+\cot\theta-\operatorname{cosec}\theta)$

We know that, $\tan\theta = \sin\theta/\cos\theta$

$\sec\theta = 1/\cos\theta$

$\cot\theta=\cos\theta/\sin\theta$

$\operatorname{cosec}\theta = 1/\sin\theta$

Now,substitutetheabovevaluesinthegivenproblem,weget

$=(1+\sin\theta/\cos\theta+1/\cos\theta)(1+\cos\theta/\sin\theta-1/\sin\theta)$

Simplifytheaboveequation,

$=(\cos\theta+\sin\theta+1)/\cos\theta\times(\sin\theta+\cos\theta-1)/\sin\theta$

$=(\cos\theta+\sin\theta)^2-1^2/(\cos\theta\sin\theta)$

$=(\cos^2\theta+\sin^2\theta+2\cos\theta\sin\theta-1)/(\cos\theta\sin\theta)$

$=(1+2\cos\theta\sin\theta-1)/(\cos\theta\sin\theta)$ (Since $\cos^2\theta+\sin^2\theta=1$)

$=(2\cos\theta\sin\theta)/(\cos\theta\sin\theta)=2$

Therefore,(1+tanθ+secθ)(1+cotθ−cosecθ)=2

(iii) (D)iscorrect.

Justification:

We know that,

SecA=1/cosA

TanA=sinA/cosA

Now,substitutetheabovevaluesinthegivenproblem,weget (secA +

tanA) (1 – sinA)

=(1/cosA+sinA/cosA)(1−sinA)

=(1+sinA/cos A)(1 – sinA)

=$(1-\sin^2 A)$/cosA

=$\cos^2 A$/cosA=cosA

Therefore,(secA+tanA)(1−sinA)=cosA

(iv) (D)iscorrect.

Justification:

We know that,

$\tan^2 A = 1/\cot^2 A$

Now,substitute thisin the given problem,we get

$1+\tan^2 A/1+\cot^2 A$

$=(1+1/\cot^2 A)/1+\cot^2 A$

$=(\cot^2 A+1/\cot^2 A)\times(1/1+\cot^2 A)$

$=1/\cot^2 A=\tan^2 A$

So,$1+\tan^2 A/1+\cot^2 A= \tan^2 A$

5. Prove the following identities, where the angles involved are acute angles for which the expressionsaredefined.

(i) $(\text{cosec}\theta - \cot\theta)^2 = (1-\cos\theta)/(1+\cos\theta)$

(ii) $\cos A/(1+\sin A)+(1+\sin A)/\cos A = 2\sec A$

(iii) $\tan\theta/(1-\cot\theta)+\cot\theta/(1-\tan\theta) = 1+\sec\theta\,\text{cosec}\theta$ [Hint :

Writetheexpressionintermsof sinθ andcos θ]

(iv) $(1+\sec A)/\sec A = \sin^2 A/(1-$

$\cos A)$[Hint:SimplifyLHS and RHSseparately]

(v) $(\cos A - \sin A+1)/(\cos A+\sin A-1) = \text{cosec}A+\cot A$,usingtheidentity$\text{cosec}^2 A$ $=1+\cot^2 A$.

(vi) $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

(vii) $(\sin\theta - 2\sin^3\theta)/(2\cos^3\theta - \cos\theta) = \tan\theta$
(viii) $(\sin A+\text{cosec}A)^2 + (\cos A+\sec A)^2 = 7+\tan^2 A+\cot^2 A$
(ix) $(\text{cosec}A - \sin A)(\sec A - \cos A) = 1/(\tan A+\cot A)$
[Hint:SimplifyLHSandRHSseparately](x)$(1+\tan^2 A/1+\cot^2 A) = (1-\tan A/1-\cot A)^2 = \tan^2 A$

Solution:

(i) $(\text{cosec}\theta - \cot\theta)^2 = (1-\cos\theta)/(1+\cos\theta)$

To prove this, first take the Left-Hand side (L.H.S) of the given equation, to prove the Right Hand Side (R.H.S)

L.H.S.$=(\text{cosec}\theta - \cot\theta)^2$

The above equation is in the form of $(a-b)^2$, and expand it

Since $(a-b)^2 = a^2 + b^2 - 2ab$

Herea$=\text{cosec}\theta$andb$=\cot\theta$

$=(cosec^2θ+cot^2θ−2cosecθcotθ)$

Now,applythe correspondinginversefunctionsandequivalentratiosto simplify

$=(1/sin^2θ + cos^2θ/sin^2θ −2cosθ/sin^2θ)$

$=(1+cos^2θ−2cosθ)/(1−cos^2θ)$

$=(1-cosθ)^2/(1−cosθ)(1+cosθ)$

$=(1-cosθ)/(1+cosθ)$=R.H.S.

Therefore,$(cosecθ−cotθ)^2=(1-cosθ)/(1+cosθ)$

Henceproved.

(ii) $(cosA/(1+sinA))+((1+sinA)/cosA)=2secA$ Now,

take the L.H.S of the given equation.

L.H.S.$=(cosA/(1+sin A)) +((1+sin A)/cosA)$

$=[cos^2A+(1+sinA)^2]/(1+sinA)cosA$

$=(cos^2A+sin^2A+1+2sinA)/(1+sinA)cosA$ Since

$cos^2A + sin^2A = 1$, we can write it as

$=(1+1+2sinA)/(1+sinA)cosA$

$=(2+2sinA)/(1+sinA)cosA$

$=2(1+sinA)/(1+sinA)cosA$

$=2/cosA=2secA$= R.H.S.

L.H.S.=R.H.S.

$(cosA/(1+sinA))+((1+sinA)/cosA)=2secA$ Hence

proved.

(iii) $tanθ/(1-cotθ)+ cotθ/(1-tanθ)=1+secθcosecθ$

L.H.S.$=tanθ/(1-cotθ)+cotθ/(1-tanθ)$

We know that tan θ =sin θ/cos θ

cot θ = cos θ/sin θ

Now,substituteitinthegivenequation,toconvertitinasimplifiedform

$=[(\sin\theta/\cos\theta)/1-(\cos\theta/\sin\theta)]+[(\cos\theta/\sin\theta)/1-(\sin\theta/\cos\theta)]$

$=[(\sin\theta/\cos\theta)/(\sin\theta-\cos\theta)/\sin\theta]+[(\cos\theta/\sin\theta)/(\cos\theta-\sin\theta)/\cos\theta]$

$=\sin^2\theta/[\cos\theta(\sin\theta-\cos\theta)]+\cos^2\theta/[\sin\theta(\cos\theta-\sin\theta)]$

$=\sin^2\theta/[\cos\theta(\sin\theta-\cos\theta)]-\cos^2\theta/[\sin\theta(\sin\theta-\cos\theta)]$

$=1/(\sin\theta-\cos\theta)\ [(\sin^2\theta/\cos\theta)-(\cos^2\theta/\sin\theta)]$

$=1/(\sin\theta-\cos\theta)\times[(\sin^3\theta-\cos^3\theta)/\sin\theta\cos\theta]$

$=[(\sin\theta-\cos\theta)(\sin^2\theta+\cos^2\theta+\sin\theta\cos\theta)]/[(\sin\theta-\cos\theta)\sin\theta\cos\theta]$

$=(1+\sin\theta\cos\theta)/\sin\theta\cos\theta$

$=1/\sin\theta\cos\theta+1$

$=1+\sec\theta\,\mathrm{cosec}\,\theta$=R.H.S.

Therefore,L.H.S.=R.H.S.

Henceproved

(iv) $(1+\sec A)/\sec A=\sin^2A/(1-\cos A)$

First find the simplified form of L.H.S

L.H.S.$=(1+\sec A)/\sec A$

Sincesecantfunctionistheinversefunctionofcosfunctionanditiswrittenas

$=(1+1/\cos A)/1/\cos A$

$=(\cos A+1)/\cos A/1/\cos A$

Therefore,$(1+\sec A)/\sec A=\cos A+1$

R.H.S.$=\sin^2A/(1-\cos A)$

Weknowthatsin$_2$A=(1−cos$_2$A),weget

=(1−cos$_2$A)/(1-cosA)

= (1-cos A)(1+cos A)/(1-cos A)

Therefore,sin$_2$A/(1-cosA)=cosA+ 1

L.H.S.=R.H.S.

Henceproved

(v) (cosA−sinA+1)/(cosA+sinA−1)=cosecA+cotA,usingtheidentitycosec$_2$A = 1+cot$_2$A.

With the help of identity function, cosec$_2$A = 1+cot$_2$A, let us prove the above equation.

L.H.S.=(cosA−sinA+1)/(cosA+sinA−1)

Dividethenumeratoranddenominatorbysin A,weget

= (cosA−sinA+1)/sinA/(cosA+sinA−1)/sinA

WeknowthatcosA/sinA=cotAand1/sinA=cosecA

=(cotA−1+cosecA)/(cotA+1−cosecA)

= (cot A− cosec$_2$A+cot$_2$A+cosec A)/(cot A+1 − cosecA) (using cosec$_2$A− cot$_2$A = 1

=[(cotA+cosecA)−(cosec$_2$A−cot$_2$A)]/(cotA+1 −cosecA)

=[(cotA+cosecA)−(cosecA+cotA)(cosecA−cotA)]/(1−cosecA+cotA)

=(cotA+cosecA)(1−cosecA+cotA)/(1−cosecA+cotA)

=cotA+cosecA=R.H.S.

Therefore,(cosA−sinA+1)/(cosA+sinA−1)=cosecA+cotA Hence

Proved

(vi) $\sqrt{\frac{1+\sin A}{1-\sin A}} = \sec A + \tan A$

L.H.S = $\sqrt{\frac{1+\sin A}{1-\sin A}}$

FirstdividethenumeratoranddenominatorofL.H.S.bycosA,

$$= \sqrt{\frac{\frac{1}{\cos A}+\frac{\sin A}{\cos A}}{\frac{1}{\cos A}-\frac{\sin A}{\cos A}}}$$

Weknowthat1/cosA=secAandsinA/cosA=tanAanditbecomes,

= √(sec A+ tan A)/(sec A-tan A)

Now using rationalization, we get

$$= \sqrt{\frac{\sec A+\tan A}{\sec A-\tan A}} \times \sqrt{\frac{\sec A+\tan A}{\sec A+\tan A}}$$

$$= \sqrt{\frac{(\sec A+\tan A)^2}{\sec^2 A-\tan^2 A}}$$

=(secA+tanA)/1

=secA+tanA=R.H.S Hence

proved

(vii) $(\sin\theta - 2\sin^3\theta)/(2\cos^3\theta - \cos\theta) = \tan\theta$

L.H.S.=$(\sin\theta - 2\sin^3\theta)/(2\cos^3\theta - \cos\theta)$

Takesinθasinnumeratorandcosθindenominatorasoutside,itbecomes

= $[\sin\theta(1 - 2\sin^2\theta)]/[\cos\theta(2\cos^2\theta - 1)]$

We know that $\sin^2\theta = 1-\cos^2\theta$

=$\sin\theta[1-2(1-\cos^2\theta)]/[\cos\theta(2\cos^2\theta-1)]$

=$[\sin\theta(2\cos^2\theta-1)]/[\cos\theta(2\cos^2\theta-1)]$

$=\tan\theta=$ R.H.S.

Henceproved

(viii) $(\sin A+\operatorname{cosec}A)^2+(\cos A+\sec A)^2=7+\tan^2A+\cot^2A$

L.H.S. $=(\sin A+\operatorname{cosec}A)^2+(\cos A+\sec A)^2$ It is of

the form $(a+b)^2$, expand it

$(a+b)^2=a^2+b^2+2ab$

$=(\sin^2A+\operatorname{cosec}^2A+2\sin A\operatorname{cosec}A)+(\cos^2A+\sec^2A+2\cos A\sec A)$

$=(\sin^2A+\cos^2A)+2\sin A(1/\sin A)+2\cos A(1/\cos A)+1+\tan^2A+1+\cot^2A$

$=1+2+2+2+\tan^2A+\cot^2A$

$=7+\tan^2A+\cot^2A=$ R.H.S.

Therefore, $(\sin A+\operatorname{cosec}A)^2+(\cos A+\sec A)^2=7+\tan^2A+\cot^2A$ Hence

proved.

(ix) $(\operatorname{cosec}A-\sin A)(\sec A-\cos A)=1/(\tan A+\cot A)$

First, find the simplified form of L.H.S

L.H.S. $=(\operatorname{cosec}A-\sin A)(\sec A-\cos A)$

Now,substitutetheinverseandequivalenttrigonometricratioforms

$=(1/\sin A-\sin A)(1/\cos A-\cos A)$

$=[(1-\sin^2A)/\sin A][(1-\cos^2A)/\cos A]$

$=(\cos^2A/\sin A)\times(\sin^2A/\cos A)$

$=\cos A\sin A$

Now,simplifytheR.H.S

R.H.S.=1/(tanA+cotA)

=1/(sinA/cos A+cosA/sinA)

=1/[(sin^2A+cos^2A)/sinAcosA]

=cosAsinA

L.H.S.=R.H.S.

(cosecA–sinA)(secA–cosA)=1/(tanA+cotA)

Henceproved

(x) (1+tan^2A/1+cot^2A)=(1-tanA/1-cotA)^2= tan^2A

L.H.S.=(1+tan^2A/1+cot^2A)

Since cotfunctionis theinverseoftanfunction,

=(1+tan^2A/1+1/tan^2A)

=1+tan^2A/[(1+tan^2A)/tan^2A]

Nowcancelthe1+tan^2Aterms,weget

=tan^2A

(1+tan^2A/1+cot^2A) = tan^2A

Similarly,

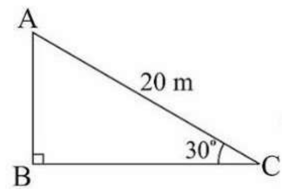
(1-tan A/1-cot A)^2=tan^2A

Hence proved

SomeApplicationsofTrigonometry

1. A circus artist is climbing a 20 m long rope, which is tightly stretched and tied from the top of a verticalpoletotheground.Findtheheightofthepole,ifthe angle madebytheropewith theground level is 30^o(see Fig.).

Solution:



Solution:

AB represents the height of the pole. In ΔABC, $\sin 30^\circ = \frac{AB}{AC}$

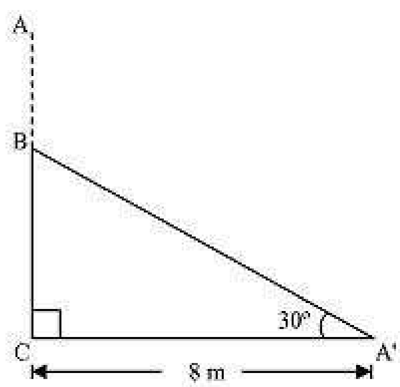
$\Rightarrow \frac{1}{2} = \frac{AB}{20}$

$AB = 10$

Hence, the height of pole is 10 m

2. A tree breaks due to storm and the broken part bends so that the top of the tree touches the ground making an angle 30° with it. The distance between the foot of the tree to the point where the top touches the ground is 8 m. Find the height of the tree.

Solution:



Let AC be the original tree and $A'B$ be the broken part which makes an angle of 30° with the ground. In $\Delta A'BC$,

$\tan 30^\circ = \frac{BC}{A'C}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{BC}{8}$

$BC = \frac{8}{\sqrt{3}}$

Again, $\cos 30^\circ = \frac{A'C}{A'B}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{8}{A'B}$

$A'B = \frac{16}{\sqrt{3}}$

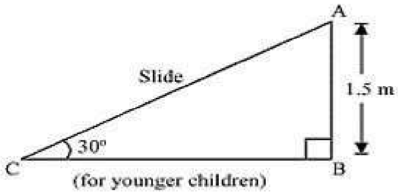
Hence, height of tree $= A'B + BC = \frac{16}{\sqrt{3}} + \frac{8}{\sqrt{3}} = \frac{24}{\sqrt{3}} = 8\sqrt{3}$ m

3. A contractor plans to install two slides for the children to play in a park. For the children below the age of 5 years, she prefers to have a slide whose top is at a height of 1.5 m, and is inclined at an angle of 30° to the ground, whereas for elder children, she wants to have a steep slide at a height of

3m,andinclined atan angleof 60^ototheground. Whatshouldbethelengthoftheslidein each case?

Solution:

Inthetwofigures,ACrepresentsslideforyoungerchildrenandPRrepresentsslideforelderchildren

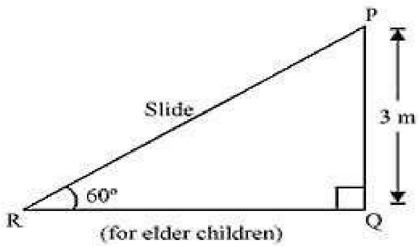


In∆ABC,

$\sin 30^o = \frac{AB}{AC}$

$\Rightarrow \frac{1}{2} = \frac{1.5}{AC}$

AC=3m



In∆PQR, $\sin 60^o = \frac{PQ}{PR}$

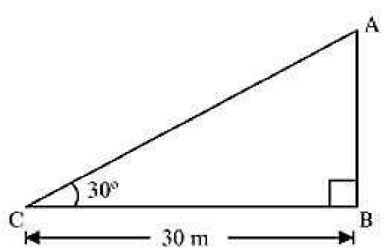
$\frac{\sqrt{3}}{2} = \frac{3}{PR}$ ⇒

$PR = \frac{6}{\sqrt{3}} = 2\sqrt{3}$m

Hence,thelengthofthetwoslidesare3m and$2\sqrt{3}$m.

4. Theangleofelevationof thetopof atowerfrom apointontheground,whichis 30mawayfrom the foot of the tower, is 30^o. Find the height of the tower.

Solution:



LetABrepresentsthe tower. In ∆ABC,

$\tan 30^o = \frac{AB}{BC}$

$\frac{1}{\sqrt{3}} = \frac{AB}{30}$

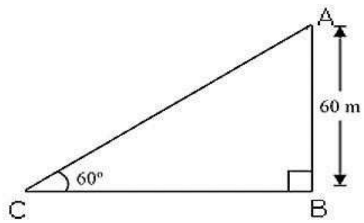
⇒

AB$= \frac{30}{\sqrt{3}} = 10\sqrt{3}$m

Hence,theheightofthetoweris$10\sqrt{3}$m

5. A kite is flying at a height of 60 m above the ground. The string attached to the kite is temporarily tiedto apointon theground.Theinclinationofthestringwiththegroundis 60^{o}.Findthelengthof the string, assuming that there is no slack in the string.

Solution:



LetArepresentsthepositionofthekiteandthestringistiedtopointContheground. In △ABC,

$\sin 60^{o} = \frac{AB}{AC}$

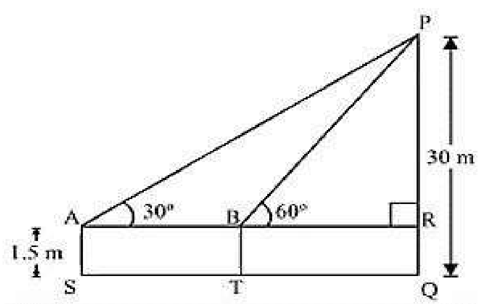
$\frac{\sqrt{3}}{2} = \frac{60}{AC}$

⇒2=AC

AC$= \frac{120}{\sqrt{3}} = 40\sqrt{3}$m

6. A1.5m tallboyis standingatsomedistancefrom a30mtallbuilding.Theangleofelevationfrom his eyes to the top of the building increases from $30^{o}$to 60^{o}as he walks towards the building. Find the distance he walked towards the building.

Solution:



LetinitiallyboywasstandingatS.Afterwalkingtowardsthebuilding,hereachedatpoint . In the figure, PQ = height of the building = 30 m

S=BT=RQ=1.5m

PR=PQ−RQ=30m−1.5m=28.5m

In△PAR,tan

$\frac{1}{\sqrt{3}} = 30^{o} = \frac{PR}{AR}$

$\Rightarrow \frac{28.5}{AR}$

AR $= 28.5\sqrt{3}$

In $\Delta PRB, \tan 60^\circ = \frac{PR}{BR}$

$\Rightarrow \sqrt{3} = \frac{28.5}{BR}$

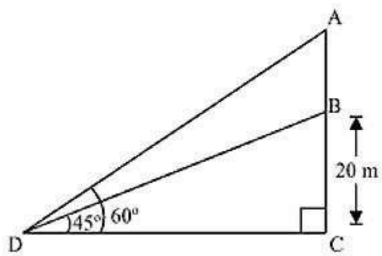
$BR = \frac{28.5}{\sqrt{3}} = 9.5\sqrt{3}$

$ST = AB = AR - BR = 28.5\sqrt{3} - 9.5\sqrt{3} = 19\sqrt{3}$

Hence, distance the boy walked towards the building $= 19\sqrt{3}$ m

7. From a point on the ground, the angles of elevation of the bottom and the top of a transmission tower fixed at the top of a 20 m high building are 45° and 60° respectively. Find the height of the tower.

Solution:



Let BC represents the building, AB represents the transmission tower, and D is the point on the ground from where elevation angles are to be measured.

In ΔBCD

$\tan 45^\circ = \frac{BC}{CD}$

$\Rightarrow 1 = \frac{20}{CD}$

$CD = 20$ m ...(i)

In $\Delta ACD, \tan 60^\circ = \frac{AC}{CD}$

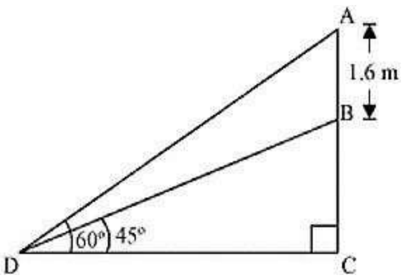
$\Rightarrow \sqrt{3} = \frac{AB + BC}{CD}$

$\Rightarrow \sqrt{3} = \frac{AB + 20}{20}$ [From(i)]

$AB = 20\sqrt{3} - 20 = 20(\sqrt{3} - 1)$ m

8. A statue, 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.

Solution:



LetABrepresentsthestatue,BCrepresentsthepedestalandDbethepointongroundfrom whereelevationangles are tobemeasurd.In $\Delta BCD, \tan 45^{o} = \frac{BC}{CD}$

$\Rightarrow 1 = \frac{BC}{CD}$

BC=CD …(i)In$\Delta ACD, \tan 60^{0} =$

$\frac{AB+BC}{CD} \Rightarrow \sqrt{3} = \frac{AB+BC}{BC}$[From(i)]

$\Rightarrow 1.6 + BC = BC\sqrt{3}$

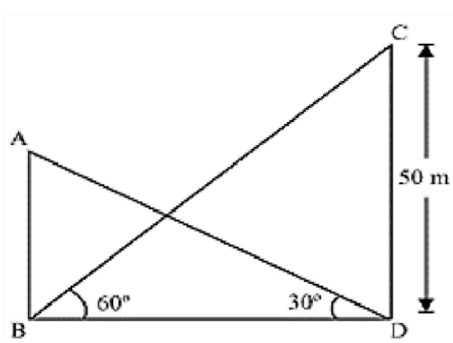
$BC = \frac{(1.6)(\sqrt{3}+1)}{(\sqrt{3}-1)(\sqrt{3}+1)}$

$= \frac{1.6(\sqrt{3}+1)}{2} = 0.8(\sqrt{3}+1)$

Hence,theheightofpedestal $= 0.8\ (\sqrt{3}+1)$m

9. The angle of elevation of the top of a building from the foot of the tower is $30^{o}$and the angle of elevation of thetop of thetower fromthefootof the building is 60^{o}. If thetower is50m high, find the height of the building.

Solution:



In$\Delta CDB, \tan 60^{o} = \frac{CD}{BD}$

$\Rightarrow \sqrt{3} = \frac{50}{BD}$

$BD = \frac{50}{\sqrt{3}}$

In$\Delta ABD, \tan 30^{o} = \frac{AB}{BD}$

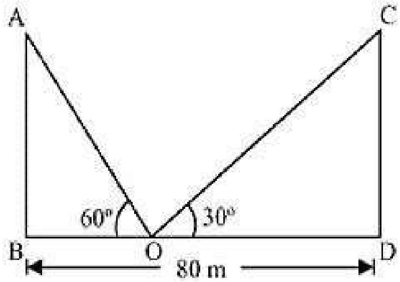
$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{BD}$

$AB = \frac{50}{\sqrt{3}} \times \frac{1}{\sqrt{3}} = \frac{50}{3} = 16\frac{2}{3}$m

Hence,heightofthebuilding=$16\frac{2}{3}$m

10. Two poles of equal heights are standing opposite each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30°, respectively. Find theheightof thepoles and thedistances of thepointfrom thepoles.

Solution:



LetABandCDrepresentthepolesandOisthepointontheroad. In

ΔABO,

$\tan 60^{\circ} =$ —

$\Rightarrow \sqrt{3} =$ —

$\Rightarrow$ = $\sqrt{3}$ …(i)

In Δ ,

$\tan 30^{\circ} = \frac{CD}{DO}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{CD}{80-BO}$

$\Rightarrow 80 - BO = CD\sqrt{3}$

$CD\sqrt{3} = 80 - \frac{AB}{\sqrt{3}}$[From(i)]

$CD\sqrt{3} + \frac{AB}{\sqrt{3}} = 80$

$CD\sqrt{3} + \frac{1}{\sqrt{3}}] = 80$ (Since,AB=CD)

$CD\frac{3+1}{\sqrt{3}}) = 80$

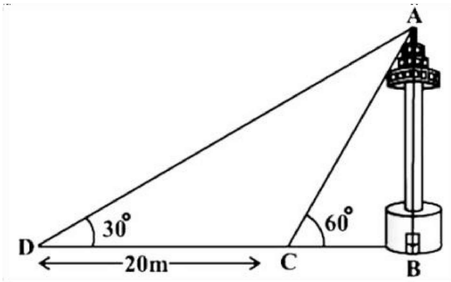
$CD = 20\sqrt{3}$

$BO = \frac{AB}{\sqrt{3}} = \frac{CD}{\sqrt{3}} = \frac{20\sqrt{3}}{\sqrt{3}} = 20$m

DO=BD−BO=80m−20m=60m

Hence, theheightofthepolesis$20\sqrt{3}$m anddistanceofthepointfromthepolesis20m and 60m.

11. A TV towerstands verticallyonabankofacanal. Fromapointon theotherbankdirectlyopposite thetower, theangleof elevation of thetop of thetower is 60°. Fromanother point20m away from this point on the line joining this point to the foot of the tower, the angle of elevation of the top of the tower is 30°(see Fig.). Find the height of the tower and the width of the canal.



Solution:

In $\Delta ABC, \tan 60^\circ = \frac{AB}{BC}$

$\Rightarrow \sqrt{3} = \frac{AB}{BC}$

$BC = \frac{AB}{\sqrt{3}} \ldots (i)$

In $\Delta ABD, \tan 30^\circ = \frac{AB}{BD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{\frac{AB}{\sqrt{3}} + 20}$ [From(i)]

$\Rightarrow \frac{AB\sqrt{3}}{AB + 20\sqrt{3}} = \frac{1}{\sqrt{3}}$

$\Rightarrow 3AB = AB + 20\sqrt{3}$

$\Rightarrow 2AB = 20\sqrt{3}$

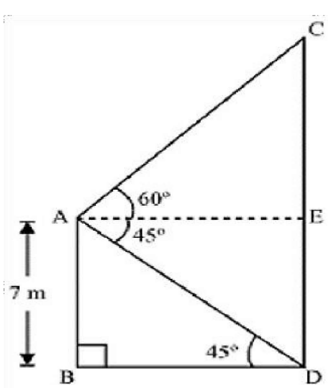
$AB = 10\sqrt{3}$

$BC = \frac{AB}{\sqrt{3}} = \frac{10\sqrt{3}}{\sqrt{3}} = 10$

Hence, the height of the tower is $10\sqrt{3}$ m and width of the canal is 10m

12. From the top of a 7m high building, the angle of elevation of the top of a cable tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

Solution:



Let AB represents the building and CD represents a cable tower.

In ΔABD,

$\tan 45^\circ = \frac{AB}{BD}$

7

$\Rightarrow 1 = \frac{}{BD}$

$BD = 7$

Hence, $AE = BD = 7$

In ΔACE,

$\tan 60^{o} = \frac{CE}{AE}$

$\Rightarrow \sqrt{3} = \frac{CE}{7}$

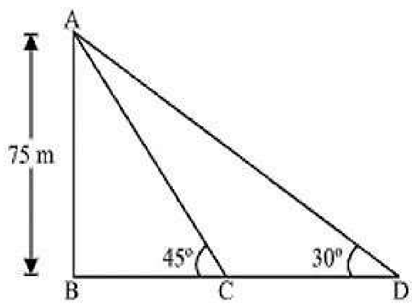
$CE = 7\sqrt{3}$

So, $CD = CE + ED = (7\sqrt{3} + 7)m = 7(\sqrt{3}+1)m$

Hence, the height of the cable tower $= 7(\sqrt{3} + 1m$

13. As observed from the top of a 75 m high lighthouse from the sea-level, the angles of depression of two ships are 30^{o} and 45^{o}. If one ship is exactly behind the other on the same side of the lighthouse, find the distance between the two ships.

Solution:



Let AB represents the lighthouse and the two ships are at point and respectively. In ΔABC, $\tan 45^{o} = \frac{AB}{BC}$

$\Rightarrow 1 = \frac{75}{BC}$

$BC = 75$

In ΔABD, $\tan 30^{o} = \frac{AB}{BD}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{75}{BC+CD}$

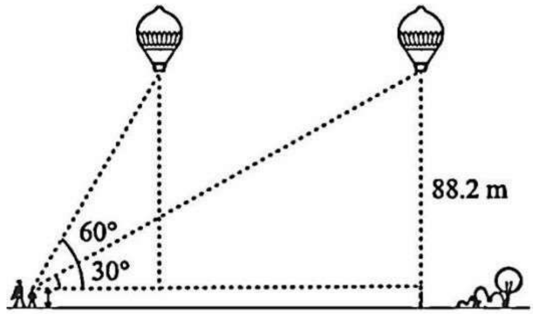
$\frac{1}{\sqrt{3}} = \frac{75}{75+CD}$ $\Rightarrow$

$\Rightarrow 75\sqrt{3} = 75 + CD$

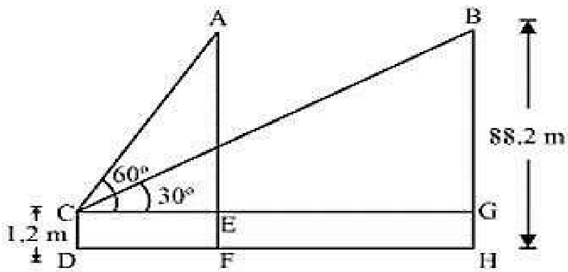
$CD = 75(\sqrt{3} - 1)$

Hence, the distance between the two ships $= 75(\sqrt{3} - 1)m$

14. A 1.2m tall girl spots a balloon moving with the wind in a horizontal line at a height of 88.2m from the ground. The angle of elevation of the balloon from the eyes of the girl at any instant is 60^{o}. After sometime,the angleof elevationreducesto30^{o}(seeFig.). Findthedistancetravelledbytheballoon during the interval.



Solution:



LetAistheinitialpositionof theballoonandB isthefinalpositionaftersome timeand CD representsthegirl.

In ΔACE,

AE=AF−EF=88.2−1.2=87

$\tan 60^{o}=\frac{AE}{CE}$

$\Rightarrow\sqrt{3}=\frac{87}{CE}$

$CE=\frac{87}{\sqrt{3}}=29\sqrt{3}$

InΔBCG,

BG=AE=87

$\tan 30^{o}=\frac{BG}{CG}$

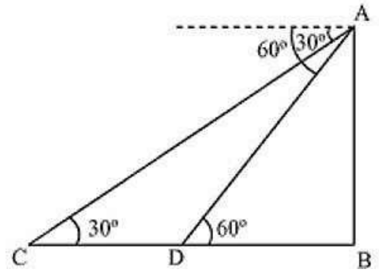
$\frac{1}{\sqrt{3}}=\frac{87}{CG}$

$CG=87\sqrt{3}$

Hence,distancetravelledbyballoon $=AB=EG=CG-CE=87\sqrt{3}-29\sqrt{3}=58\sqrt{3}$ m

15. Astraighthighwayleadstothefootofatower.Amanstandingatthe topofthe towerobservesacar at an angle of depression of 30^{o}, which is approaching the foot of the tower with a uniform speed. Sixseconds later, theangleofdepressionofthecar is foundtobe60^{o}.Findthetimetakenbythecar to reach the foot of the tower from this point.

Solution:



Let AB represents the tower. C is the initial position of the car and D is the final position after six seconds.

in ΔADB,

$\tan 60^{o} = \frac{AB}{DB}$

$\Rightarrow \sqrt{3} = \frac{AB}{DB}$

$DB = \frac{AB}{\sqrt{3}}$ ....(i)

In ΔABC, $\tan 30^{o} = \frac{AB}{BC}$

$\Rightarrow \frac{1}{\sqrt{3}} = \frac{AB}{BD+DC}$

$AB\sqrt{3} = BD+DC$

$AB\sqrt{3} = \frac{Ab}{\sqrt{3}} + DC$ [From(i)]

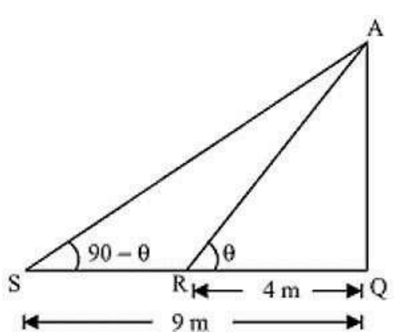
$\Rightarrow DC = AB\sqrt{3} - \frac{AB}{\sqrt{3}} = AB\left(\sqrt{3} - \frac{1}{\sqrt{3}}\right) = \frac{2AB}{\sqrt{3}}$

Since, time taken by car to travel distance $DC \left(= \frac{2AB}{\sqrt{3}}\right) = 6$ seconds

Hence, time taken by car to travel distance $DB \left(= \frac{AB}{\sqrt{3}}\right) = \frac{6}{\frac{2AB}{\sqrt{3}}} \times \frac{AB}{\sqrt{3}} = 3$ seconds (Since, speed is uniform)

16. The angles of elevation of the top of a tower from two points at a distance of 4 m and 9 m from the base of the tower and in the same straight line with it are complementary. Prove that the height of the tower is 6 m.

Solution:



Let AQ represents the tower and R, S are the points which are 4 m, 9 m away from base of the tower respectively.

Let $\angle ARQ = \theta$, then $\angle ASQ = 90^{o} - \theta$

(Since,theanglesarecomplementary)InΔAQR,tanθ=
AQQR

$\tan\theta = \frac{AQ}{4}$ …(i)

InΔAQS,

$\tan(90^0-\theta) = \frac{AQ}{SQ}$

$\cot\theta = \frac{AQ}{9}$ ….(ii)

Multiplyingequations(i)and(ii),

$\frac{AQ}{4}\frac{AQ}{9} = (\tan\theta).(\cot\theta)$

$\Rightarrow \frac{AQ^2}{36} = 1$

$AQ = \sqrt{36} = 6$

Hence, heightof thetower is6m

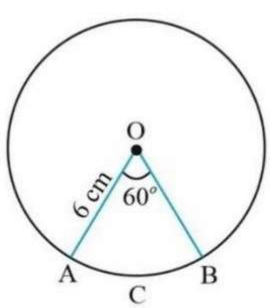
CHAPTER12

AreasRelatedtoCircles

EXERCISE 12.1

1. Findtheareaofasectorofacirclewithradius6cm ifangleofthesectoris60°.

Solution:



LetOACBbeasectorofcirclemaking60°angleatcentreOofcircle.Weknow,areaofsector of angle

$= \frac{\theta}{360} \times \pi r^2$

So,areaofsector OACB=$\frac{60}{360} \times \frac{22}{7} \times (6)^2$

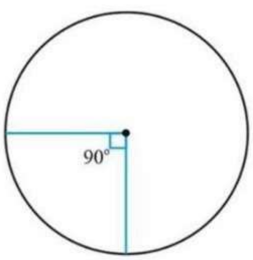
360

$$=\frac{1}{6}\times\frac{22}{7}\times 6\times 6=\frac{132}{7}\,cm^2$$

Therefore, area of sector of circle making 60° at centre of circle is $18.85\,cm^2$

2. Find the area of a quadrant of circle whose circumference is 22cm.

Solution:



Given circumference of the circle = 22cm Let radius of circle be .

$\Rightarrow 2\quad = 22$

$$r=\frac{22}{2}$$

$$\Rightarrow\quad =\frac{7}{2}$$

We know, quadrant of circle will subtend 90° angle at centre of circle. So, area of such quadrant of circle

$$=\frac{90^\circ}{360}\times\quad\times\quad^2$$

$$=\frac{1}{4}\times\quad\times\left(\frac{7}{2}\right)^2$$

$$=\frac{1}{4}\times\frac{22}{7}\times\frac{49}{4}$$

$$=\frac{77}{8}\,cm^2$$

Therefore, the area of a quadrant of circle whose circumference is 22cm is $\frac{77}{8}\,cm^2$

3. Thelengthoftheminutehandofaclockis 14cm.Findtheareasweptbytheminutehandin5minutes.

Solution:

Weknow thatinonehour,minutehandrotates 360^o.

So, in 5 minutes, minutehand willrotate= $\frac{360^o}{60}\times 5 = 30°$

Weknow,area ofsectorofangle $= \frac{\quad}{360}{}^o\times \quad {}^2$

Areaofsector of $30^o = \frac{30^o}{360^o}\times\frac{22}{7}\times 14\times 14$

$=\frac{22}{12}\times 2\times 14$

$=\frac{11\times 14}{3}$

$=\frac{154}{3}\ cm^2$

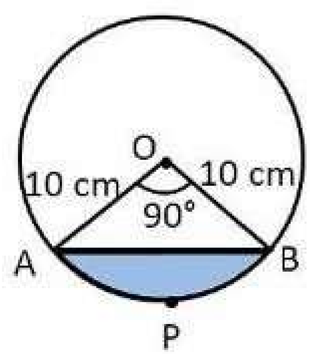
So,areasweptbyminutehandin5minutesis $51.3cm^2$.

4. Achordofacircleofradius 10cmsubtendsaright angleatthe centre. Find theareaofthe corresponding:

(i) Minorsegment

(ii) Majorsector

(Use =3.14)**Solution:**



Given achordofacircleofradius 10cmsubtendsarightangleat thecentreLetABbethe cordof circle subtending 90^oangle at centre O of circle.

(i) Areaof minor sector Weknow, areaof sector ofangle $= \frac{\quad}{360}\times \quad {}^2$

Therefore,areaofminorsector(OAPB)= $\frac{90^o}{360^o}\times \quad {}^2$

$=\frac{1}{4}\times\frac{22}{7}\times 10\times 10$

$=\frac{1100}{14}=78.5cm^2$

Area of $\Delta OAB=\frac{1}{2}\times OA\times OB=\frac{1}{2}\times 10\times 10$

$=50cm^2$

Area of minor segment APB = Area of minor sector $OAPB$ − Area of ΔOAB

$=78.5-50=28.5cm^2$

(ii) Area of major sector $=\left(\frac{360^\circ-90^\circ}{360^\circ}\right)\times\ {}^2=\left(\frac{270^\circ}{360^\circ}\right)\ {}^2$

$=\frac{3}{4}\times\frac{22}{7}\times 10\times 10$

$=\frac{3300}{14}cm^2=235.7cm^2$

5. In a circle of radius $21cm$, an arc subtends an angle of 60° at the centre. Find:

(i) The length of the arc

(ii) Area of the sector formed by the arc

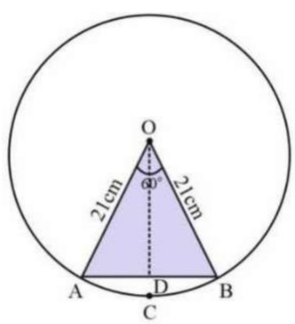
(ii) Area of the segment formed by the corresponding chord

Solution:

Given radius ( ) of circle $=21cm$

And angle subtended by given arc $=60^\circ$

We know, length of an arc of a sector of angle $=\frac{}{360^\circ}\times 2$



(i) Length of arc $ACB=\frac{60}{360}\times 2\times\frac{22}{7}\times 21$

$=\frac{1}{6}\times 2\times 22\times 3$

$=22cm$

(ii) Area of sector formed by the arc

We know, area of sector of angle $=\frac{}{360}\times\ {}^2$

⇒AreaofsectorOACB —— =36060ₒₒ2×

$= \frac{1}{6} \times \frac{22}{7} \times 21 \times 21$

=231cm2

(iii) Areaofthesegmentformedbythecorrespondingchord In

ΔOAB

OAB= OBA (asOA=OB)

Weknow,sumofanglesinatriangle=180°

OAB+ A0B+ OBA=180°

2 OAB+60o =180o

OAB=60°

So, ΔOAB is an equilateral triangle.

Therefore,areaofΔOAB= $\frac{\sqrt{3}}{4} \times (\text{side})^2$

$= \frac{\sqrt{3}}{4} \times (21^2) = \frac{441\sqrt{3}}{4}\ \text{cm}^2$

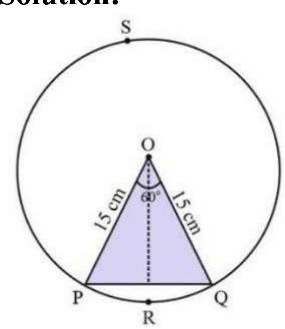
Now,areaofsegmentACB=AreaofsectorOACB−AreaofΔOAB

$= \left(231 - \frac{441\sqrt{3}}{4}\right)\ \text{cm}^2$

6. A chord of a circle of radius 15 cm subtends an angle of 60° at the centre. Findtheareasofthecorrespondingminorandmajorsegmentsofthecircle.

(Use π = 3.14 and $\sqrt{3}$ = 1.73)

Solution:



Givenradius( )ofcircle =15cmWe know,area ofsectorofangle

= ____ × 2

360

AreaofsectorOPRQ= 60ₒₒ× 2

360

$= \frac{1}{6} \times \frac{22}{7} \times (15)^2$

$= \frac{11 \times 75}{7} = \frac{825}{7}$

$= 117.85 cm^2$

In ΔOPQ

OPQ= OQP (as OP=OQ)

OPQ+ OQP+ POQ= 180°

2 OPQ = 120°

OPQ=60°

ΔOPQ is an equilateral triangle

Area of equilateral ΔOPQ $= \frac{\sqrt{3}}{4}(\text{side})^2$

$= \frac{\sqrt{3}}{4} \times (15)^2 = \frac{225\sqrt{3}}{4}\ \text{cm}^2$

$= 56.25\sqrt{3}$

$= 97.3125 cm^2$

Area of segment PRQ = Area of sector OPRQ − Area of ΔOPQ

$= 117.85 - 97.3125$

$= 20.537 cm^2$

Area of major segment PSQ = Area of circle − Area of segment PRQ

$= (15)^2 - 20.537$

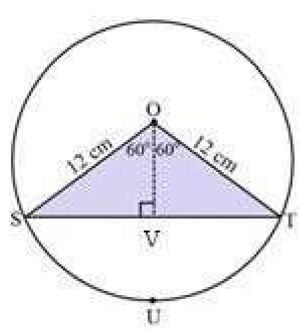
$= \frac{225 \times 22}{7} - 20.537$

$= \frac{4950}{7} - 20.537 = 707.14 - 20.537$

$= 686.605 cm^2$

7. A chord of a circle of radius 15cm subtends an angle of 120^o at the centre. Find the areas of the corresponding segment of the circle. (Use = 3.14 and $\sqrt{3} = 1.73$)

Solution:



Given radius of circle $= 15$ cm

Draw a perpendicular OV on chord ST. It will bisect the chord ST. SV = VT

In ΔOVS

$\frac{OV}{OS} = \cos 60^o$

$\Rightarrow \frac{1}{2} = \frac{OV}{12}$

$\Rightarrow OV = 6$

And $\sin 60^o = \frac{\sqrt{3}}{2} = \frac{SV}{SO}$

$\Rightarrow \frac{SV}{12} = \frac{\sqrt{3}}{2}$

$SV = 6\sqrt{3}$

$\therefore ST = 2SV = 2 \times 6\sqrt{3} = 12\sqrt{3}$

Now, area of $\Delta OST = \frac{1}{2} \times ST \times OV$

$= \frac{1}{2} \times 12\sqrt{3} \times 6$

$= 36\sqrt{3} = 36 \times 1.73 = 62.28$

We know, area of sector of angle $\theta = \frac{\theta}{360} \times \pi r^2$

Here $\theta = 120^o$

Area of sector OSUT $= \frac{120^o}{360^o} \times \pi (12)^2$

$= \frac{1}{3} \times \frac{22}{7} \times 144 = 150.72$

Area of segment SUT = Area of sector OSUT − Area of ΔOST

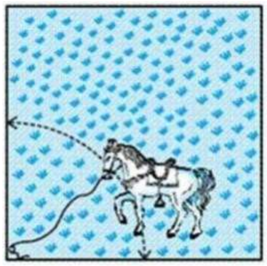
$= 150.72 - 62.28$

$= 88.44 \text{ cm}^2$

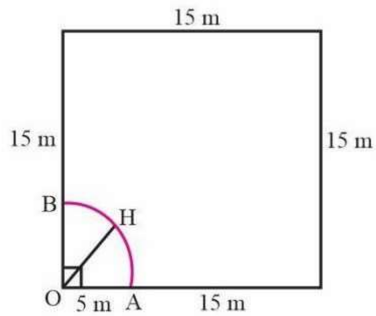
8. A horse is tied to a peg at one corner of a square shaped grass field of side 15m by means of a 5m long as given in figure. Find

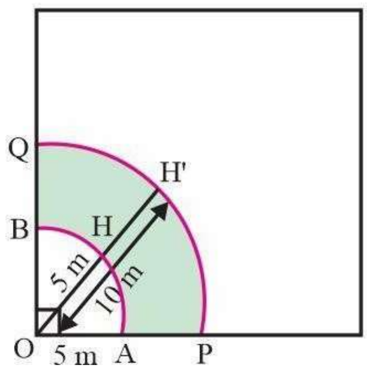
(i) The area of that part of the field in which the horse can graze.

(ii) The increase in the grazing area if the rope were 10m long instead of 5m. (use $\pi = 3.14$)



Solution:





Givenhorseis tired witharopeoflength5m.

Horsecangraze asectorof$90°$inacircleof5mradius.

(i) Theareaofthatpartofthefieldinwhich thehorsecangraze Theareathatcanbe grazed by horse = area of sector

$= \frac{90^o}{360^o}{}_2 = \frac{1}{4} \times (5)^2$

$= \frac{25}{4} = 19.62 m^2$

(iii) Areathat canbegrazedbyhorsewhenlengthofropeis 10mlongTherefore,radiusof sector becomes 10 m Area that can be grazed by horse = $\frac{90}{360} \times \times (10)^2$

$= \frac{1}{4} \times \times 100$

$=25$ Hence increase ingrazingarea$=25 \quad -_{25}$______

$=25\quad(1-\frac{1}{4})$

$=25\times3.14\times\frac{3}{4}=58.87\ cm^2$

9. Abroochismadewithsilverwire intheformof acirclewithdiameter 35mm.Thewireisalsoused in making 5 diameters which divide the circle into 10 equal sectors as shown in figure. Find. (i) Thetotallengthofthesilverwirerequired.

(ii) Theareaofeach sectorofthebrooch

Solution:

(i) Thetotallengthofthesilverwirerequired.

Here,totallengthofwirerequiredwillbe=lengthof 5diameters+circumferenceofbrooch. Given diameter of circle = 35 mm

⇒Radiusofcircle= $\frac{35}{2}$ mm

Weknow,circumferenceofcircle=2

Therefore,circumferenceofbrooch=2 $(\frac{35}{2})$

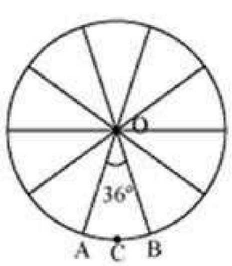
$=2\times\frac{22}{7}\times(\frac{35}{2})$

=110mm

Totallength of silverwirerequired=110+5×35

=110+175=285mm

(ii) Eachof10sectorsofcirclesubtend $\frac{360^o}{10}=36^o$ atcentre ofcircle.



Weknow,areaofsectorofangle =360 × 2

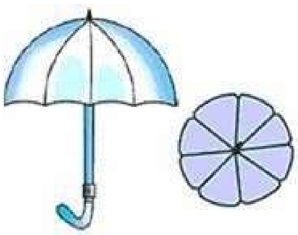
So,areaofeachsector =360 36oo× 2

$$= \frac{1}{10} \times \frac{22}{7} \times \left(\frac{35}{2}\right) \times \left(\frac{35}{2}\right)$$

$$= \frac{385}{4}\ mm^2$$

$$= 96.25\ mm^2$$

10. An umbrella has 8 ribs which are equally spaced as shown in figure. Assuming umbrella to be a flat circle of radius 45 cm, find the area between the two consecutive ribs of the umbrella.



Solution:

Given there are 8 ribs in umbrella.

Each of 8 sectors of circle subtend $\frac{360^\circ}{8} = 45^\circ$ at centre of circle.

We know, area of sector of angle $\theta = \frac{\theta}{360} \times \pi r^2$

So, area between two consecutive ribs of circle $= \frac{45^\circ}{360^\circ} \times \pi r^2$

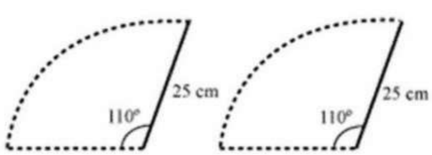
$$= \frac{1}{8} \times \frac{22}{7} \times (45)^2$$

$$= \frac{11}{28} \times 2025 = \frac{22275}{28}\ cm^2$$

$$= 795.8\ cm^2$$

11. A car has two wipers which do not overlap. Each wiper has blade of length 25 cm sweeping through an angle of 115°. Find the total area cleaned at each sweep of the blades.

Solution:



Given that each blade of wiper will sweep an area of a sector of 115° in a circle of 25 cm radius. We know, area of sector of angle $\theta = \frac{\theta}{360} \times \pi r^2$

Here $\theta = 115^\circ$ and $r = 25$

Areaofsuchsector= $\frac{115}{360} \times \times (25)^2$

$= \frac{23}{72} \times \frac{22}{7} \times 25 \times 25$

$= \frac{158125}{252} cm^2$

Hence area swept by 2blades $= 2 \times \frac{158125}{252}$

$= \frac{158125}{126} cm^2$

$= 1254.96 cm^2$

12. Towarnshipsforunderwaterrocks,alighthousespreadsaredcolouredlightoverasectorofangle 80° to a distance of 16.5 km. Find the area of the sea over which the ships warned. (Use $=3.14$)

Solution:

Givenlighthousespreadslightover asector ofangle $80°$ in acircleof $16.5 km$ radius

Weknow,area ofsectorofangle $= \frac{}{360} \times {}^2$

Here $=80^\circ$ and $=16.5$

Areaofsector OACB= $\frac{80^{\circ}}{360^{\circ}} \times {}^2$

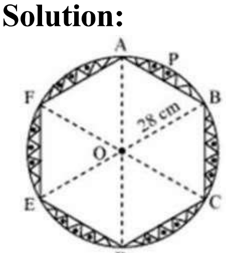
$= \frac{2}{9} \times 3.14 \times 16.5 \times 16.5$

$= 189.97 km^2$

Therefore,areaoftheseaovershipswarnedis $189.97 km^2$

13. Aroundtablecoverhassixequaldesignsasshowninfigure.Iftheradiusofthecoveris 28cm, find the cost of making the designs at the rate of Rs. 0.35 per cm^2.(Use $\sqrt{3} = 1.7$)

Solution:



Designsaresegmentsofcircle.

Consider segment APB. Chord AB is a side of hexagon. Each chord will subtend $\frac{360^\circ}{6}=60^\circ$ at centreof circle.

In ΔOAB

∠OAB = ∠OBA (as OA=OB)

And ∠AOB = 60°

Weknow,sumofinterioranglesoftriangle=180°

∠OAB+ ∠OBA+ ∠AOB=180°

2∠OAB=180°−60°=120°

∠OAB=60°

So, ΔOAB is an equilateral triangle

HenceareaofΔOAB= $\frac{\sqrt{3}}{4}\times(\text{side})^2$

$$= \frac{\sqrt{3}}{4}\times(28)^2 = 196\sqrt{3}\ \text{cm}^2 = 333.2\ \text{cm}^2.$$

Weknow,area ofsectorofangle $\theta = \frac{\theta}{360}\times\pi r^2$

Here $\theta=60^\circ$ and $r=28$

AreaofsectorOAPB= $\frac{60}{360}\times\pi r^2$

$= \frac{1}{6}\times\frac{22}{7}\times 28\times 28$

$=\frac{1232}{3}=410.66\text{cm}^2$

AreaofsegmentAPB=AreaofsectorOAPB−AreaofΔOAB

=410.66− 333.2

$=77.46\text{cm}^2$

Hence,totalareaofdesigns= 6 ×77.46=464.76 cm^2

Costoccurredinmaking 1cm^2 designs=Rs.0.35

Cost occurred in making 464.76 cm^2 designs =464.76 ×0.35 =162.66 So, cost of making such designs is Rs. 162.66.

14. Tickthecorrectanswerinthefollowing:

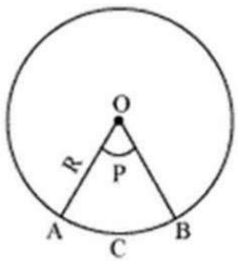
Areaofasectorofangle p (indegrees)ofacirclewithradiusRis

(A) $\frac{}{180} \times 2 \quad R$

(B) $\frac{}{180} \times \quad R^2$

(C) $\frac{}{360} \times 2 \quad R$

(D) $\frac{}{720} \times 2 \quad R^2$



We know that area of sector of angle $= \frac{}{360}$

$^{o} \quad r^2$

Here $=$ and $r = R$ Hence, Area of sector of angle $=$

$_{o}(R^2)$

360

$=(720 \quad \frac{}{}_{o})(2 \quad R^2)$

Chapter13
SURFACEAREAANDVOLUMES
EXERCISE 13.1

(Unlessstatedotherwise,take$\pi=\frac{22}{7}$)

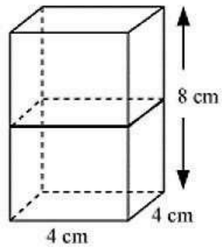
1. 2cubeseach of volume $64cm^3$ arejoinedend to end. Findthesurfaceareaoftheresulting cuboid.

Solution:

Volumeofcube$=64cm^3$. Letthesidelength of thecubebe . Then

$\Rightarrow \quad ^3=64$

$\Rightarrow \quad =4cm$



Ifcubesarejoinedendto endasshownintheadjacentfigure,thenthedimensionsofresulting cuboid will be 4 cm, 4cm, 8 cm.

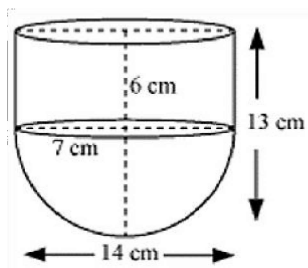
We know surface area of cuboid=2(+ h+ h) where is length, is breadth and h is height of the cuboid. Hence area of the cuboid = 2(4 × 4 + 4×8+4×8)

=2(16+ 32+ 32)

=2×80=160cm2

2. A vessel isin the form of a hollow hemisphere mounted by a hollowcylinder. The diameter of thehemisphere is 14 cm and the total height of the vessel is 13 cm. Findtheinnersurfaceareaofthevessel.

Solution:



Lettheradius ofthecylindricalpartas wellas hemisphericalpartbe .

Giventhediameterofthehemispherepartis14cm.

⇒2 =14

=7cm

Herewecanalsosayheightofhemisphereis7cmwhichis radius ofthehemisphere.

Now lettheheightofthecylindricalpartbe h.Given heightofthevesselis 13cm,henceheightof the cylindrical part = 13 − 7 = 6cm.

Weknow,innersurfaceareaofthehemisphere=$2\pi r^2$and innersurfaceareaof thecylinder= $2\pi rh$.

Total innersurface areaof thevessel=Innersurface areaofthehemisphere+Innersurfaceareaof the cylindrical part

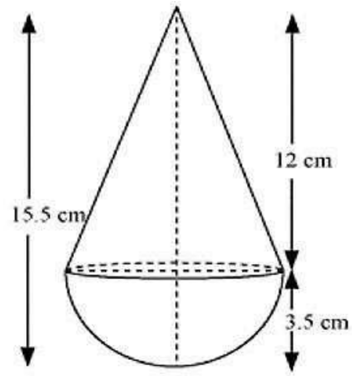
Totalinnersurfaceareaofthevessel=$2\pi r^2+2\pi rh$.

Totalinnersurfaceareaof thevessel$=2\times\frac{22}{7}\times7\times7+2\times\frac{22}{7}\times7\times6$

$=44(7+6)=572cm^2$.

3. A toy is in the form of a cone of radius 3.5cm mounted on a hemisphere of same radius. The total height of the toy is 15.5 cm. Find the total surface area of the toy.

Solution:



Clearly radius of cone as well hemisphere is same. Let it be r=3.5cm Height of the hemisphere = radius of hemisphere (r)=3.5cm. Height of toy is given 15.5 cm.

Hence height of cone (h)= 15.5−3.5= 12cm

Slant height ( )of cone $= \sqrt{r^2 + h^2}$

$= \sqrt{\left(\frac{7}{2}\right)^2 + (12)^2} = \sqrt{\frac{49}{4} + 144} = \sqrt{\frac{49 + 576}{4}}$

$= \sqrt{\frac{625}{4}} = \frac{25}{2} = 12.5$

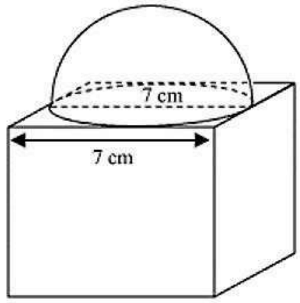
Total surface area of toy = CSA of cone + CSA of hemisphere Total surface area of toy = $\pi rl + 2\pi r^2$

$= \frac{22}{7} \times \frac{7}{2} \times \frac{25}{2} + 2 \times \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2}$

$= 137.5 + 77 = 214.5 cm^2$

4. A cubical block of side 7cm is surmounted by a hemisphere. What is the greatest diameter the hemisphere can have? Find the surface area of the solid.

Solution:



Given a cubical block is surmounted by a hemisphere as in given figure, hence we can say the greatest diameter will be equal to cube's edge = 7 cm.

Hence radius(r) of hemisphere $= \frac{7}{2} = 3.5$ cm (i)

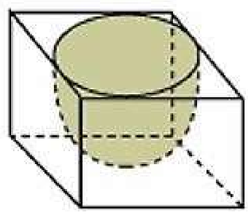
Total surface area of solid = surface area of cube + CSA of hemisphere − area of base of hemisphere

$= 6(\text{edge})^2 + 2\pi r^2 - \pi r^2 = 6(\text{edge})^2 + \pi r^2$

Total surface area of solid $= 6(7)^2 + \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = 294 + 38.5 = 332.5\ \text{cm}^2$

5. A hemispherical depression is cut out from one face of a cubical wooden block such that the diameter of the hemisphere is equal to the edge of the cube.
Determine the surface area of the remaining solid.

Solution:



Clearly diameter of hemisphere = edge of cube = andradius of hemisphere =.2

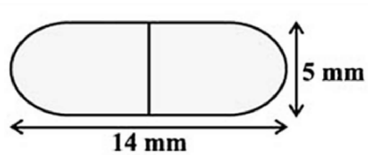
Total surface area of solid = surface area of cube + CSA of hemisphere − area of base of hemisphere

$= 6(\text{edge})^2 + 2\pi r^2 - \pi r^2 = 6(\text{edge})^2 + \pi r^2$

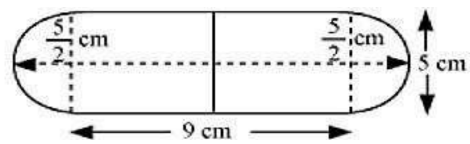
Total surface area of solid $= 6 \quad ^2 + \quad \times \quad 22$

$= 6 \quad ^2 + \frac{\pi \quad ^2}{4}$ unit$^2 = (24 + \frac{\quad ^2}{4}) \quad ^2$.

6. A medicine capsule is in the shape of a cylinder with two hemispheres stuck to each of its ends (see Fig.). The length of the entire capsule is 14mm and the diameter of the capsule is 5mm. Find its surface area.



Solution:



Wecanobservethatinmedicinecapsuleradiusofcylinderisequaltoradiusofhemisphere.

Givendiameterof capsuleis 5mm,henceradiusofcylinder=radiusofhemisphere(r)=$\frac{5}{2}$mm. Let the length of the cylinder be h.

Givenlengthoftheentirecapsuleis14mm.

Hence lengthofcylinder(h)=lengthoftheentire capsule−(2×r)

=14−5=9cm

Surfaceareaofcapsule=2×CSAofhemisphere+CSAofcylinder

$=2\times2\pi r^2+2\pi rh$

$= 4 \frac{5^2}{2} + \frac{5}{2}(9)$

=25 +45

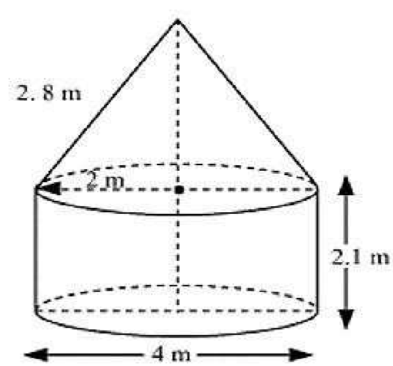
$=70 \ mm^2$

$= 70 \times \frac{22}{7}$

$=220mm^2$

7. A tent is in the shape of a cylinder surmounted by a conical top. If the height and diameter of the cylindricalpartare2.1mand4mrespectively, andtheslantheightofthetopis2.8m,findthearea of the canvas used for making the tent. Also, find the cost of the canvas of the tent at the rate of ₹ 500 per m^2. (Note that the base of the tent will not be covered with canvas.)

Solution:



Itisgiventhat,

Theheight(h)ofthecylindricalpart=2.1 m

Thediameter of thecylindricalpart=4m

Therefore,theradius ofthecylindricalpart=2m

Theslantheight(l)ofconicalpart=2.8m

Theareaofcanvasusedformakingthetent=CSAofconicalpart+CSAofcylindricalpart

$=\pi rl+2\pi rh$

$=\quad \times 2\times 2.8+2 \quad \times 2\times 2.1$

$=2[2.8+2\times 2.1]=2[2.8+4.2]=2\times \frac{22}{7}\times 7$

$=44m^2$

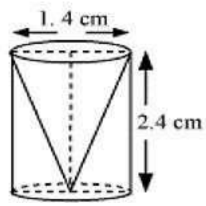
Costof1m^2canvas= 500

Costof44m^2canvas=44×500= 22000

So,itwillcost 22000formakingsuchatent.

8. From a solid cylinder whose height is 2.4 cm anddiameter 1.4 cm, a conical cavity of the same height andsamediameterishollowedout. Findthetotalsurfaceareaoftheremainingsolid tothenearest cm^2.

Solution:



Itisgiventhat,

Theheight(h)oftheconicalpart=height(h)ofthecylindricalpart=2.4cm

Thediameter of thecylindricalpart=1.4cm

Therefore, the radius (r)of the cylindrical part =0.7cm The slant height (l)of conical part = $\sqrt{r^2+h^2}$

$$=\sqrt{(0.7)^2+(2.4)^2}=\sqrt{0.49+5.76}=\sqrt{6.25}=2.5cm$$

Thetotalsurfaceareaoftheremainingsolidwillbe=CSAofcylindricalpart +CSAofconicalpart+areaofcylindricalbase

$=2\pi rh+\pi rl+\pi r^2$

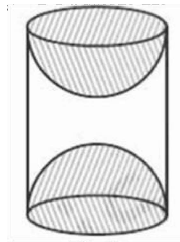
$=2\times \frac{22}{7}\times 0.7\times 2.4+\frac{22}{7}\times 0.7\times 2.5+\frac{22}{7}\times 0.7\times 0.7$

$=(4.4\times 2.4)+(2.2\times 2.5)+(2.2\times 0.7)$

$=10.56+5.50+1.54=17.60cm^2$

Itisclear thatthetotalsurfaceareaof theremainingsolid to thenearestcm^2is18cm^2.

9. Awoodenarticlewasmadebyscoopingoutahemispherefrom eachendof asolidcylinder, asshownin figure. If the height of the cylinder is 10 cm, and its base is of radius 3.5 cm, find the total surface area of the article.



Solution:

Fromthegivendiagramit canbeclearlyobservedthatradiusofhemisphere issameasradiusof cylinder.

Givenradiusofcylindricalpart=radiusofhemisphericalpart=r=3.5cm. Also, height of the cylinder = 10 cm.

Thetotalsurfaceareaofthearticle=(CSAofcylinder)+(2×CSAofhemisphere)

$=(2\pi rh)+(2\times 2\pi r^2)$

$=(2\ \ \times 3.5\times 10)+(2\times 2\ \ \times 3.5\times 3.5)$

$=70\ \ +49$

$= 119\times\frac{22}{7}$

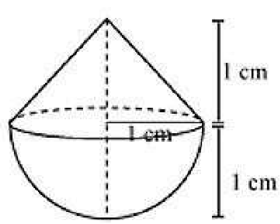
$=17\times 22=374 cm^2$

EXERCISE 13.2

(Unlessstatedotherwise,take$\pi=\frac{22}{7}$)

1. Asolidisintheshapeof aconestandingonahemispherewithboththeirradiibeingequalto 1cmand the height of the cone is equal to its radius. Find the volume of the solid in terms of π.

Solution:



Letthe$h$betheheightandrbetheradius ofthecone.As pergivencondition$h$= =1cm

………(i)

Also,inthegivensolidwecan easilyobserve thatradiusofthecone andradiusofhemisphereare equal.

Radiusofhemisphere=radiusofcone= =1cm (ii)

Inthehemispherewecanalsosaythatradiusas wellasheightofhemisphereareequal.

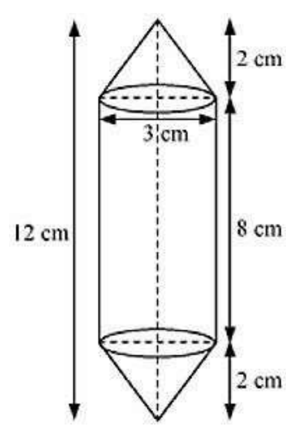
Radiusofhemisphere( )=heightofhemisphere(h)
=1cm (iii)

Volumeofsolid=volumeofcone+volumeofhemisphere

$=\frac{1}{3}\quad{}^{2}h+\frac{2}{3}\quad{}^{3}$

$=\frac{1}{3}(1)^2(1)+\frac{2}{3}(1)=\quad\frac{}{3}+\frac{}{3}=\quad cm^3$

2. Rachel, an engineering student, was asked to make a model shaped like a cylinder with two cones attached at its two ends by using a thin aluminum sheet. The diameter of the model is 3 cm and its lengthis12cm.Ifeachconehasaheightof2cm,findthevolumeofair containedinthemodelthat Rachel made. (Assume the outer and inner dimensions of the model to be nearly the same.)**Solution:**



Let$h_1$betheheightof thecone and $h_2$betheheight ofthecylinder.Inthegivenmodelradius of cone and cylinder are same, hence let it be .

Given heightofthecone h_1=2cm (i)

Givenheightofthemodelis12cm,henceheight(h_2)ofcylinder

=12−2×heightof cone

=12−2×2=8cm (ii)

Radiusof cylinder= Radiusof cone=( )=$\frac{3}{2}$cm (iii)

Volume of air present in the model = volume of cylinder $+2\times$ volume of cones

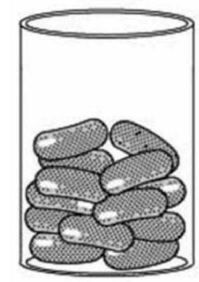
$= \pi r^2 h_2 + 2 \times \frac{1}{3} \pi r^2 h_1$

$= \left(\frac{3}{2}\right)^2 (8) + 2 \times \frac{1}{3} \left(\frac{3}{2}\right)^2 (2)$

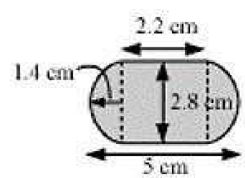
$= \times \frac{9}{4} \times 8 + \frac{2}{3} \times \frac{9}{4} \times 2$

$= 18 + 3 = 21 = 66 cm^2$

3. A , contains sugar syrup up to about 30% of its volume. Find approximately how much syrup would be found in 45 , each shaped like a cylinder with two hemispherical ends with length 5 cm and diameter 2.8 cm (see Fig.).



Solution:



Radius( ) of cylindrical part = radius( ) of hemispherical part $= \frac{2.8}{2} = 1.4 cm$

Length (h) of cylindrical part $= 5 - 2\times$ length of hemispherical part

$= 5 - 2 \times 1.4 = 2.2 cm$

Volume of one gulabjamun = volume of cylindrical part $+2\times$ volume of hemispherical part

$= \pi r^2 h + 2 \times \frac{2}{3} \pi r^3 = \pi r^2 h + \frac{4}{3} \pi r^3 = \pi r$

$= \times (1.4)^2 \times 2.2 + \frac{4}{3} (1.4)^3$

$= \frac{22}{7} \times 1.4 \times 1.4 \times 2.2 + \frac{4}{3} \times \frac{22}{7} \times 1.4 \times 1.4 \times 1.4$

$= 13.55 + 11.50 = 25.05 cm^3$

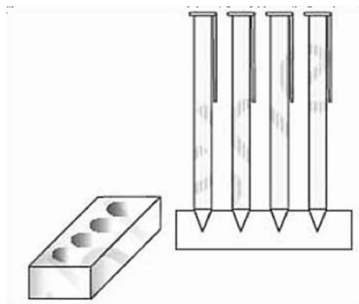
Volume of 45 gulabjamus $= 45 \times 25.05 = 1,127.25 cm^3$

Volume of sugar syrup = 30% of volume

$= \frac{30}{100} \times 1{,}127.279$

$\approx 338 cm^3$

4. A pen stand made of wood isin the shape of a cuboid with four conical depressions to hold pens.Thedimensionsofthecuboidare15cmby10cmby3.5cm.Theradiusofeachofthedepressions is 0.5cm and the depth is 1.4 cm. Find the volume of wood in the entire stand (see figure).



Solution:

Itisgiventhat,

The depth (h_1) of each conical depression depression =1.4cm The radius ( ) of

each conical depression = 0.5 cm

The length ( ) of cuboid = 15

cm The breadth ( ) of cuboid

=10cm The height (h) of cuboid =

3.5 cm

Volumeofwoodinstand = volumeofcuboid−(4×volumeofcones)

$=(lbh)-(4 \times \frac{1}{3} \pi r^2 h_1)$

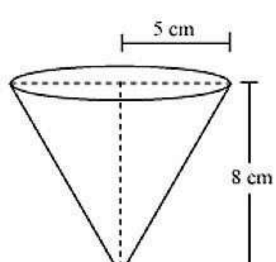
$=(15\times10\times3.5)-(4\times\frac{1}{3}\times\frac{22}{7}\times\frac{1}{2}^2\times1.4)$

=525−1.466

$=523.53 cm^3$

5. Avesselisintheformof aninvertedcone.Itsheight is 8cmandtheradiusofitstop,whichisopen, is5cm. It isfilled with water upto the brim. When lead shots, each of which isasphere of radius 0.5cm are dropped into the vessel, one-fourth of the water flows out. Find the number of lead shots dropped in the vessel.

Solution:



Itisgiventhat,

The height (h) of conical vessel = 8 cm

Theradius ($_1$) of conicalvessel=5cm

The radius ($_2$) of lead shot = 0.5 cm

Let leadshotsweredroppedinthevessel

Thenthe volumeofwaterspilled=volumeofdroppedleadshots

Which implies, $\frac{1}{4}$ × volume of cone = $\frac{4}{3}$ × $_3$ $_2$ (Given:Whenleadshotsare dropped, one-fourth of thewaterinvesselflowsout)

$$\frac{1}{4} \times \frac{1}{3}\pi r_1^2 h = \times \frac{4}{3} \quad {}_2{}^3$$

$$_1{}^2 h = n \times 16 \quad {}_2{}^3$$

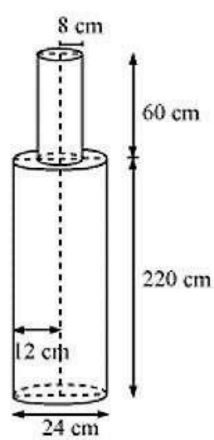
$$5^2 \times 8 = \times 16 \times (0.5)^3$$

$$n = \frac{25 \times 8}{16 \times \frac{1}{2}{}^3} = 100$$

Therefore,thenumberofleadshotsdroppedinthevesselare100.

6. A solid iron pole consists of a cylinder of height 220cm and base diameter 24cm, which is surmounted by another cylinder of height 60cm and radius 8cm. Find the mass of the pole, given that 1 cm^3 of iron has approximately 8g mass. (Use π = 3.14)

Solution:



Fromthefigurewehaveheight(h_1)oflargercylinder= 220cm

Itisgiventhat,

Theradius($_1$)oflargercylinder=$\frac{24}{2}$=12cm

Theheight(h_2)ofsmallercylinder=60cm

Theradius($_2$)ofsmallercylinder=8cm

Thetotalvolumeofironpole =volumeoflargercylinder +volumeofsmallercylinder

$= \quad {}_{12}h_1 + \quad {}_{22}h_2$

$=(12)_2\times220+(8)_2\times60$

$=[144\times220+64\times60]$

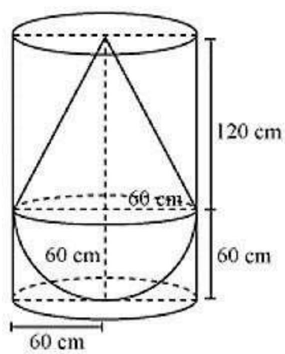
$=35520\times3.14=1,11,532.8cm_3$

Massof1cm_3iron=8g

Therefore,themassof111532.8cm_3iron=111532.8×8=892262.4g=892.26kg.

7. A solid consisting of a right circular cone of height 120cm and radius 60cm standing on a hemisphere of radius60cm is placed upright in a right circular cylinder full of water such that it touches the bottom. Find the volume of water left in the cylinder, if the radius of the cylinder is60 cm and its height is 180 cm.

Solution:



From the figure,the radius( )ofhemispherical part=radius( )ofconical part
=60cm

Itisgiventhat,

Theheight(h)ofrightcircularcone=120cm

Theheight(h_1)ofcylinder= 180cm

Theradius( )ofcylinder= 60cm

Thevolumeofwaterleftinthecylinder=volumeofcylinder−volumeofsolid

=volumeofcylinder−(volumeofcone+volume ofhemisphere)

$=\pi r_2h_1-\frac{1}{3}\pi r_2h+\frac{2}{3}\pi r_3$

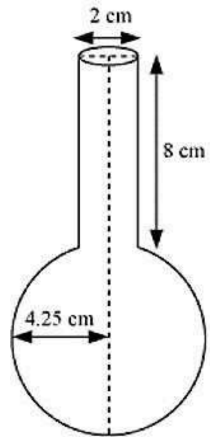
$=(60)_2(180)-\frac{1}{3}(60)_2\times120+\frac{2}{3}(60)_3$

$=(60)_2[(180)-(40+40)]$

$=(3600)(100)=360000\pi cm_3=1131428.57cm_3=1.131m_3$(approx.)

8. A spherical glass vessel has a cylindrical neck 8cm long, 2cm in diameter; the diameter of the spherical part is 8.5cm. By measuring the amount of water it holds, a child finds its volume to be $345 cm^3$. Check whether she is correct, taking the above as the inside measurements, and π = 3.14.

Solution:



Itisgiventhat,

Theheight(h)ofcylindricalpartofglassvessel=8cm

The radius ($_2$) of cylindrical part of glass vessel= $\frac{2}{2}$= 1 cm

Theradius ($_1$) of sphericalpartof glass vessel = $\frac{8.5}{2}$=4.25cm

Volume of vessel = volume of sphere + volume of cylinder

$$=\frac{4}{3}\,_1^3 + \,_2^2h$$

$$=\frac{4}{3}\left(\frac{8.5}{2}\right)^3 + (1)^2(8)$$

=321.39+ 25.12

$=346.51 cm^3$

Hence,thevolumeobtainedbythechildisnotcorrect.Thecorrectansweris $346.51 cm^3$.

Chapter14

Statistics

1. Asurveywasconductedbyagroupofstudentsasa partoftheirenvironmentawarenessprogram,in which they collected the following data regarding the number of plants in 20 houses in a locality. Find the mean number of plants per house.

Numberof plants	0−2	2−4	4−6	6−8	8−10	10−12	12−14
Numberof houses	1	2	1	5	6	2	3

Whichmethoddidyouuseforfindingthemean,andwhy?

Solution:

Firstofall,wehavetofindclassmarks( )foreachinterval.

Weknowthat,classmark( )= $\frac{\text{upperclasslimit+lowerclasslimit}}{2}$

Nowwemaycompute and asfollowing

Numberofplants	Numberofhouses( )		
0−2	1	1	1×1=1
2−4	2	3	2×3=6
4−6	1	5	1×5=5
6−8	5	7	5×7=35
8−10	6	9	6×9=54
10−12	2	11	2×11=22
12−14	3	13	3×13=39
Total			

Fromthetablewecanseethat

Σ =20

Σ =162

Mean $\bar{}$ = $\frac{\Sigma}{\Sigma}$

$= \frac{162}{20} = 8.1$

So,meannumberofplants perhouseis8.1.

Directmethodisusedhereasvaluesofclassmarks( )and aresmall.

2. Consider thefollowingdistributionofdailywagesof50workerofafactory.

Dailywages(in ₹)	500−520	520−540	540−560	560−580	580−600
Numberof workers	12	14	8	6	10

Findthemeandailywages oftheworkersofthefactorybyusinganappropriatemethod.

Solution:

Firstofall,wehavetofind classmarkforeachinterval. We know that,

$= \frac{\text{upperclasslimit+lowerclasslimit}}{}$

Classsize(h)ofthisdata=20

letustake550asassuredmean(   ).Now,wecancalculate   ,   and   asfollowing.

Daily wages (in₹)	Numberof workers ( )		=	− =	� �
500-520	12	510	−40	−2	−24
520−540	14	530	−20	−1	−14
540−560	8	550	0	0	0
560−580	6	570	20	1	6
580−600	10	590	40	2	20
Total	**50**				−

Fromthetablewecanseethat

Σ   =50

Σ   =−12

Mean ̄=   + $\frac{\Sigma}{\Sigma}$ h

$=550+\frac{-12}{50}\,20$

$=550-\frac{24}{5}$

=550−4.8

=545.2

So,meandailywagesoftheworkersofthefactoryis₹545.20

3. Thefollowingdistributionshowsthedailypocket allowanceofchildrenofalocality.Themean pocket allowance is ₹ 18. Find the missing frequency   .

Dailypocket allowance (in₹)	11−13	13−15	15−17	17−19	19−21	21−23	23−25
Numberof workers	7	6	9	13		5	4

Solution:

Firstofall,wehavetofindclassmarkforeachinterval.Weknow that,

$$\text{Class mark}(\quad) = \frac{\text{upper class limit} + \text{lower class limit}}{\quad}$$

Meanpocketallowance $\bar{x}$=₹18………(Given)

Let us take 18 as assured mean ( ). Now we can calculate , and as following.

Dailypocket allowance(in ₹)	Number of children	Class mark	= −	� �
11−13	7	12	−6	−42
13−15	6	14	−4	−24
15−17	9	16	−2	−18
17−19	13	18	0	0
19−21		20	2	2
21−23	5	22	4	20
23−25	4	24	6	24
Total	∑ = +			−

Fromthetablewecanseethat

=44 + (1)

=2 −40 (2) $\bar{x}$= +

_ (3)

Nowputthevaluesof and fromequations(1)and(2)toequation(3)

$18=18+\frac{2\quad -40}{44+}$

Onsolving, weget

$0=\frac{2\quad -40}{44+}$

2 −40=0

2 =40

=20

Hencethemissingfrequency is20.

4. Thirty women were examined in a hospital by a doctor and the number of heart beats per minute wererecordedandsummarizedasfollows.Findthe meanheartbeatsperminuteforthesewomen, choosing a suitable method.

Number of heart beatsper minute	65−68	68−71	71−74	74−77	77−80	80−83	83−86
Number ofwomen	2	4	3	8	7	4	2

Solution:

Firstofall,wehavetofind classmarkforeachinterval. We know that,

Class mark( ) = $\frac{\text{upperclasslimit} + \text{lowerclasslimit}}{2}$

Classsizehofthisdata=3

Letustake75.5asassuredmean( ).Nowwecancalculate , and asfollowing.

Numberof heartbeats per minute	Numberof women		= − .	− . =	� �
65−68	2	66.5	−9	−3	−6
68−71	4	69.5	−6	−2	−8
71−74	3	72.5	−3	−1	−3
74−77	8	75.5	0	0	0
77−80	7	78.5	3	1	7
80−83	4	81.5	6	2	8
83−86	2	84.5	9	3	6
Total					

Now wecanseefromtablethat

=30

=4

Mean $\bar{}$ = + $\frac{\Sigma}{\quad} h\Sigma$

$=75.5+\frac{4}{30}\times 3$

$=75.5+0.4=75.9$

Someanheartbeatsperminuteforthesewomenare75.9beatsperminute.

5. In aretailmarket, fruitvendors wereselling mangoes keptin packing boxes. Theseboxes contained varyingnumberof mangoes.Thefollowingwasthedistributionofmangoesaccordingtothenumber of boxes.

Numberof mangoes	50−52	53−55	56−58	59−61	62−64
Numberof boxes	15	110	135	115	25

Findthemeannumberof mangoeskeptinapacking box.Whichmethodoffindingthemeandidyou choose?

Solution:

Numberofmangoes	**Numberofboxes**
50−52	15
53−55	110
56−58	135
59−61	115
62−64	25

Thereisagapof1between anytwoconsecutiveclassintervals.So,wecansaythatclassintervals are not continuous. In order to remove this problem, we haveto add $\frac{1}{2}$to upper class limit and subtract $\frac{1}{2}$from lower class limit of each interval.

Andclassmark( )maybe obtainedbyusingtherelation

$$\text{Class mark } (\quad) = \frac{\text{upper class limit} + \text{lower class limit}}{2}$$

Classsize(h)ofthisdata=3

Letustake57asassuredmean( ).Nowwecancalculate , and asfollowing.

Class interval			=	=	
49.5− 52.5	15	51	−6	−2	−30
52.5− 55.5	110	54	−3	−1	−110
55.5− 58.5	135	57	0	0	0
58.5− 61.5	115	60	3	1	115

61.5− 64.5	25	63	6	2	50
Total	� �				

Nowwecanseethat

=400

=25

Mean ‾= +

$=57+\frac{25}{400}\times 3$

$=57+\frac{3}{16}=57+0.1875$

=57.1875

=57.19

So,meannumberofmangoes keptinapackingboxis57.19.

Here, we chose step deviation method as values of , are big and also there is a common multiple (3) between all .

6 Thetablebelowshowsthedailyexpenditureonfoodof25householdsinalocality.

Daily expenditure (in ₹)	100−150	150−200	200−250	250−300	300−350
Numberof households	4	5	12	2	2

Findthemeandaily expenditureonfood byasuitablemethod.

Solution:

Firstofall,wehavetofind classmarkforeachinterval. We know that,

$$\text{Class mark}(\quad) = \frac{\text{upperclasslimit}+\text{lowerclasslimit}}{2}$$

Classsize=50

Letustake225asassuredmean( ).Nowwecancalculate , and asfollowing.

Daily expenditure (in ₹)			= −	− =______	

100−150	4	125	−100	−2	−8
150−200	5	175	−50	−1	−5
200−250	12	225	0	0	0
250−300	2	275	50	1	2
300−350	2	325	100	2	4
Total					−

Nowwecanseethat -

=25

=−7

Mean $\overline{}$ = $+ \frac{\Sigma}{\Sigma} h$

$=225+-\frac{7}{25}\times(50)$

$=225-14$

$=211$

So,meandailyexpenditureonfoodis₹211.

7. Tofindouttheconcentrationof $_2$inthe air(in partspermillion, i.e.,ppm), thedatawas collected for 30 localities in a certain city and is presented below:

Concentrationof (inpmm)	**Frequency**
0.00− 0.04	4
0.04− 0.08	9
0.08− 0.12	9
0.12− 0.16	2
0.16− 0.20	4
0.20− 0.24	2

Findthemeanconcentrationof $_2$intheair.

Solution:

Firstofall,wehavetofindclassmarkforeachinterval.Weknow that,

Class mark( ) = $\frac{\text{upper class limit} + \text{lower class limit}}{}$

2

Letustake0.14asassuredmean( ).Nowwecancalculate , and asfollowing.

Concentration of (inppm)	Frequency	Class mark	= −	=	
0.00− 0.04	4	0.02	−0.12	−3	−12
0.04− 0.08	9	0.06	−0.08	−2	−18
0.08− 0.12	9	0.10	−0.04	−1	−9
0.12− 0.16	2	0.14	0	0	0
0.16− 0.20	4	0.18	0.04	1	4
0.20− 0.24	2	0.22	0.08	2	4
Total					−

Forthetablewecanseethat

=30

=−31

Mean, $\bar{} = + \frac{\Sigma}{\Sigma} h$

$= 0.14 + -\frac{31}{30} \times (0.04)$

$= 0.14 - 0.04133 = 0.09867$

$= 0.099$ppm

Hence,meanconcentrationof $_2$inthe airis0.099ppm.

8. Aclassteacherhasthefollowingabsenteerecordof 40 studentsofaclassforthewholeterm. Find the mean number of days a student was absent.

Number of days	0−6	6−10	10−14	14−20	20−28	28−38	38−40
Number of students	11	10	7	4	4	3	1

Solution:

Firstofall,wehavetofind classmarkforeachinterval. We know that,

upperclasslimit+lowerclasslimit

Classmark( )=______________________________

2

Letustake17asassumedmean(a),wecancalculate and asfollowing

Numberofdays	Numberof students		= −	
0−6	11	3	−14	−154
6−10	10	8	−9	−90
10−14	7	12	−5	−35
14−20	4	17	0	0
20−28	4	24	7	28
28−38	3	33	16	48
38−40	1	39	22	22
Total				− � �

Nowwecanseethat

=40

=−181

Mean ‾= +Σ/Σ

$=17+\frac{-181}{40}$

=17−4.525

=12.475

=12.48

Hence,meannumberofdaysforwhichastudentwas absentis 12.48days.

9. Thefollowingtablegivestheliteracy rate(in percentage) of35cities.Find themean literacy rate.

Literacy rate(in%)	45−55	55−65	65−75	75−85	85−95
Numberof cities	3	10	11	8	3

Solution:

Firstofall,wehavetofind classmarkforeachinterval. We

know that,

upperclasslimit+lowerclasslimit

Classmark( )=______________________________

2

Classsize( )forthisdata=10

Let us take 70 as assumed mean ( ), we can calculate , , and

as following

Literacyrate (in %)	Numberof cities		=	=	� �
45−55	3	50	−20	−2	−6
55−65	10	60	−10	−1	−10
65−75	11	70	0	0	0
75−85	8	80	10	1	8
85−95	3	90	20	2	6
� �					−

Nowwecanseethat

=35

=−2

Mean $\bar{\ } = \ + \frac{\Sigma}{\Sigma} h$

$=70+-\frac{2}{35}\times(10)$

$= 70 - \frac{20}{35}$

=70−0.57

=69.43

So,meanliteracyrate is69.43%.

EXERCISE14.2

1. Thefollowingtableshows theages ofthepatients admittedinahospitalduringayear:

Age(inyears)	5−15	15−25	25−35	35−45	45−55	55−65

Numberofpatients	6	11	21	23	14	5

Findthemodeandthemeanof thedatagiven above. Compareand interpretthetwomeasuresof central tendency.

Solution:

Firstofall,wehavetofind classmarkforeachinterval. We

know that,

$$= \frac{\text{upperclasslimit} + \text{lowerclasslimit}}{2}$$

Now,letustake30asassumedmean( ),wecancalculate and asfollowing.

Age(inyears)	Numberof patients	Class mark	= −	� �
5−15	6	10	−20	−120
15−25	11	20	−10	−110
25−35	21	30	0	0
35−45	23	40	10	230
45−55	14	50	20	280
55−65	5	60	30	150

Fromthetablewemayobservethat

Σ =80

Σ =430Mean ‾= +

$$= 30 + \frac{430}{80}$$

=30+5.375

=35.375

≈35.38

Clearly, meanof this datais 35.38whichmeans that theaverageageof apatient admitted tohospital was 35.38 years.

Aswecanseethatmaximumclassfrequency is23whichbelongstoclassinterval35−45.

So,modalclass=35−45

Lowerlimit(l)ofmodalclass=35Frequency($_1$)ofmodal

class = 23

Classsize(h)=10

Frequency($_0$)ofclassprecedingthemodalclass=21

Frequency($_2$)ofclasssucceedingthemodalclass=14

Now mode=l+__________ $_1$− $_0$× h

2 $_1$− $_0$− $_2$

$$=35+\frac{23-21}{2(23)-21-14}\times 10$$

$$=35+\frac{20}{11}$$

=35+1.81

=36.8

Clearly,modeis36.8.Itrepresents thatmaximumnumberofpatients admittedinhospitalwereof 36.8years.

2. Thefollowingdatagivestheinformationontheobservedlifetimes (inhours)of

electricalcomponents:

Lifetimes(in hours)	0−20	20−40	40−60	60−80	80 −100	100−120
Frequency	10	35	52	61	38	29

Determinethe modallifetimesofthecomponents.

Solution:

Fromthegivendata,itis clearthatmaximumclassfrequencyis61whichbelongstoclass interval 60−80.

So,modalclass=60−80

Lowerclasslimit( )ofmodalclass=60

Frequency($_1$)ofmodalclass=61

Frequency($_0$)ofclassprecedingthemodalclass=52

Frequency($_2$)ofclasssucceedingthemodalclass=38 Class

size (h) = 20

Mode= +_______ $_1$− $_0$×h

2 $_1$− $_0$− $_2$

$$=60+\frac{61-52}{2(61)-52-38}\times(20)$$

$$=60+\frac{9}{122-90}(20)$$

$$=60+\frac{9}{32}(20)$$

$=60+\frac{90}{16}=60+5.625$

$=65.625$

So,modallifetimeofelectricalcomponents is 65.625hours.

3. Thefollowingdatagives thedistributionoftotalmonthlyhouseholdexpenditureof familiesofavillage.Findthemodalmonthlyexpenditureofthefamilies. Also,

find the mean monthly expenditure.

Expenditure (in ₹)	**Numberoffamilies**
1000−1500	24
1500−2000	40
2000−2500	33
2500−3000	28
3000−3500	30
3500−4000	22
4000−4500	16
4500−5000	7

Solution:

Fromthe givendata,it isclearthat maximum classfrequencyis40whichbelongsto1500−2000 intervals.

So, modal class = 1500 − 2000

Lowerlimit( )ofmodalclass=1500

Frequency ($_1$) of modal class = 40

Frequency($_0$)ofclassprecedingmodalclass=24

Frequency($_2$)ofclasssucceedingmodalclass=33Classsize(h)=500

$$\text{Mode}= \quad + \frac{\quad_1 - \quad_0}{2\quad_1 - \quad_0 - \quad_2} \times h$$

$$=1500+\frac{40-24}{2(40)-24-33}\times(500)$$

$=1500+\frac{16}{80-57}(500)$

$=1500+\frac{8000}{23}$

=1500+347.826

=1847.826=1847.83

Somodalmonthlyexpenditurewas₹1847.83

Now,wehaveto findclassmark

$\text{Classmark}=\frac{\text{upperclasslimit}+\text{lowerclasslimit}}{2}$

Classsize(h)ofgivedata=500

Letustake2750asassumedmean( ),wecancalculate and asfollowing

Expenditure(in₹)	Numberof families		= − � �	− � � =	
1000−1500	24	1250	−1500	−3	−72
1500−2000	40	1750	−1000	−2	−80
2000−2500	33	2250	−500	−1	−33
2500−3000	28	2750	0	0	0
3000−3500	30	3250	500	1	30
3500−4000	22	3750	1000	2	44
4000−4500	16	4250	1500	3	48
4500−5000	7	4750	2000	4	28
Total	� �				− � �

Now,fromthetableitisclearthat Σ =200

Σ =−35

$\overline{\ }(\text{mean}) = a+\frac{\Sigma}{\Sigma}\times h$

$\overline{\ }=2750+\frac{-35}{200}\times 500$

=2750−87.5

=2662.5

So,meanmonthlyexpenditurewas₹2662.50.

4. Thefollowingdistributiongivesthestate-wiseteacher-studentratioinhighersecondaryschoolsof India. Find the mode and mean of this data. Interpret the two measures.

Numberofstudentsper teacher	**Numberofstates/U.T**
15−20	3
20−25	8
25−30	9
30−35	10
35−40	3
40−45	0
45−50	0
50−55	2

Solution:

Fromthegivendata, itisclear thatmaximumclassfrequencyis 10 whichbelongstoclass interval 30 − 35. So, modal class= 30 − 35

Classsize(h)= 5

Lowerlimit( )ofmodalclass=30

Frequency($_1$)ofmodalclass=10

Frequency($_0$)ofclassprecedingmodalclass=9

Frequency($_2$)ofclasssucceedingmodalclass=3

$$\text{Mode} = \; + \frac{{}_1 - {}_0}{2\;{}_1 - {}_0 - {}_2} \times h$$

$$= 30 + \frac{10 - 9}{2(10) - 9 - 3} \times (5)$$

$$= 30 + \frac{1}{20 - 12} 5$$

$$= 30 + \frac{5}{8} = 30.625$$

Mode=30.6

Itrepresents thatteacherstudentratioofmostofstates/U.Tis30.6

Now,wehavetofindoutclass markbyusingtherelation

$$\text{Classmark} = \frac{\text{upperclasslimit} + \text{lowerclasslimit}}{2}$$

Letustake32.5asassumedmean( ),wecancalculate , and asfollowing.

Number of students perteacher	Numberof states/U.T ( )		= - .	− . =______	� �
15−20	3	17.5	−15	−3	-9
20−25	8	22.5	−10	−2	−16
25−30	9	27.5	−5	−1	−9
30−35	10	32.5	0	0	0
35−40	3	37.5	5	1	3
40−45	0	42.5	10	2	0
45−50	0	47.5	15	3	0
50−55	2	52.5	20	4	8
Total				−	

Nowmean ¯= + _ h

$=32.5+-\frac{23}{35}\times5$

$=32.5-\frac{23}{7}=32.5-3.28$

$=29.22$

So mean of datais29.2whichmeansthaton anaverageteacher-studentratiowas29.2

5. Thegivendistributionshowsthenumberofrunsscoredbysometopbatsmenoftheworldinone- day international cricket matches.

Numberofstudents perteacher	**Numberofstates/U.T**
3000−4000	4
4000−5000	18
5000−6000	9

6000−7000	7
7000−8000	6
8000−9000	3
9000−10000	1
10000−11000	1

Findthemodeofthedata.

Solution:

Itisclear fromthegiven datathatmaximumclassfrequency is18which belongsto classinterval 4000−5000.

So,modalclass= 4000−5000

Lowerlimit( )ofmodalclass=4000

Frequency($_1$)ofmodalclass=18

Frequency($_0$)ofclassprecedingmodalclass=4

Frequency($_2$)ofclasssucceedingmodalclass=9Classsize(h)=1000

$$\text{NowMode}=l+\frac{_1-\quad _0}{2\ _1-\ _0-\ _2}\times(h)$$

$$=4000+\frac{18-4}{2(18)-4-9}\times 1000$$

$$=4000+\frac{14000}{23}$$

=4000+608.695

=4608.695

Somodeofgivendatais4608.7runs.

6. Astudentnotedthenumberofcarspassingthroughaspotonaroadfor periodseachof minutesandsummariseditinthetablegivenbelow.Findthemodeofthedata:

Numberof cars	0−10	10−20	20−30	30−40	40−50	50−60	60−70	70−80
Frequency	7	14	13	12	20	11	15	8

Solution:

Itisclearfromthegivendatathatmaximumclassfrequencyis20whichbelongsto40 -50class intervals.

So,modalclass=40−50

Lowerlimit( )ofmodalclass=40

Frequency($_1$)ofmodalclass=20

Frequency($_0$)ofclassprecedingmodalclass=12

Frequency($_2$)ofclasssucceedingmodalclass=11Classsize=10

$$\text{Mode}= \quad + \frac{_1 - \quad _0}{2 \quad _1 - \quad _0 - \quad _2} \times (h)$$

$$=40+\frac{20-12}{2(20)-12-11}\times 10$$

$$=40+\frac{80}{40-23}$$

$$=40+\frac{80}{17}$$

=40+4.7

Somode ofthisdatais44.7cars.

EXERCISE 14.3

1. Thefollowingfrequencydistributiongivesthemonthlyconsumptionofelectricityof68consumers of a locality. Find the median, mean and mode of the data and compare them.

Monthlyconsumption(in units)	**Numberofconsumers**
65 -85	4
85 -105	5
105 -125	13
125 -145	20
145 -165	14
165 -185	8
185 -205	4

Solution:

Firstofall,wehavetofind classmarkforeachinterval. We know that,

$$\text{Classmark}=\frac{\text{Upperclasslimit+Lowerclasslimit}}{2}$$

Letustake135asassumedmean(a),accordingtostepdeviationmethodwemayfind , , × ,asfollowing

Monthly consumption(in units)	**Numberof consumers ( )**	**class mark**	= −	=___	
65−85	4	75	−60	−3	−12

85 −105	5	95	−40	−2	−10
105−125	13	115	−20	−1	−13
125−145	20	135	0	0	0
145−165	14	155	20	1	14
165−185	8	175	40	2	16
185−205	4	195	60	3	12

Wecanobservefromthetable

=7

=68

Classsize(h)= 20

Mean $\bar{}$ = $+\frac{\Sigma}{\Sigma}\times h$

$=135+\frac{7}{68}\times 20$

$=135+\frac{140}{68}$

=137.058

Itisclearfromthetablethatmaximum classfrequencyis20whichbelongsto classinterval125 - 145.

Modalclass =125 -145

Lowerlimit(l)ofmodalclass=125

Class size (h) = 20

Frequency($_1$)ofmodalclass=20

Frequency ($_0$) of class preceding modal class = 13

Frequency($_2$)ofclasssucceedingthemodalclass=14

Mode= $+\frac{_1-\ _0}{2\ _1-\ _0-\ _2}\times h$

$=125+\frac{20-13}{2(20)-13-14}\times 20$

$=125+\frac{7}{13}\times 20$

$$=125+\frac{140}{13}=135.76$$

Weknow that

3median=mode+2mean

= 135.76 +2 (137.058)

= 135.76 +274.116

= 409.876

Median $=\frac{409.876}{3}$

Median=136.625

Somedian,mode,meanofgiven datais 136.625,135.76,137.05 respectively.

2. Ifthemedianofthedistributionisgivenbelow is 28.5,findthevalues ofxandy.

Classinterval	Frequency
0 -10	5
10 -20	x
20 -30	20
30 -40	15
40 -50	y
50 -60	5
Total	**60**

Solution:

Firstofallwehavetofindcumulativefrequencyofthegivendata

Classinterval	Frequency	Cumulativefrequency
0 -10	5	5
10 -20	x	5 +x

20 -30	20	25 +x
30 -40	15	40 +x
40 -50	y	40 +x +y
50 -60	5	45 +x +y
Total(n)	**60**	

Itisclearthatn=60 45 +

+ = 60

+ =15……..(1)

Median of data = 28.5 ……… (given)

Median of datalies in interval 20−30.

So, median class = 20 − 30

Lowerlimit( )ofmedianclass=20

Cumulativefrequency(cf)ofclass precedingthemedianclass=5+

Frequency(f)ofmedianclass=20

Classsize(h)=10

Now,median= $+ \frac{\frac{}{2} -}{} \times h$

$$28.5 = 20 + \frac{\frac{60}{2} - (5+\ )}{20} \times 10$$

$$8.5 = \frac{25-}{2}$$

17=25−

=8

Putthevalueofxinequation(1)

8+ =15

=7

Hencevaluesofxandyare8and7respectively.

3. A life insurance agent found the following data for distribution of ages of 100 policy holders. Calculate themedianage,ifpoliciesaregivenonlytopersonshavingage18yearsonwardsbutless than 60 year.

Age(inyears)	Number ofpolicy holders
Below20	2
Below25	6
Below30	24
Below35	45
Below40	78
Below45	89
Below50	92
Below55	98
Below60	100

Solution:

We can see from above table that class width is not same. We don’t need to adjust the frequencies according toclassintervals.Nowabovefrequencytableisrepresentedwithupperclasslimitsand is of less than type. As policies were given only to persons having age 18 years onwards but less than 60 years, we can define class intervals with their respective cumulative frequency as given below.

Age(inyears)	Numberof policyholders ( )	Cumulativefrequency(cf)
18 – 20	2	2
20 – 25	6−2=4	6
25 – 30	24−6=18	24
30 -35	45−24=21	45
35 -40	78−45=33	78
40 -45	89−78=11	89
45 – 50	92−89=3	92
50 – 55	98−92=6	98
55 – 60	100−98=2	100
Total(n)		

Wecanseefromthe tablethat =100.

$\frac{}{2}$ =50

Cumulativefrequency(cf)justgreater thanis78. ($\frac{}{2}$ =50)

Itbelongstointerval35–40 So,

median class = 35 – 40

Lowerlimit(l)ofmedianclass=35

Classsize(h)= 5

Frequency(f)ofmedianclass=33

Cumulativefrequency(cf)ofclassprecedingmedianclass=45

$\frac{-}{}$ median= $+_2 \times h$

$=35+\frac{50-45}{33}\times 5$

$=35+\frac{25}{33}$ =35.76

So,medianageis 35.76years.

4. Thelengthsof40leavesofaplant aremeasuredcorrectto thenearestmillimeter,andthedata obtained is represented in the following table:

Length(inmm)	Numberor leaves( )
118 – 126	3
127 – 135	5
136 – 144	9
145 – 153	12
154 – 162	5
163 -171	4
172 -180	2

Findthemedianlengthoftheleaves.

(Hint: The formula is for continuous classes only so we have to convert the given data into continuousclassesforfindingthe median. Theclassesthenchangeto117.5 -126.5,126.5-135.5-171.5 - 180.5)

Solution:

The given data does not have continuous class intervals. We can observe from the table that differencebetween twoclassintervalsis1.So,wehavetoadd 0.5 toupperclasslimitsandsubtract 0.5tolowerclass limits.

Nowcontinuousclassintervalswithrespectivecumulativefrequenciescanberepresentedasbelow.

Length(inmm)	Numberor leaves( )	Cumulativefrequency
117.5 -126.5	3	3

126.5 -135.5	5	3 +5 =8
135.5 -144.5	9	8 +9 =17
144.5 -153.5	12	17 +12 =29
153.5 -162.5	5	29 +5 =34
162.5 -171.5	4	34 +4 =38
171.5 -180.5	2	38 +2 =40

Here,n=40

So,=20

2

Wecanobservefromthetablethatcumulativefrequencyjustgreaterthenis 29.

2Itbelongstoclassinterval144.5-153.5. So,

Median class = 144.5 - 153.5

Lowerlimit(l)ofmedianclass=144.5 Class

size (h) = 9

Frequency(f)ofmedianclass=12

Cumulativefrequency(cf)ofclassprecedingmedianclass=17

Median= +2— — $\times h$

$=144.5+\frac{20-17}{12}\times 9$

$=144.5+\frac{9}{4}=146.75$

So,medianlengthofleavesis146.75mm.

5. Findthefollowingtablesgivesthedistributionofthelifetimeof400neonlamps:

Lifetime (inhours)	Numberoflamps
1500 -2000	14
2000 -2500	56
2500 -3000	60
3000 -3500	86
3500 -4000	74
4000 -4500	62
4500 -5000	48

Findthemedianlifetimeofalamp.

Solution:

Firstofall,wehavetofindcumulativefrequencieswiththeirrespectiveclass intervals

Lifetime	Numberoflamps( )	Cumulativefrequency
1500 -2000	14	14
2000 -2500	56	14+56=70
2500 -3000	60	70 +60=130
3000 -3500	86	130+86=216
3500 -4000	74	216+74=290
4000 -4500	62	290+62=352
4500 -5000	48	352+48=400
Total(n)	**400**	

Here, =400

So, $\frac{}{2}$ =200

From thetablewecanobservethatcumulativefrequencyjustgreaterthan

$\frac{-}{2}$ (. $\frac{400}{2}$, =200)is216.

It belongs to class interval 3000 −3500.

So, Median class = 3000 – 3500

Lowerlimit(l)ofmedianclass=3000

Frequency(f)ofmedianclass=86

Cumulativefrequency(cf)ofclassprecedingmedianclass=130

Classsize(h)= 500

Median= +2= ——— × h

$=3000\times\frac{200-130}{86}\times500$

$=3000+\frac{70\times500}{86}$

=3000+406.976

=3406.976

So,medianlifetimeoflampsis3406.98hours.

6. 100 surnames were randomly picked up from a local telephone directory and the frequency distributionof thenumberoflettersin theEnglishalphabetsinthesurnameswasobtainedas follows:

Numberof letters	1 -4	4 -7	7 -10	10 -13	13 -16	16 -19
Numberof surnames	6	30	40	16	4	4

Determinethemediannumberoflettersinthesurnames.Find themeannumberoflettersin the surnames? Also, find the modal size of the surnames.

Solution:

Firstofall,wehavetofindcumulativefrequencieswiththeirrespectiveclass intervals

Numberofletters	**Frequency( )**	**Cumulativefrequency**
1 -4	6	6
4 -7	30	30+6=36
7 -10	40	36+40=76
10 -13	16	76+16=92
13 -16	4	92+4=96
16 -19	4	96+4=100
Total(n)	**100**	

Here, =100.

So, =50

2

Fromthetable,wecanobservethatcumulativefrequencyjustgreater thanis76.Itbelongstoclass interval 7 - 10.

So,Medianclass=7–10

Lowerlimit(l)ofmedianclass=7

Cumulativefrequency(cf)ofclassprecedingmedianclass=36

Frequency(f)ofmedianclass=40

Classsize(h)= 3

–

Median= +2 ×h

$$=7+\frac{50-36}{40}\times 3$$

$$=7+\frac{14\times 3}{40}$$

=7+1.05

=8.05

Nowwehavetofindclass marksofgivenclass intervalsbyusingrelation

$$\text{Classmark}=\frac{\text{Upperclasslimit}+\text{lowerclasslimit}}{2}$$

Taking11.5asassumedmean(a)wecanfinddi,uiandfiuiaccordingtostepdeviation method as below.

Numberofletters	**Number of surnames�** **�**		–	=___ –	� �
1 – 4	6	2.5	−9	−3	−18
4 – 7	30	5.5	−6	−2	−60
7 – 10	40	8.5	−3	−1	−40
10 – 13	16	11.5	0	0	0
13 – 16	4	14.5	3	1	4
16 – 19	4	17 .5	6	2	8
Total					–

=−106

=100

Σ

Mean ‾ = +(______)h

Σ

$=11.5+\frac{-106}{100}\times 3$

$= 11.5-3.18=8.32$

Weknow that

3median=mode+2mean

3(8.05)=mode+2(8.32)

24.15−16.64=mode

7.51=mode

So,mediannumberandmeannumberof lettersinsurnamesis8.05and8.32respectivelywhile modal size of surnames is 7.51.

7. Thedistributionbelowgivestheweightsof30studentsofaclass.Findthe medianweightof the students.

Weight (inkg)	40 -45	45 -50	50 -55	55 -60	60 -65	65 -70	70 -75
Numberof students	2	3	8	6	6	3	2

Solution:

Wemayfindcumulativefrequencieswiththeirrespective classintervalsasbelow

Weight(inkgs)	**Frequency( )**	**Cumulativefrequency**
40-45	2	2
45-50	3	2+3=5
50-55	8	5+8=13
55-60	6	13+6=19
60-65	6	19+6=25
65-70	3	25+3=28
70-75	2	28+2=30
Total(n)		

Here, =30

So, =15

2

Cumulativefrequencyjustgreaterthan i.e.,−15 $\frac{30}{2}$ is 19.

2

Itbelongs toclass interval55-60.

So,Medianclass=55–60

Lowerlimit(l)ofmedianclass=55

Frequency(f)ofmedianclass=6

Cumulativefrequency(cf)ofmedian class=13

Classsize(h)= 5

Median= $+2\frac{-}{\quad}\times h$

$=55+\frac{15-13}{6}\times 5$

$= 55 + \frac{10}{6}$

=56.666

So,medianweightis56.67kg.

EXERCISE 14.4

1. Thefollowingdistributiongives thedailyincomeof50workers ofafactory.

Dailyincome (in ₹)	100-120	120−140	140−160	160−180	180−200
Numberofworkers	12	14	8	6	10

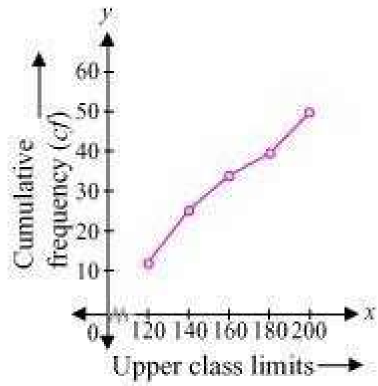
Converttheabovedistribution toalessthantypecumulativefrequencydistribution,anddrawits ogive.

Solution:

Wecan convertthegivendistributiontableintofrequencydistributiontableof lessthantypeas following

Dailyincome (in₹) (upperclass limits)	**Cumulativefrequency**
Lessthan 120	12
Lessthan 140	12+14=26
Lessthan 160	26+8=34
Lessthan 180	34+6=40
Lessthan 200	40+10=50

Now,onx-axiswetakeupperclasslimitsofclassintervalsandtheirrespectivecumulative frequencies on y- axis we can draw its ogive as following



2. Duringthemedicalcheck-upof35students ofaclass,theirweightswererecordedas follows:

Weight(inkg)	Numberofstudents
Lessthan 38	0
Lessthan 40	3
Lessthan 42	5
Lessthan 44	9
Lessthan 46	14
Lessthan 48	28
Lessthan 50	32
Lessthan 52	35

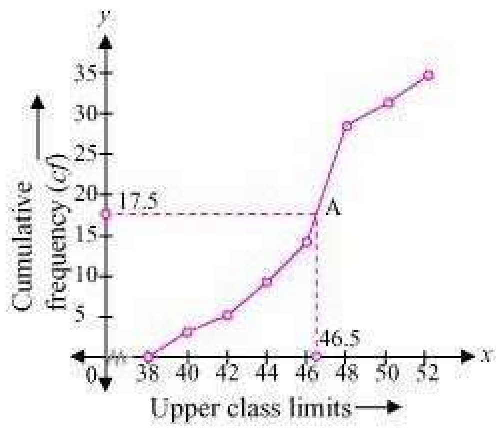
Drawalessthantypeogiveforthegivendata.Henceobtain themedianweightfromthegraph verify the result by using the formula.

Solution:

Thecumulativefrequencydistributionsoflessthantypeisgivenas

Weight (in kg) upperclasslimits	Numberofstudents(cumulative frequency)
Lessthan 38	0
Lessthan 40	3
Lessthan 42	5
Lessthan 44	9
Lessthan 46	14
Lessthan 48	28
Lessthan 50	32
Lessthan 52	35

Now,onx-axiswetakeupperclasslimitsofclassintervalsandtheirrespectivecumulative frequencies on y-axis we can draw its ogive as following



NowmarkthepointAwhoseordinateis 17.5its abscissais46.5.Somedianofthis datais 46.5.

Wecanseefromthe tablethatdifferencebetweentwoconsecutiveupper-classlimitsis2.Nowwe have to obtain class marks with their respective frequencies as below

Weight (inkg)	**Frequency(f)**	**Cumulativefrequency**
Lessthan 38	0	0
38 -40	3−0=3	3
40 -42	5−3=2	5
42 -44	9−5=4	9
44 -46	14−9=5	14
46 -48	28−14=14	28
48 -50	32−28=4	32
50 -52	35−32=3	35
Total(n)		

Here,n=35.

So, $\frac{n}{2}$ =17.5

Nowthecumulativefrequencyjustgreaterthan $\frac{n}{2}$ . .,= $\frac{35}{2}$ 17.5is28.

2Itbelongs to class interval

46 – 48

So,Medianclass=46–48

Lowerclasslimit(l)ofmedianclass=46

Frequency(f)ofmedianclass=14

Cumulativefrequency(cf)ofclassprecedingmedianclass=14

Classsize(h)= 2

Median= +2=______ × h

$=46+\frac{17.5-14}{14}\times 2$

$= 46+\frac{3.5}{7}$

=46+0.5

=46.5

Somedianofthisdatais46.5

Hence,valueofmedianisverified.

3. Thefollowingtablegives productionyieldperhectareofwheatof100farmsofavillage.

Production yield(inkg/ha)	50 -55	55 -60	60 -65	65 -70	70 -75	75 -80
Numberof farms	2	8	12	24	38	16

Changethedistributionto amorethantypedistributionanddrawogive.

Solution:

Wecanobtaincumulativefrequencydistributionofmorethantypeasfollowing

Productionyield(lower class limits)	**Cumulativefrequency**
morethan orequalto50	100
morethan orequalto55	100 −2=98
morethan orequalto60	98−8=90
morethan orequalto65	90−12=78
morethan orequalto70	78−24=54
morethan orequalto75	54−38=16

Now,onx-axiswetakeupperclasslimitsofclassintervalsandtheirrespectivecumulative frequencies on y-axis we can draw its ogive as following.