Holyfaith presentation school.
Rawalpora Srinagar

CLASS:10TH

SUBJECT:Mathematics

SESSION 2024– 2025

Term1st

Chapter5–ArithmeticProgressions

Exercise5.1

1. Inwhichofthefollowingsituations,doesthelistofnumbersinvolvedmakeas arithmetic progression and why?

(i) ThetaxifareaftereachkmwhenthefareisRs15forthefirstkmandRs8for each additional km.

Solution:

Wecan writethegiven condition as;

Taxi fare for 1 km = 15

Taxi fare for first 2 kms = 15+8 = 23

Taxi fare for first 3 kms = 23+8 = 31

Taxi fare for first 4 kms = 31+8 = 39

And so on……

Thus,15,23,31,39…formsanA.P.becauseeverynexttermis8morethanthe precedingterm.

(ii) Theamount ofairpresent inacylinder whenavacuumpumpremoves1/4of theairremaininginthecylinderatatime.

Solution:

LetthevolumeofairinacylinderinitiallybeVlitres.

In each stroke, the vacuum pumpremoves 1/4th of air remaining in the cylinder atatime.Orwecansay,aftereverystroke,1-1/4=3/4thpartofairwillremain.

Therefore,volumeswillbeV,3V/4,$(3V/4)_2$,$(3V/4)_3$…andsoon

Clearly,wecanseehere,theadjacent termsofthisseriesdonot have thecommon difference between them. Therefore, this series is not an A.P.

(iii) The cost of digging a well after every metre of digging, when it costs Rs 150 forthe first metre and rises by Rs 50 for each subsequent metre.

Solution:

Wecanwritethegivenconditionas;

Costofdiggingawellforfirstmetre=Rs.150

Cost of digging a well for first 2 metres = Rs.150+50 = Rs.200

Cost of digging a well for first 3 metres = Rs.200+50 = Rs.250

Cost of digging a well for first 4 metres =Rs.250+50 = Rs.300

And so on..

Clearly,150,200,250,300…formsanA.P.withacommondifferenceof50 betweeneachterm.

(iv) Theamountofmoneyintheaccounteveryyear,whenRs10000isdepositedat compound interest at 8% per annum.

Solution:

We know that if Rs. P is deposited at r% compound interest per annum for n years, the amount of money will be:

$P(1+r/100)_n$

Therefore, after each year, the amount of money will be;

$10000(1+8/100),10000(1+8/100)_2,10000(1+8/100)_3$……

Clearly,thetermsofthis seriesdo not havethecommondifferencebetweenthem. Therefore, this is not an A.P.

2. WritefirstfourtermsoftheA.P.whenthefirsttermaandthecommon difference are given as follows:

(i) a=10,d=10
(ii) a=-2,d=0
(iii) a=4,d=–3

(iv) a=-1d=1/2

(v) a=–1.25,d=–0.25

Solutions:

(i) a= 10,d= 10

Letusconsider,theArithmeticProgressionseriesbea_1,a_2,a_3,a_4,a_5...

a_1=a=10

a_2 =a_1+d=10+10=20 a_3

=a_2+d=20+10=30 a_4

=a_3+d=30+10=40

a_5=a_4+d=40+10=50

Andsoon...

Therefore,theA.P.serieswillbe10,20,30,40,50...

AndFirstfourtermsofthisA.P.willbe10,20,30,and40.

(ii) a=−2,d=0

Letusconsider,theArithmeticProgressionseriesbea_1,a_2,a_3,a_4,a_5...

a_1=a=-2

a_2 =a_1+d=−2+0=−2 a_3

=a_2+d=−2+0 =−2

a_4=a_3+d=−2+0=−2

Therefore,theA.P.serieswillbe −2,−2,−2,−2...

And,FirstfourtermsofthisA.P.willbe−2,−2,−2and−2.

(iii) a=4,d=−3

Letusconsider,theArithmeticProgressionseriesbea_1,a_2,a_3,a_4,a_5...

$a_1=a=4$

$a_2 = a_1+d = 4-3 = 1$

$a_3 =a_2+d=1-3=-2$

$a_4=a_3+d=-2-3=-5$

Therefore,theA.P.serieswillbe4,1,−2−5…

And,first fourtermsofthisA.P.willbe4,1, −2and − 5.

(iv) a=−1,d=1/2

Letusconsider,theArithmeticProgressionseriesbea_1,a_2,a_3,a_4,a_5…

$a_2 = a_1+d = -1+1/2 = -1/2$

$a_3 = a_2+d = -1/2+1/2 = 0$

$a_4 = a_3+d = 0+1/2 = 1/2$

Thus,theA.P.serieswillbe-1,-1/2,0,1/2

AndFirstfourtermsofthisA.P.willbe-1,-1/2,0and1/2.

(v) a=−1.25,d=−0.25

Letusconsider,theArithmeticProgressionseriesbea_1,a_2,a_3,a_4,a_5…

$a_1= a = -1.25$

$a_2= a_1+ d = - 1.25-0.25 = - 1.50$

$a_3= a_2+ d = - 1.50-0.25 = - 1.75$

$a_4=a_3+d=-1.75-0.25=-2.00$

Therefore,theA.Pserieswillbe1.25,− 1.50,−1.75,−2.00……..

Andfirstfourterms ofthisA.P.willbe−1.25,−1.50,−1.75and−2.00.

3. ForthefollowingA.P.s,writethefirsttermandthecommondifference.

(i) 3,1,–1,–3…

(ii) -5,–1,3,7…

(iii) 1/3,5/3,9/3,13/3….

(iv) 0.6,1.7,2.8,3.9…

Solutions

(i) Given series,

3, 1, – 1, – 3 …

Firstterm,a=3

Commondifference,d=Secondterm−Firstterm

⇒1−3=-2

⇒d=-2

(ii) Givenseries,–5,–1,3,7…

Firstterm,a=-5

Commondifference,d=Secondterm−Firstterm

⇒(−1)-(−5)=−1+5=4

(iii) Givenseries,1/3,5/3,9/3,13/3….

Firstterm,a=1/3

Commondifference,d=Secondterm−Firstterm

⇒5/3−1/3=4/3

(iv) Givenseries,0.6,1.7,2.8,3.9…

Firstterm,a=0.6

Commondifference,d=Secondterm−Firstterm

⇒1.7−0.6

⇒1.1

4. Which of the following are APs? If they form an A.P. find the common difference d and write three more terms.

(i) 2,4,8,16…

(ii) 2,5/2,3,7/2….

(iii) -1.2,-3.2,-5.2,-7.2…

(iv) -10,–6,–2,2…

(v) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}$

(vi) 0.2,0.22,0.222,0.2222….

(vii) 0,–4,–8,–12…

(viii) -1/2,-1/2, -1/2, -1/2 ….

(ix) 1,3,9,27…

(x) a,2a,3a,4a…

(xi) a, a^2, a^3, a^4…

(xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}$…

(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}$…

(xiv) $1^2, 3^2, 5^2, 7^2$…

(xv) $1^2, 5^2, 7^2, 7^3$…

Solution

(i) Givento us,

2,4,8,16…

Here,thecommondifferenceis;

$a_2 - a_1 = 4 - 2 = 2$

$a_3 - a_2 = 8 - 4 = 4$

$a_4 - a_3 = 16 - 8 = 8$

Since, $a_{n+1} - a_n$ orthe common difference is not the same every time.

Therefore, the given series are not forming an A.P.

(ii) Given,2,5/2,3,7/2….

Here,

$a_2 - a_1 = 5/2 - 2 = 1/2$

$a_3 - a_2 = 3 - 5/2 = 1/2$

$a_4 - a_3 = 7/2 - 3 = 1/2$

Since, $a_{n+1} - a_n$ orthecommon differenceissame everytime.

Therefore, d = 1/2 and the given series are in A.P.

Thenextthreetermsare; a_5

= 7/2+1/2 = 4

a_6 = 4 +1/2 = 9/2

a_7=9/2+1/2=5

(iii) Given,-1.2,–3.2,-5.2,-7.2…

Here,

$a_2 - a_1$ =(-3.2)-(-1.2)=-2

$a_3 - a_2$ =(-5.2)-(-3.2)=-2

$a_4 - a_3$ =(-7.2)-(-5.2)=-2

Since, $a_{n+1} - a_n$ orcommondifferenceis sameeverytime.

Therefore,d=-2andthegivenseriesareinA.P.

Hence,nextthreetermsare; a_5

= – 7.2-2 = -9.2

a_6=−9.2-2=−11.2

a_7=−11.2-2=−13.2

(iv) Given,-10,–6,–2,2…

Here, the terms and their difference are;

$a_2 - a_1$= (-6)-(-10)= 4

$a_3 - a_2$=(-2)-(-6)= 4

$a_4 - a_3$=(2-(-2)=4

Since, $a_{n+1} - a_n$ or the common difference is same everytime.

Therefore, d = 4 and the given numbers are in A.P.

Hence, next three terms are; a_5

= 2+4 = 6

$a_6=6+4=10$

$a_7=10+4=14$

(v) Given, 3, 3+√2, 3+2√2, 3+3√2

Here,

$a_2-a_1=3+\sqrt{2}-3=\sqrt{2}$

$a_3-a_2=(3+2\sqrt{2})-(3+\sqrt{2})=\sqrt{2}$

$a_4-a_3=(3+3\sqrt{2})-(3+2\sqrt{2})=\sqrt{2}$

Since, $a_{n+1} - a_n$ or the common difference is same everytime.

Therefore, d = √2 and the given series forms a A.P.

Hence, next three terms are; a_5

$=(3+\sqrt{2})+\sqrt{2}=3+4\sqrt{2}$ a_6 =

$(3+4\sqrt{2})+\sqrt{2} = 3+5\sqrt{2}$ a_7 =

$(3+5\sqrt{2})+\sqrt{2} = 3+6\sqrt{2}$

(vi) 0.2, 0.22, 0.222, 0.2222….

Here,

$a_2-a_1=0.22-0.2=0.02$

$a_3-a_2=0.222-0.22=0.002$

$a_4-a_3=0.2222-0.222=0.0002$

Since,$a_{n+1}-a_n$orthecommondifference isnotsame everytime.

Therefore,andthegivenseriesdoesn'tformsaA.P.

(vii) 0,-4,-8,-12…

Here,

$a_2 - a_1 = (-4)-0 = -4$

$a_3-a_2=(-8)-(-4)=-4$

$a_4-a_3=(-12)-(-8)=-4$

Since,$a_{n+1} - a_n$orthe common differenceissame everytime.

Therefore,d=-4andthegivenseriesformsaA.P.

Hence,nextthreetermsare; a_5

$= -12-4 = -16$

$a_6=-16-4=-20$

$a_7=-20-4=-24$

(viii) -1/2,-1/2, -1/2, -1/2 ….

Here,

$a_2-a_1=(-1/2)-(-1/2)=0$

$a_3-a_2=(-1/2)-(-1/2)=0$

$a_4-a_3=(-1/2)-(-1/2)=0$

Since,$a_{n+1} - a_n$orthecommon differenceissame everytime.

Therefore, d = 0 and the given series forms a A.P.

Hence,nextthreetermsare; a_5

$= (-1/2)-0 = -1/2$

a_6=(-1/2)-0=-1/2

a_7=(-1/2)-0=-1/2

(ix) 1,3,9,27…

Here,

$a_2 - a_1$ = 3-1 = 2 a_3

$- a_2$ = 9-3 = 6

a_4-a_3=27-9=18

Since,$a_{n+1}-a_n$orthecommondifference isnotsame everytime.

Therefore,andthegivenseriesdoesn'tformaA.P.

(x) a,2a,3a,4a…

Here,

$a_2 - a_1$ = 2a–a = a

$a_3 - a_2$ = 3a-2a= a

a_4-a_3=4a-3a=a

Since,$a_{n+1} - a_n$orthecommon differenceissame everytime.

Therefore, d = a and the given series forms a A.P.

Hence,nextthreetermsare; a_5

= 4a+a = 5a

a_6=5a+a=6a

a_7=6a+a=7a

(xi) a,a_2,a_3,a_4…

Here,

$a_2-a_1=a_2-a=a(a-1)$

$a_3 - a_2 = a_3 - a_2 = a_2(a-1)$

$a_4 - a_3 = a_4 - a_3 = a_3(a-1)$

Since, $a_{n+1} - a_n$ or the common difference is not same everytime.

Therefore, the given series doesn't forms a A.P.

(xii) √2,√8,√18,√32…

Here,

$a_2 - a_1 = \sqrt{8} - \sqrt{2} = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$ $a_3 - a_2 =$

$\sqrt{18} - \sqrt{8} = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$ $a_4 - a_3 =$

$4\sqrt{2} - 3\sqrt{2} = \sqrt{2}$

Since, $a_{n+1} - a_n$ or the common difference is same everytime.

Therefore, $d = \sqrt{2}$ and the given series forms a A.P.

Hence, next three terms are;

$a_5 = \sqrt{32} + \sqrt{2} = 4\sqrt{2} + \sqrt{2} = 5\sqrt{2} = \sqrt{50}$

$a_6 = 5\sqrt{2} + \sqrt{2} = 6\sqrt{2} = \sqrt{72}$

$a_7 = 6\sqrt{2} + \sqrt{2} = 7\sqrt{2} = \sqrt{98}$

(xiii) √3,√6,√9,√12…

Here,

$a_2 - a_1 = \sqrt{6} - \sqrt{3} = \sqrt{3} \times \sqrt{2} - \sqrt{3} = \sqrt{3}(\sqrt{2} - 1)$

$a_3 - a_2 = \sqrt{9} - \sqrt{6} = 3 - \sqrt{6} = \sqrt{3}(\sqrt{3} - \sqrt{2})$

$a_4 - a_3 = \sqrt{12} - \sqrt{9} = 2\sqrt{3} - \sqrt{3} \times \sqrt{3} = \sqrt{3}(2 - \sqrt{3})$

Since, $a_{n+1} - a_n$ or the common difference is not same everytime. Therefore,

the given series doesn't form a A.P.

(xiv) $1_2, 3_2, 5_2, 7_2$…

Or,1,9,25,49.....

Here,

$a_2 - a_1 = 9 - 1 = 8$

$a_3 - a_2 = 25 - 9 = 16$

$a_4 - a_3 = 49 - 25 = 24$

Since, $a_{n+1} - a_n$ orthecommondifference isnotsame everytime.

Therefore,thegivenseriesdoesn'tformaA.P.

(xv) $1_2, 5_2, 7_2, 73$...

Or1,25,49,73...

Here,

$a_2 - a_1 = 25 - 1 = 24$

$a_3 - a_2 = 49 - 25 = 24$

$a_4 - a_3 = 73 - 49 = 24$

Since, $a_{n+1} - a_n$ orthecommon differenceissame everytime.

Therefore, d = 24 and the given series forms a A.P.

Hence,nextthreetermsare; a_5

$= 73 + 24 = 97$

$a_6 = 97 + 24 = 121$

$a_7 = 121 + 24 = 145$

Exercise5.2

1. Fillintheblanksinthefollowingtable,giventhataisthefirstterm,dthe commondifferenceand a_n the n_{th} termoftheA.P.

	a	d	n	a_n
(i)	7	3	8	…...
(ii)	− 18	…..	10	0
(iii)	…..	− 3	18	− 5
(iv)	− 18.9	2.5	…..	3.6
(v)	3.5	0	105	…..

Solutions:

(i) Given,

Firstterm,a=7

Commondifference,d=3

Number of terms, n = 8,

We have to find the nth term, a_n= ?

As we know, for an A.P.,

$a_n = a+(n-1)d$

Puttingthevalues,

=>7+(8−1)3

=>7+(7)3

=>7+21=28

Hence,a_n=28

(ii) Given,

First term, a = -18

Commondifference,d=?

Number of terms, n = 10

Nth term, a_n= 0

As we know, for an A.P.,

$a_n = a+(n-1)d$

Puttingthevalues,

$0=-18+(10-1)d$

$18=9d$

$d=18/9=2$

Hence,commondifference,d=2

(iii) Given,

Firstterm,a=?

Commondifference,d=-3

Number of terms, n = 18

Nth term, a_n= -5

As we know, for an A.P.,

$a_n = a+(n-1)d$

Puttingthevalues,

$-5=a+(18-1)(-3)$

$-5=a+(17)(-3)$

$-5=a-51$

$a=51-5=46$

Hence,a=46

(iv) Given,

First term, a = -18.9

Commondifference,d=2.5

Number of terms, n = ?

Nthterm,a_n=3.6

As we know, for an A.P.,

$a_n = a + (n - 1)d$

Puttingthevalues,

$3.6 = -18.9 + (n-1)2.5$

$3.6 + 18.9 = (n-1)2.5$

$22.5 = (n-1)2.5$

$(n-1) = 22.5/2.5$ n −

1 = 9

n=10

Hence,n=10

(v) Given,

First term, a = 3.5

Common difference, d = 0

Number of terms, n = 105

Nth term, a_n= ?

As we know, for an A.P.,

$a_n = a + (n - 1)d$

Putting the values,

$a_n = 3.5 + (105-1)0$

$a_n=3.5+104\times0$

$a_n =$

3.5Hence,a_n=3.

5

2. Choosethecorrectchoiceinthefollowingandjustify:

(i) 30thtermoftheA.P:10,7,4,…,is

(A)97(B)77(C)−77(D)−87

(ii) 11thtermoftheA.P.-3,-1/2,,2….is

(A)28(B)22(C)−38(D)

$-48\frac{1}{2}$

Solutions:

(i) Givenhere,

A.P.=10,7,4,…

Therefore, we can find,

First term, a = 10

Commondifference,$d=a_2-a_1=7-10=-3$

As we know, for an A.P.,

$a_n= a +(n-1)d$

Putting the values;

$a_{30}=10+(30-1)(-3)$

$a_{30}=10+(29)(-3)$

$a_{30}=10-87=-77$

Hence, thecorrectanswerisoptionC.

(ii) Givenhere,

A.P.=-3,-1/2,,2…

Therefore, we can find,

First term a = – 3

Commondifference,d=$a_2 - a_1$=(-1/2)-(-3)

⇒(-1/2)+3=5/2

As we know, for an A.P.,

a_n = a+(n−1)d

Putting the

values;a_{11} = -3+(11-

1)(5/2)

a_{11}=-3+(10)(5/2)

a_{11} = -3+25

a_{11}=22

Hence,the answerisoptionB.

3. InthefollowingAPsfindthemissingtermintheboxes.

(i) 2, $\square$, 26

(ii) $\square$, 13, $\square$, 3

(iii) 5, $\square$, $\square$, $9\frac{1}{2}$

(iv) -4, $\square$, $\square$, $\square$, $\square$, 6

(v) $\square$, 38, $\square$, $\square$, $\square$, -22

Solutions:

(i) ForthegivenA.P.,2,2,26

The first and third term are;

$a = 2$

$a_3=26$

As we know, for an A.P.,

$a_n = a+(n-1)d$

Therefore,puttingthevalueshere, a_3

$= 2+(3-1)d$

$26=2+2d$

$24=2d$

$d=12$

$a_2=2+(2-1)12$

$=14$

Therefore,14is themissingterm.

(ii) ForthegivenA.P.,,13,,3

$a_2 = 13$ and

$a_4 = 3$

As we know, for an A.P.,

$a_n = a+(n-1) d$

Therefore,puttingthevalueshere, a_2

$= a +(2-1)d$

$13=a+d$ **(i)**

$a_4=a+(4-1)d$

$3=a+3d$ **(ii)**

Onsubtractingequation**(i)**from**(ii)**,weget,

−10=2d d

= − 5

From equation **(i)**, putting the value of d,we get

13 = a+(-5)

a=18

a_3=18+(3-1)(-5)

=18+2(-5)=18-10=8

Therefore,themissingtermsare18and8respectively.

(iii) ForthegivenA.P., a

= 5 and

a_4=19/2

As we know, for an A.P.,

a_n = a+(n−1)d

Therefore,puttingthevalueshere, a_4

= a+(4-1)d

19/2 = 5+3d (19/2)

− 5 = 3d

3d=9/2 d

= 3/2

a_2=a+(2-1)d

a_2= 5+3/2a_2

= 13/2

a_3=a+(3-1)d

a_3 =5+2×3/2

a_3 = 8

Therefore,themissingtermsare13/2and8respectively.

(iv) ForthegivenA.P., a

= −4 and

a_6=6

As we know, for an A.P.,

a_n = a +(n−1) d

Therefore,puttingthevalueshere, a_6

= a+(6−1)d

6=−4+5d

10=5d

d=2

a_2=a+d=−4+2=−2

a_3 =a+2d=−4+2(2)=0

a_4=a+3d=−4+3(2)=2 a_5

=a+4d=−4+4(2)=4

Therefore,themissingtermsare−2,0,2,and4respectively.

(v) ForthegivenA.P.,

a_2 = 38

a_6=−22

Asweknow,foranA.P.,

$a_n=a+(n-1)d$

Therefore,puttingthevalueshere, a_2

$= a+(2-1)d$

$38=a+d$ **(i)**

$a_6=a+(6-1)d$

$-22=a+5d$ **(ii)**

Onsubtractingequation**(i)**from**(ii)**,weget

$\sim 22-38=4d$

$-60=4d$

$d=-15$

$a=a_2-d=38-(-15)=53$

$a_3=a+2d=53+2(-15)=23$

$a_4=a+3d=53+3(-15)=8$

$a_5=a+4d=53+4(-15)=-7$

Therefore,themissingtermsare53,23,8,and−7respectively.

4. WhichtermoftheA.P.3,8,13,18,…is78?

Solutions:

GiventheA.P.seriesas3,8,13,18,…

Firstterm,a=3

Commondifference,$d=a_2-a_1=8-3=5$

Let the n_{th}term of given A.P. be 78. Now as we know,

$a_n = a+(n-1)d$

Therefore,

78=3+(n−1)5

75=(n−1)5

(n−1)=15

n=16

Hence,16thtermofthisA.P.is78.

5. FindthenumberoftermsineachofthefollowingA.P.

(i) 7,13,19,…,205

(ii) 18, $15\frac{1}{2}$, 13... -47

Solutions:

(i) Given,7,13,19,…,205istheA.P

Therefore

Firstterm,a=7

Commondifference,$d=a_2-a_1=13-7=6$

Let there are n terms in this A.P.

$a_n = 205$

As we know, for an A.P.,

$a_n= a + (n - 1) d$

Therefore,205=7 +(n−1)6

198= (n − 1)6

33=(n−1)

n=34

Therefore,thisgivenserieshas34termsinit.

(ii) Given, 18, $15\frac{1}{2}$, 13... -47 is the A.P.

Firstterm,a=18

Commondifference,d=a_2-a_1=

$15\frac{1}{2} - 18$

d=(31-36)/2=-5/2

Let there are n terms in this A.P.

a_n = -47

As we know, for an A.P.,

a_n = a+(n−1)d

-47=18+(n-1)(-5/2)

-47-18=(n-1)(-5/2)

-65=(n-1)(-5/2)

(n-1) = -130/-5

(n-1) = 26

n=27

Therefore,thisgivenA.P.has27termsinit.

6. Checkwhether-150isatermoftheA.P.11,8,5,2,…

Solution:

Forthe given series, A.P. 11, 8, 5, 2..

First term, a = 11

Commondifference,d=a_2−a_1 =8−11=−3 Let

−150 be the n_{th}term of this A.P.

Asweknow,foranA.P.,

$a_n = a + (n-1)d$

-150=11+(n-1)(-3)

-150=11-3n+3

-164=-3n n

= 164/3

Clearly, n is not an integer but a fraction.

Therefore, −150 is nota term of this A.P.

7. Findthe31sttermofanA.P.whose11thtermis38andthe16thtermis73.

Solution:

Giventhat,

11thterm, $a_{11} = 38$

and 16thterm, $a_{16} = 73$

We know that,

$a_n = a + (n-1)d$

$a_{11} = a + (11-1)d$

38=a+10d **(i)**

Inthesameway,

$a_{16} = a + (16-1)d$

73=a+15d **(ii)**

On subtracting equation **(i)** from **(ii)**, we get

35 = 5d

d=7

Fromequation**(i)**,wecanwrite,

$38=a+10\times(7)$

$38-70=a$

$a=-32$

$a_{31}=a+(31-1)d$

$=-32+30(7)$

$=-32+210$

$=178$

Hence,31_{st}termis178.

8. AnA.P.consistsof50termsofwhich3_{rd}termis12andthelasttermis106. Findthe29_{th}term.

Solution: Given that,

3_{rd}term, $a_3=$

1250_{th}term, a_{50} =

106 We know that,

$a_n=a+(n-1)d$

$a_3=a+(3-1)d$

$12=a+2d$ **(i)**

Inthesameway,

$a_{50}=a+(50-1)d$

$106=a+49d$ **(ii)**

On subtracting equation **(i)** from **(ii)**, we get

$94 = 47d$

d=2=commondifference

From equation **(i)**, we can write now,

$12 = a+2(2)$

$a = 12-4 =$

$8a_{29}=a+(29-1)d$

$a_{29} = 8+(28)2$

$a_{29}=8+56=64$

Therefore,29thtermis64.

9. Ifthe3rdandthe9thtermsofanA.P.are4and−8respectively.Whichtermof thisA.P.iszero.

Solution:

Giventhat,

3rdterm,$a_3=4$

and9thterm,$a_9=-8$

We know that,

$a_n=a+(n-1)d$

Therefore,

$a_3=a+(3-1)d$

$4=a+2d$ **(i)**

$a_9=a+(9-1)d$

$-8=a+8d$ **(ii)**

Onsubtractingequation**(i)**from**(ii)**,wewillgethere,

$-12=6d$

$d=-2$

Fromequation**(i)**,wecanwrite,

$4=a+2(-2)$

$4=a-4$

$a=8$

Let n_{th} term of this A.P. be zero.

$a_n = a+(n-1)d$

$0=8+(n-1)(-2)$

$0=8-2n+2$

$2n = 10$

$n=5$

Hence,5_{th}termofthisA.P.is0.

10. If17$_{th}$termofanA.P.exceedsits10$_{th}$termby7.Findthecommondifference.

Solution:

We knowthat,foran A.P series; a_n

$= a+(n-1)d$

$a_{17}=a+(17-1)d$

$a_{17}=a +16d$

Inthesameway,

$a_{10} = a+9d$

Asitisgiveninthequestion, a_{17}

$- a_{10} = 7$

Therefore,

$(a+16d)-(a+9d)=7$

$7d = 7$

d=1

Therefore,thecommondifferenceis1.

11. WhichtermoftheA.P.3,15,27,39,..willbe132morethanits54thterm?

Solution:

GivenA.P.is3,15,27,39,…

firstterm,a=3

commondifference,$d=a_2-a_1=15-3=12$

We know that,

$a_n=a+(n-1)d$

Therefore,

$a_{54}=a+(54-1)d$

$\Rightarrow 3+(53)(12)$

$\Rightarrow 3+636=639$

$a_{54}=639+132=771$

We have to find the term of this A.P. which is 132 more than a_{54}, i.e.771.

Let nth term be 771.

$a_n = a+(n-1)d$

$771=3+(n-1)12$

$768=(n-1)12$

$(n-1)=64$

$n=65$

Therefore,65thtermwas132morethan54thterm.

Oranothermethodis;

Let n_{th} termbe 132morethan 54_{th}term. n =

54 + 132/2

=54+11=65_{th}term

12. Two APs havethe same common difference. The difference between their 100_{th}termis100,whatisthedifferencebetweentheir1000_{th}terms?

Solution:

Let, the first term of two APs be $a_1$and $a_2$respectively

And the common difference of these APs be d.

For the first A.P.,we know,

$a_n = a+(n-1)d$

Therefore,

$a_{100}=a_1+(100-1)d$

$=a_1+99d$

$a_{1000} =a_1+(1000-1)d$

$a_{1000} = a_1+999d$

ForsecondA.P.,weknow, a_n

$= a+(n-1)d$

Therefore,

$a_{100}=a_2+(100-1)d$

$=a_2+99d$

$a_{1000}=a_2+(1000-1)d$

$=a_2+999d$

Given that, difference between 100_{th} term of the two APs = 100

Therefore, $(a_1+99d) - (a_2+99d) = 100$

$a_1 - a_2 = 100$… **(i)**

Difference between 1000_{th} terms of the two APs

$(a_1+999d) - (a_2+999d) = a_1 - a_2$

Fromequation**(i)**,

Thisdifference,$a_1 - a_2 = 100$

Hence,thedifferencebetween1000_{th}termsofthetwoA.P.willbe100.

13. Howmanythreedigitnumbersaredivisibleby7?

Solution:

First three-digit number that is divisible by 7 are;

First number = 105

Secondnumber=105+7=112

Thirdnumber=112+7=119

Therefore,105,112,119,…

All are three digit numbers are divisible by 7 and thus, all these are terms of an A.P. having first term as 105 and common difference as 7.

As we know, the largest possible three-digit number is 999.

When we divide 999 by 7, the remainder will be 5.

Therefore, 999-5 = 994 is the maximum possible three-digit number that isdivisible by 7.

Nowtheseriesisasfollows.

105,112, 119,…,994

Let994bethenthtermofthisA.P.

first term, a = 105

commondifference,d =7

a_n = 994

n=?

As we know,

$a_n=a+(n-1)d$

$994=105+(n-1)7$

$889=(n-1)7$

$(n-1)=127$

$n=128$

Therefore,128three-digitnumbersaredivisibleby7.

14. How manymultiples of4liebetween10and250?

Solution:

The first multiple of 4 that is greater than 10 is 12.

Next multiple will be 16.

Therefore,theseriesformedas;

12,16,20,24, …

All these are divisible by 4 and thus, all these are terms of an A.P. with first term as 12 and common difference as 4.

Whenwedivide250by4,theremainderwillbe2.Therefore,250−2=248is divisibleby4.

Theseriesisasfollows,now;

12,16,20,24,…,248

Let248bethen$_{th}$termofthisA.P.

first term, a = 12

commondifference,d =4

a_n = 248

As we know,

$a_n=a+(n-1)d$

248=12+(n-1)×4

236/4=n-1

59=n-1

n=60

Therefore,thereare60multiplesof4between10and250.

15. Forwhatvalueofn,arethen$_{th}$termsoftwoAPs63,65,67,and3,10,17,… equal?

Solution:

GiventwoAPsas;63,65,67,…and3,10,17,….

TakingfirstAP,

63,65,67,…

Firstterm,a=63

Common difference, d = a_2-a_1= 65−63 = 2

Weknow,n$_{th}$termofthisA.P.=$a_n=a+(n-1)d$ a_n=

63+(n−1)2 = 63+2n−2

a_n=61+2n **(i)**

TakingsecondAP,

3,10,17,…

Firstterm,a=3

Commondifference,$d=a_2-a_1=10-3=7$

Weknowthat,

n_{th}termofthisA.P.$=3+(n-1)7$ a_n

$= 3+7n-7$

$a_n=7n-4$ **(ii)**

Given,n_{th}termoftheseA.P.sareequaltoeachother.

Equating both these equations, we get,

$61+2n=7n-4$

$61+4=5n$

$5n = 65$

$n=13$

Therefore,13_{th}termsofboththeseA.P.s areequaltoeachother.

16. Determine the A.P.whosethird term is16and the 7_{th}termexceedsthe 5_{th}term by 12.

Solutions:

Given,

Third term, $a_3 = 16$

As we know,

$a+(3-1)d=16$

$a+2d=16$ (i)

Itisgiventhat,7_{th}termexceedsthe5_{th}termby12.

$a_7 - a_5 = 12$

$[a+(7-1)d]-[a+(5-1)d]=12$

$(a+6d)-(a+4d)=12$

$2d=12$

$d=6$

Fromequation(i),weget,

$a+2(6) = 16$

$a+12=16$

$a=4$

Therefore,A.P.willbe4,10,16,22,...

17. Findthe20$_{th}$term fromthelasttermof theA.P.3,8,13,…,253.

Solution:

GivenA.P.is3,8,13,...,253

Commondifference,d=5.

Therefore,wecanwritethegivenAPinreverseorderas;

253,248, 243,...,13, 8,5

NowforthenewAP, first

term, a = 253

andcommondifference,$d=248-253=-5$

$n=20$

Therefore, using nth term formula, we get,

$a_{20} = a+(20-1)d$

$a_{20}=253+(19)(-5)$

$a_{20}=253-95$

$a=158$

Therefore,20_{th}termfromthelasttermoftheAP3,8, 13,...,253.is158.

18. Thesumof4_{th}and8_{th}termsofanA.P.is24andthesumofthe$6_{th}$and 10_{th}termsis44.FindthefirstthreetermsoftheA.P.

Solution:

We know that, the nth term of the AP is;

$a_n = a+(n-1)d$

$a_4=a+(4-1)d$

$a_4=a+3d$

In the same way, we can write,

$a_8 = a+7d$

$a_6 = a+5d$

$a_{10}=a+9d$

Giventhat,

$a_4+a_8=24$

$a+3d+a+7d=24$

$2a+10d=24$

$a+5d=12$ **(i)**

$a_6+a_{10}=44$

$a+5d+a+9d=44$

$2a+14d = 44$

$a+7d=22$ **(ii)**

On subtracting equation **(i)** from **(ii)**, we get,

2d = 22 − 12

2d=10

d=5

From equation **(i)**, we get,

a+5d = 12

a+5(5)= 12

a+25=12

a=−13

a_2=a+d=−13+5=−8

a_3=a_2+d=−8+5=−3

Therefore, thefirstthreetermsofthisA.P.are−13, −8,and−3.

19. Subba Rao started work in 1995 at an annual salary of Rs 5000 and received an increment of Rs 200 each year. In which year did his income reach Rs 7000?

Solution:

Itcanbeseenfromthegivenquestion,thattheincomesofSubbaRaoincreases every year by Rs.200 and hence, forms an AP.

Therefore,after1995,thesalariesofeachyearare;

5000,5200,5400,…

Here,firstterm,a=5000

and common difference,d=200

Let after n_{th}year, his salary be Rs 7000.

Therefore, by the n_{th}term formula of AP,

a_n=a+(n−1)d

$7000=5000+(n-1)200$

$200(n-1)=2000$

$(n-1)=10$

$n=11$

Therefore,in11thyear,hissalarywillbeRs7000.

20. Ramkali saved Rs 5 in the first week of a year and then increased her weeklysavingbyRs1.75.Ifinthen_{th}week,herweeklysavingsbecomeRs20.75,findn.

Solution:

Given that, Ramkali saved Rs.5 in first week and then started saving each week by Rs.1.75.

Hence,

Firstterm,a=5

and common difference, d = 1.75

Also given,

$a_n=20.75$

Find,n=?

As we know, by the n_{th}term formula,

$a_n = a+(n-1)d$

Therefore,

$20.75=5+(n-1)\times1.75$

$15.75=(n-1)\times1.75$

$(n-1)=15.75/1.75=1575/175$

$=63/7=9$

$n-1=9$

n=10

Hence,nis10.

Exercise5.3

1. FindthesumofthefollowingAPs.

(i) 2,7,12,….,to10terms.

(ii) −37,−33,−29,…,to12terms

(iii) 0.6,1.7,2.8,……..,to100terms

(iv) 1/15,1/12,1/10,……,to11terms

Solutions:

(i) Given,2,7,12,…,to10terms

For this A.P.,

first term, a = 2

Andcommondifference,d=a_2-a_1=7−2=5

n=10

We know that, the formula for sum of nth term in AP series is,

S_n= n/2 [2a +(n-1)d]

S_{10}=10/2[2(2)+(10-1)×5]

=5[4+(9)×(5)]

=5×49=245

(ii) Given,−37, −33,−29,…,to12terms

ForthisA.P.,

firstterm,a=−37

Andcommondifference,d =a_2-a_1

$d=(-33)-(-37)$

$=-33+37=4$

$n=12$

We know that, the formula for sum of nth term in AP series is,

$S_n = n/2\ [2a+(n-1)d]$

$S_{12}=12/2[2(-37)+(12-1)\times 4]$

$=6[-74+11\times 4]$

$=6[-74+44]$

$=6(-30)=-180$

(iii) Given,0.6,1.7,2.8,…,to100terms

ForthisA.P.,

firstterm,a=0.6

Commondifference,$d=a_2-a_1=1.7-0.6=1.1$

$n=100$

We know that, the formula for sum of nth term in AP series is,

$S_n = n/2[2a +(n-1)d]$

$S_{12}=50/2[1.2+(99)\times 1.1]$

$=50[1.2+108.9]$

$=50[110.1]$

$=5505$

(iv) Given,1/15,1/12,1/10,……,to11terms

ForthisA.P.,

Firstterm,a=1/5

Common difference, $d = a_2 - a_1 = (1/12)-(1/5) = 1/60$

And number of terms n = 11

We know that, the formula for sum of nth term in AP series is,

$S_n = n/2\ [2a + (n - 1)\ d]$

$$Sn = \frac{11}{2}\ [2\left(\frac{1}{15}\right) + \frac{(11\ -\ 1)\ 1}{60}]$$

=11/2(2/15+10/60)

=11/2(9/30)

=33/20

2. Findthesumsgivenbelow:

(i) $7 + \mathbf{10\frac{1}{2}} + \mathbf{14} + \ldots\ldots\ldots\ldots\ldots\ldots + \mathbf{84}$

(ii) 34+32+30+……….+10

(iii) −5+(−8)+(−11)+…………+(−230)

Solutions:

(i)

For this given A.P., $7+10\frac{1}{2} + 14 + \ldots\ldots\ldots\ldots\ldots\ldots +84$,

First term, a = 7

n_{th}term, $a_n = 84$

Common difference, $d = a_2 - a_1 = \mathbf{10\frac{1}{2}}\ \mathbf{-}\ 7 = \frac{21}{2} - 7 = \frac{7}{2}$

Let 84 be the n_{th} term of this A.P., thenas per the n_{th}term formula, a_n

= a(n-1)d

84=7+(n−1)×7/2

77= (n-1)×7/2

22=n−1

n=23

We know that, sum of n term is;

S_n= n/2 (a + l) , l = 84
S_n=23/2(7+84)

S_n=(23×91/2)=2093/2

$S_n = 1046\frac{1}{2}$

(ii) Given,34+32+30+………..+10

ForthisA.P.,

firstterm,a=34

commondifference,d=a_2−a_1=32−34=−2

n_{th}term,a_n=10

Let 10 be the n_{th} term of this A.P., therefore,

a_n= a +(n−1)d

10=34+(n−1)(−2)

−24=(n−1)(−2)

12=n−1

n=13

We know that, sum of n terms is;

S_n= n/2 (a +l) , l = 10

=13/2(34+10)

=(13×44/2)=13×22

=286

(iii) Given,(−5)+(−8)+(−11)+…………+(−230)

ForthisA.P.,

Firstterm,$a=-5$

nthterm,$a_n=-230$

Commondifference,$d=a_2-a_1=(-8)-(-5)$

$\Rightarrow d=-8+5=-3$

Let -230 be the n_{th}term of this A.P., and by the n_{th}term formula weknow, $a_n=$

$a+(n-1)d$

$-230=-5+(n-1)(-3)$

$-225=(n-1)(-3)$

$(n-1)=75$

$n=76$

And, Sum of n term,

$S_n= n/2\ (a + l)$

$=76/2[(-5)+(-230)]$

$=38(-235)$

$=-8930$

3. InanAP

(i) Givena=5,d=3,a_n=50,findnandS$_n$.

(ii) Givena=7,a_{13}=35,finddandS$_{13}$.

(iii) Givena_{12}=37,d=3,findaandS$_{12}$.

(iv) Givena_3=15,S_{10}=125,finddanda_{10}.

(v) Givend=5,S_9=75,findaanda_9.

(vi) Givena=2,d=8,S_n=90,findnanda_n.

(vii) Givena=8,a_n=62,S_n=210,findnandd.

(viii) Givena_n=4,d=2,S_n=−14,findnanda.

(ix) **Givena=3,n=8,S=192,findd.**

(x) **Givenl=28,S=144andtherearetotal9terms.Finda.**

Solutions:

(i) Given that,$a=5,d=3,a_n=50$

As we know, from the formula of the nth term in an

AP,$a_n= a+(n-1)d$,

Therefore,puttingthegivenvalues,weget,

⇒$50=5+(n-1)\times3$

⇒$3(n-1)= 45$

⇒$n-1=15$

⇒$n=16$

Now, sum of n terms,

$S_n= n/2\ (a+a_n)$

$S_n=16/2(5+50)=440$

(ii) Giventhat, $a=7,a_{13}=35$

As we know, from the formula of the nth term in an

AP,$a_n = a+(n-1)d$,

Therefore,puttingthegivenvalues,weget,

⇒$35=7+(13-1)d$

⇒$12d=28$

⇒$d=28/12=2.33$

Now,$S_n=n/2(a+a_n)$

$S_{13}=13/2(7+35)=273$

(iii) Giventhat,$a_{12}=37$,$d=3$

As we know, from the formula of the n_{th}term in an AP,a_n

$= a+(n-1)d$,

Therefore,puttingthegivenvalues,weget,

$\Rightarrow a_{12}=a+(12-1)3$

$\Rightarrow 37=a+33$

$\Rightarrow a=4$

Now, sum of nth term,

$S_n = n/2\ (a+a_n)$

$S_n=12/2(4+37)$

$=246$

(iv) Giventhat,$a_3=15$,$S_{10}=125$

As we know, from the formula of the nth term in an

AP,$a_n= a+(n-1)d$,

Therefore,puttingthegivenvalues,weget, a_3

$= a+(3-1)d$

$15=a+2d$ **(i)**

Sum of the nth term,

$S_n=n/2\ [2a+(n-1)d]$

$S_{10} = 10/2\ [2a+(10-1)d]$

$125 = 5(2a+9d)$

$25=2a+9d$ **(ii)**

Onmultiplyingequation**(i)**by**(ii)**,wewillget;

$30=2a+4d$ **(iii)**

Bysubtractingequation**(iii)**from**(ii)**,weget,

$-5=5d$

$d=-1$

From equation **(i)**,

$15 = a+2(-1)$

$15=a-2$

$a = 17$ = First term

$a_{10} = a+(10-1)d$

$a_{10} = 17+(9)(-1)$

$a_{10}=17-9=8$

(v) Given that, $d = 5$, $S_9 = 75$

As, sum of n terms in AP

is,$S_n= n/2\ [2a +(n -1)d]$

Therefore,the sum of first nine terms are;

$S_9 = 9/2\ [2a +(9-1)5]$

$25=3(a+20)$

$25=3a+60$

$3a=25-60$

$a=-35/3$

As we know, the n_{th}term can be written as;

$a_n = a+(n-1)d$

$a_9=a+(9-1)(5)$

=-35/3+8(5)

=-35/3+40

=(35+120/3) =85/3

(vi) Given that, a = 2, d = 8, S_n= 90

As, sum of n terms in an AP is,

S_n= n/2 [2a +(n -1)d] 90

= n/2 [2a +(n -1)d]

⇒180=n(4+8n-8)=n(8n-4)=$8n_2$-4n

⇒$8n_2$-4n–180=0

⇒$2n_2$–n-45=0

⇒$2n_2$-10n+9n-45=0

⇒2n(n-5)+9(n-5)=0

⇒(n-5)(2n+9)=0

So,n=5(asnonlybeapositiveinteger)

∴a_5=8+5×4=34

(vii) Given that, a = 8, a_n= 62, S_n= 210

As, sum of n terms in an AP is,

S_n= n/2 (a + a_n)

210 = n/2 (8 +62)

⇒35n =210

⇒n = 210/35 = 6 Now,

62 = 8+5d

⇒5d=62-8=54

$\Rightarrow d=54/5=10.8$

(viii) Giventhat,n_{th}term,$a_n=4$,commondifference,$d=2$,sumofnterms,$S_n=$
-14.

As we know, from the formula of the n_{th}term in an AP,a_n

$= a+(n-1)d$,

Therefore,puttingthegivenvalues,weget, 4 =

$a+(n-1)2$

$4=a+2n-2$

$a+2n=6$

$a=6-2n$ **(i)**

Aswe know, thesum ofnterms is; S_n

$= n/2\ (a+a_n)$

$-14=n/2(a+4)$

$-28=n(a+4)$

$-28=n(6-2n+4)$\{Fromequation**(i)**\}

$-28=n(-2n+10)$

$-28 = -2n^2+10n$

$2n^2 -10n - 28 = 0$

$n^2-5n-14=0$

$n^2-7n+2n-14=0$

$n(n-7)+2(n-7)=0$

$(n-7)(n+2)=0$

Eithern$-7=0$orn$+2=0$

n=7orn=−2

However, n can neither be negative norfractional.

Therefore, n = 7

From equation **(i)**, we get

a = 6−2n

a=6−2(7)

=6−14

=−8

(ix) Given that, first term, a = 3,

Number of terms, n = 8

And sum of n terms, S = 192

As we know,

S_n= n/2 [2a+(n -1)d]

192=8/2[2×3+(8-1)d]

192=4[6+7d]

48=6+7d

42=7d

d=6

(x) Giventhat,l=28,S=144andtherearetotalof9terms.

Sum of n terms formula,

S_n= n/2 (a + l)

144=9/2(a+28)

(16)×(2)=a+28

$32=a+28$

$a=4$

4. HowmanytermsoftheAP.9,17,25…mustbetakentogiveasumof636?

Solutions:

LettherebentermsoftheAP.9,17,25…

For this A.P.,

First term, a = 9

Commondifference,$d=a_2-a_1=17-9=8$

As,thesumofnterms,is;

$S_n= n/2\ [2a+(n\ -1)d]636$

$= n/2\ [2\times a+(8-1)\times 8]$

$636=n/2[18+(n-1)\times 8]$

$636=n[9+4n-4]$

$636=n(4n+5)$

$4n_2+5n-636=0$

$4n_2+53n-48n-636=0$

$n(4n+53)-12(4n+53)=0$

$(4n+53)(n-12)=0$

Either $4n+53 = 0$ or $n-12 = 0$

$n = (-53/4)$ or $n = 12$

ncannotbenegativeorfraction,therefore,n=12only.

5. ThefirsttermofanAPis5,thelasttermis45andthesumis400.Findthe number of terms and thecommon difference.

Solution:

Giventhat,

first term, a = 5

lastterm,l=45

SumoftheAP,S_n=400

As we know, the sum of AP formula is;

$S_n = n/2\ (a+l)$

$400=n/2(5+45)$

$400=n/2(50)$

Numberofterms,n=16

As we know, the last term of AP series can be written as;l

$= a+(n-1)d$

$45=5+(16-1)d$

$40=15d$

Commondifference,d=40/15=8/3

6. ThefirstandthelasttermofanAPare17and350respectively.Ifthecommon differenceis9,howmanytermsarethereandwhatistheirsum?

Solution:

Giventhat,

First term, a = 17

Last term, l = 350

Commondifference,d=9

LettherebentermsintheA.P.,thustheformulaforlasttermcanbewrittenas;

$l = a+(n-1)d$

$350=17+(n-1)9$

$333=(n-1)9$

$(n-1)=37$

$n=38$

$S_n=n/2(a+l)$

$S_{38}=38/2(17+350)$

$=19\times367$

$=6973$

Thus,thisA.P.contains38termsandthesumofthetermsofthisA.P.is6973.

7. Findthe sum offirst 22 terms of an AP in whichd= 7 and22ndterm is 149. Solution:

Given,

Commondifference,d =7

22ndterm, $a_{22}= 149$

Sum of first 22 term, $S_{22} = ?$

By the formula of nth

term,$a_n = a+(n-1)d$

$a_{22}=a+(22-1)d$

$149 = a+21\times7$

$149=a+147$

a= 2 = First term

Sum of n terms,

$S_n = n/2(a + a_n)$

$S_{22} = 22/2(2+149)$

$=11\times151$

$=1661$

8. Findthesumoffirst51termsofanAPwhosesecondandthirdtermsare14 and 18 respectively.

Solution:

Giventhat,

Second term, $a_2 = 14$

Third term, $a_3 = 18$

Commondifference,$d = a_3 - a_2 = 18 - 14 = 4$

$a_2 = a + d$

$14 = a + 4$

$a = 10$ = First term

Sum of n terms;

$S_n = n/2[2a + (n-1)d]$

$S_{51} = 51/2[2\times10(51-1)4]$

$=51/2[20+(50)\times4]$

$=51\times220/2$

$=51\times110$

$=5610$

9. Ifthesumoffirst7termsofanAPis49andthatof17termsis289,findthe sum of first n terms.

Solution:

Giventhat,

$S_7 = 49$

$S_{17}=289$

Weknow,Sum ofnterms;

$S_n= n/2 [2a + (n - 1)d]$

Therefore,

$S_7= 7/2 [2a +(n -1)d]$ S_7

$=7/2[2a+(7-1)d]$ $49 =$

$7/2 [2a +6d]$

$7=(a+3d)$

$a+3d=7$ **(i)**

Inthesameway,

$S_{17} = 17/2 [2a+(17-1)d]$

$289 = 17/2 (2a +16d)$

$17=(a+8d)$

$a+8d=17$ **(ii)**

Subtracting equation **(i)** from equation **(ii)**,

$5d = 10$

$d=2$

From equation **(i)**, we can write it as;

$a+3(2) = 7$

$a+6=7$

$a=1$

Hence,

S_n=n/2[2a+(n-1)d]

=n/2[2(1)+(n−1)×2]

=n/2(2+2n-2)

=n/2(2n)

=n_2

10. Showthata$_1$,a$_2$…,a$_n$,…formanAPwherea$_n$isdefinedasbelow

(i) a_n= 3+4n
(ii) a_n=9−5n
Also find the sum of the first 15 terms ineach case.

Solutions:

(i) a_n=3+4n

a_1 =3+4(1)=7

a_2 =3+4(2)=3+8=11

a_3=3+4(3)=3+12=15

a_4=3+4(4)=3+16=19

Wecanseehere,thecommondifferencebetweenthetermsare; a_2 −

a_1 = 11−7 = 4

a_3−a_2=15−11=4

a_4−a_3=19−15=4

Hence,a_{k+1} −a_k isthesame valueeverytime.Therefore,thisisan APwith common difference as 4 and first term as 7.

Now,weknow,thesumofnthtermis;

$S_n=n/2[2a+(n-1)d]$

$S_{15}=15/2[2(7)+(15-1)\times 4]$

$=15/2[(14)+56]$

$=15/2(70)$

$=15\times 35$

$=525$

(ii) $a_n=9-5n$

$a_1=9-5\times 1=9-5=4$

$a_2=9-5\times 2=9-10=-1$

$a_3=9-5\times 3=9-15=-6$

$a_4=9-5\times 4=9-20=-11$

Wecanseehere,thecommondifferencebetweenthetermsare; a_2

$-a_1 =-1-4=-5$

$a_3-a_2=-6-(-1)=-5$

$a_4-a_3=-11-(-6)=-5$

Hence,$a_{k+1}-a_k$issameeverytime.Therefore,thisisanA.P.withcommon difference as −5 and first term as 4.

Now, we know, the sum of nth term is;

$S_n= n/2 [2a +(n-1)d]$

$S_{15}=15/2[2(4)+(15-1)(-5)]$

$=15/2[8+14(-5)]$

$=15/2(8-70)$

$=15/2(-62)$

=15(-31)

=-465

11. Ifthesumofthefirstntermsofan APis $4n-n^2$,whatisthefirstterm(that is S_1)? What is the sum of first two terms? What is the second term? Similarly find the 3^{rd},the 10^{th}andthe n^{th}terms.

Solution:

Given that,

$S_n=4n-n^2$

Firstterm,$a=S_1=4(1)-(1)^2=4-1=3$

Sumoffirsttwoterms=$S_2=4(2)-(2)^2=8-4=4$

Second term, $a_2 = S_2 - S_1 = 4-3 = 1$

Commondifference,$d=a_2-a=1-3=-2$ N^{th}

term, $a_n = a+(n-1)d$

$=3+(n-1)(-2)$

$=3-2n+2$

$=5-2n$

Therefore,$a_3=5-2(3)=5-6=-1$

$a_{10}=5-2(10)=5-20=-15$

Hence,thesumoffirsttwotermsis4.Thesecondtermis1.

The3^{rd},the10^{th},andthen^{th}termsare$-1,-15$,and$5-2n$respectively.

12. Findthesumoffirst40positiveintegersdivisibleby6.

Solution:

Thepositiveintegersthataredivisibleby6are6,12,18,24....

We can see here, that this series forms an A.P. whose first term is 6 and common difference is 6.

a=6

d = 6

S_{40}=?

By the formula of sum of n terms, we know,

S_n= n/2 [2a +(n – 1)d]

Therefore, putting n = 40, we get,

S_{40} = 40/2 [2(6)+(40-1)6]

=20[12+(39)(6)]

=20(12+234)

=20×246

=4920

13. Findthesumoffirst15multiplesof8.

Solution:

Themultiplesof8are8,16,24,32…

TheseriesisintheformofAP,havingfirsttermas8andcommondifferenceas8.

Therefore,a=8 d

= 8

S_{15}=?

By the formula of sum of nth term, we know,

S_n= n/2 [2a+(n-1)d]

S_{15}=15/2[2(8)+(15-1)8]

$=15/2[16+(14)(8)]$

$=15/2[16+112]$

$=15(128)/2$

$=15\times 64$

$=960$

14. Findthesumoftheoddnumbersbetween0and50.

Solution:

The oddnumbersbetween0 and50 are1, 3, 5, 7,9 … 49. Therefore, we

can see that these odd numbers are in the form of A.P. Hence,

Firstterm,a=1

Commondifference,d =2

Last term, l = 49

By the formula of last term, we know,l

$= a+(n-1)\ d$

$49=1+(n-1)2$

$48=2(n-1)$

$n-1=24$

n=25=Number ofterms

By the formula of sum of nth term, we know,

$S_n= n/2(a +l)$

$S_{25}=25/2(1+49)$

$=25(50)/2$

=(25)(25)

=625

15. A contract on construction job specifies a penalty for delay of completion beyond acertaindateasfollows:Rs.200forthefirstday,Rs.250fortheseconddday,Rs. 300forthethirdday,etc.,thepenaltyforeachsucceedingdaybeingRs.50more thanfortheprecedingday.Howmuchmoneythecontractorhastopayaspenalty, if he has delayed the work by 30 days.

Solution:

We can see, that the given penalties are in the form of A.P. having first term as 200 and common difference as 50.

Therefore,a=200andd=50

Penalty that has to be paid if contractor has delayed thework by 30 days = S_{30}

By the formula of sum of nth term, we know,

S_n = n/2[2a+(n -1)d]

Therefore,

S_{30}=30/2[2(200)+(30−1)50]

=15[400+1450]

=15(1850)

=27750

Therefore,thecontractorhastopayRs27750aspenalty.

16. A sum of Rs 700 is to be used to give seven cash prizes to students of a school for their overall academic performance.If each prize is Rs 20 less than its preceding prize, find the value of each of the prizes.

Solution:

Letthecostof1_{st}prize beRs.P.

Costof2ndprize=Rs.P−20

Andcostof 3rdprize= Rs.P−40

Wecanseethat thecost oftheseprizesareintheformofA.P.,having common difference as −20 and first term as P.

Thus, a = P and d = −20

Giventhat,S_7=700

By the formula of sum of nth term, we know,

S_n= n/2 [2a + (n – 1)d]

7/2[2a+(7−1)d]=700

$$\frac{[2a+(6)(-20)]}{2}=100$$

a+3(−20)=100

a−60=100

a=160

Therefore, the value of each of the prizes was Rs 160, Rs 140, Rs 120, Rs 100, Rs 80, Rs 60, and Rs 40.

17. In a school, students thought of planting trees in and around the school to reduce air pollution. It was decided that the number of trees, that each section of each class will plant, will be the same as the class, in which they arestudying, e.g., a section of class I will plant 1 tree, a section of class II will plant 2 trees and so ontill class XII.There are three sections of each class. How many trees will be planted by the students?

Solution:

Itcanbe observedthatthe numberof treesplantedbythe studentsisin anAP.

1,2,3,4,5… 12

Firstterm,a=1

Commondifference,d=2−1=1

S_n=n/2[2a+(n-1)d]

S_{12}=12/2[2(1)+(12-1)(1)]

=6(2+11)

=6(13)

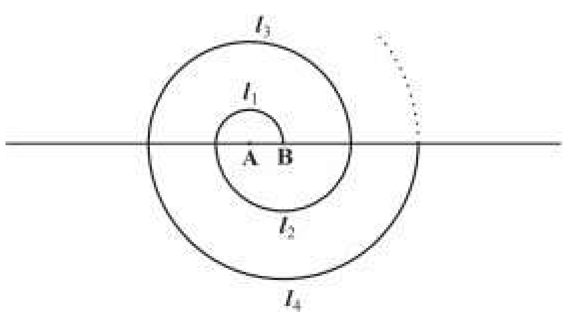
=78

Therefore,numberoftreesplantedby1 sectionoftheclasses=78

Numberoftrees plantedby3sectionsof the classes= 3×78= 234

Therefore, 234 trees will be planted by the students.

18. A spiral is made up of successive semicircles, withcentres alternately at A andB, startingwithcentreatAofradii0.5,1.0cm,1.5cm,2.0cm,………asshownin figure. What is the total length of such a spiral made up of thirteen consecutive semicircles? (Take π = 22/7)



Solution:

Weknow,

Perimeter of a semi-circle = πr

Therefore,

P_1=π(0.5)=π/2cm

P_2=π(1)=πcm

P_3=π(1.5)=3π/2cm

Where, P_1, P_2, P_3 are the lengths of the semi-circles.

Hence we got a series here, as,

$\pi/2, \pi, 3\pi/2, 2\pi, \ldots$

$P_1 = \pi/2$cm

$P_2 = \pi$cm

Commondifference,d $= P_2 - P_1 = \pi - \pi/2 = \pi/2$

Firstterm$= P_1 = a = \pi/2$cm

By the sum of n term formula, we know,

$S_n = n/2\ [2a + (n - 1)d]$

Therefor,Sumofthelengthof13consecutivecirclesis;

$S_{13} = 13/2[2(\pi/2) + (13-1)\pi/2]$

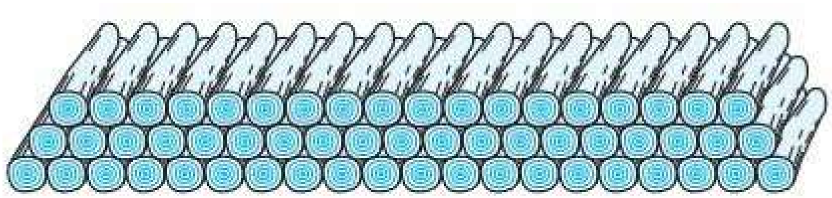
$= 13/2[\pi + 6\pi]$

$= 13/2(7\pi)$

$= 13/2 \times 7 \times 22/7$

$= 143$cm

19. 200logsarestackedinthefollowingmanner:20logsinthebottomrow,19in thenextrow,18intherownexttoitandsoon.Inhowmanyrowsarethe200 logs placed and how many logs are in the top row?



Solution:

WecanseethatthenumbersoflogsinrowsareintheformofanA.P.20,19,18…

ForthegivenA.P.,

Firstterm,a = 20 andcommondifference,d= $a_2 - a_1$= 19−20 = −1

Let a total of 200 logs be placed in n rows.

Thus, S_n = 200

By the sum of nth term formula,

S_n= n/2 [2a +(n -1)d]

S_{12} =12/2[2(20)+(n-1)(-1)] 400

= n (40−n+1)

400=n(41-n)

400=41n−n_2

n_2−41n+400 =0

n_2−16n−25n+400=0

n(n−16)−25(n−16)=0

(n−16)(n−25)=0

Either(n −16) = 0 orn−25 = 0

n = 16 or n = 25

By the nth term formula,

a_n = a+(n−1)d

a_{16}=20+(16−1)(−1)

a_{16}=20−15

a_{16}=5

Similarly, the 25$_{th}$term could be written as;

a_{25} = 20+(25−1)(−1)

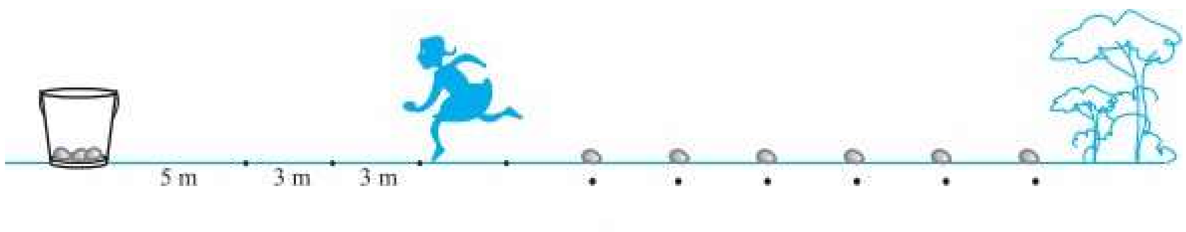
a_{25}=20−24

=−4

It can be seen, the number of logs in 16^{th}row is 5 as the numbers cannot be negative.

Therefore, 200 logs can be placed in 16 rows and the number of logs in the 16^{th}row is 5.

20. In a potato race, a bucket is placed at the starting point, which is 5 m from the firstpotato and otherpotatoes are placed 3 m apartin a straightline. There areten potatoes in the line.



A competitor starts from the bucket, picks up the nearest potato, runs back with it, drops it in the bucket, runs back to pick up the next potato, runs to the bucket to dropitin,andshecontinuesinthesamewayuntilallthepotatoesareinthebucket. What is the total distance the competitor has to run?
[Hint:topickupthefirstpotatoandthesecondpotato,thetotaldistance(inmetres) run by a competitor is 2×5+2×(5+3)]

Solution:

Thedistancesofpotatoesfromthebucketare5,8,11,14…,whichisintheformof AP.

Given, the distance run by the competitor for collecting these potatoes are two times of the distance at which the potatoes have been kept.

Therefore,distancestoberunw.r.tdistancesofpotatoes,couldbewrittenas;

10,16,22,28, 34,……….

Hence, the firstterm, a=10andd=16−10=6

S_{10}=?

By the formula of sum of n terms, we know,

S_{10}= 10/2 [2(10)+(10 -1)(6)]

=5[20+54]

=5(74)

=370

Therefore,thecompetitorwillrunatotaldistanceof370m.

C00rdinategeometry

exercise 7.1

1. Findthedistancebetweenthefollowingpairsofpoints:

(i) (2,3),(4,1)

(ii) (-5,7),(-1,3)

(iii) (a, b), (- a, – b)

Solution:

Distance formula to find the distance between two points (x_1, y_1) and (x_2, y_2) is, say d,

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad \text{OR } d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

(i) Distance between (2, 3), (4, 1)

$$d = \sqrt{(4-2)^2 + (1-3)^2} = \sqrt{(2)^2 + (-2)^2} = \sqrt{8} = 2\sqrt{2}$$

(ii) Distance between (-5, 7), (-1, 3)

$$d = \sqrt{(-1+5)^2 + (3-7)^2} = \sqrt{(4)^2 + (-4)^2} = \sqrt{32} = 4\sqrt{2}$$

(iii) Distance between (a, b), (- a, - b)

$$d = \sqrt{(-a-a)^2 + (-b-b)^2} = \sqrt{(-2a)^2 + (-2b)^2} = \sqrt{4a^2 + 4b^2} = 2\sqrt{a^2 + b^2}$$

2. Find the distance between the points (0, 0) and (36, 15). Can you now find the distancebetweenthetwotowns,AandB,discussedinSection7.2?

Solution:

Let us consider town A at point (0, 0). Therefore, townB will be at point (36,

15).Distance between points (0, 0) and (36, 15)

$$d = \sqrt{(36-0)^2 + (15-0)^2} = \sqrt{(36)^2 + (15)^2} = \sqrt{1296 + 225} = \sqrt{1521} = 39$$

Insection7.2,Ais(4,0)andBis(6,0)
$AB^2=(6-4)^2-(0-0)^2=4$

The distance between towns A and B will be 39 km. The distance between the two towns, A and B, discussed in Section 7.2, is 4 km.

3. Determineifthepoints(1,5),(2,3)and(-2,-11)arecollinear.

Solution: If the sum of the lengthsof anytwo line segmentsis equal to the length of the third line segment, then all three points are collinear.

Consider,A=(1,5)B=(2,3)andC=(-2,-11)

Findthedistancebetweenpoints:sayAB,BCandCA

$$AB = \sqrt{(2-1)^2 + (3-5)^2} = \sqrt{(1)^2 + (-2)^2} = \sqrt{1+4} = \sqrt{5}$$
$$BC = \sqrt{(-2-2)^2 + (-11-3)^2} = \sqrt{(-4)^2 + (-14)^2} = \sqrt{16+196} = \sqrt{212}$$
$$CA = \sqrt{(-2-1)^2 + (-11-5)^2} = \sqrt{(-3)^2 + (-16)^2} = \sqrt{9+256} = \sqrt{265}$$

SinceAB+BC≠CA

Therefore,thepoints(1,5),(2,3),and(-2,−11)arenotcollinear.

4. Checkwhether(5,– 2), (6, 4) and(7,– 2) aretheverticesofanisosceles triangle.

Solution:

Sincetwosidesofanyisoscelestriangleareequal,tocheckwhethergivenpointsare vertices of an isosceles triangle, we will find the distance between all the points.

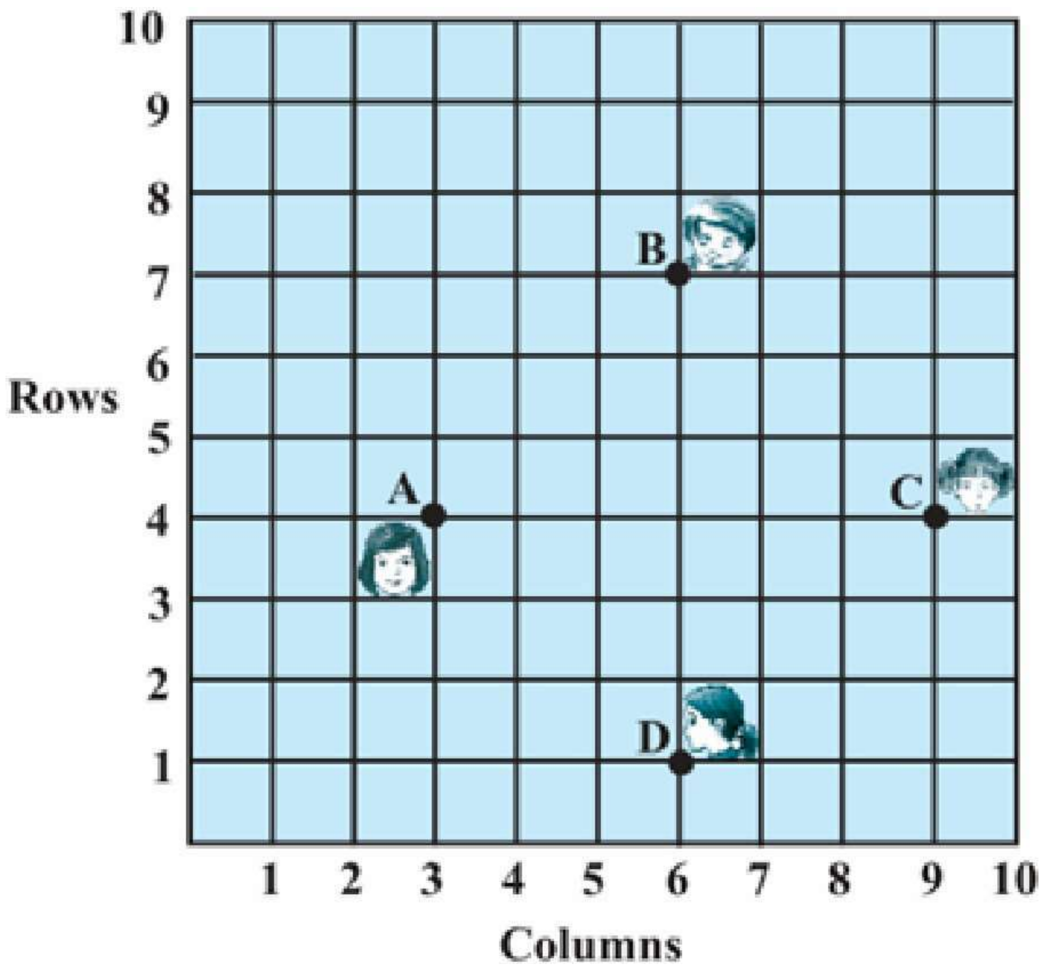
Letthe points (5, – 2), (6, 4),and (7, –2) represent the vertices A, Band C, respectively.

$$AB = \sqrt{(6-5)^2 + (4+2)^2} = \sqrt{(-1)^2 + (6)^2} = \sqrt{37}$$
$$BC = \sqrt{(7-6)^2 + (-2-4)^2} = \sqrt{(-1)^2 + (6)^2} = \sqrt{37}$$
$$CA = \sqrt{(7-5)^2 + (-2+2)^2} = \sqrt{(-2)^2 + (0)^2} = 2$$

Here AB = BC = $\sqrt{37}$

thisimplieswhethergivenpointsareverticesofanisoscelestriangle.

5. In a classroom, 4 friends areseatedatpoints A, B, CandD, as showninFig. 7.8. Champa and Chameli walk into the class, and after observing for a few minutes, ChampaasksChameli,"Don'tyouthinkABCDisasquare?"Chamelidisagrees.Using the distance formula, find which of them is correct.



Solution:

Fromthefigure,thecoordinatesofpointsA,B,CandDare(3,4),(6,7),(9,4) and(6,1).

Find the distancebetween pointsusingthe distance formula,we get

$$AB = \sqrt{(6-3)^2 + (7-4)^2} = \sqrt{9+9} = 3\sqrt{2}$$

$$BC = \sqrt{(9-6)^2 + (4-7)^2} = \sqrt{9+9} = 3\sqrt{2}$$

$$CD = \sqrt{(6-9)^2 + (1-4)^2} = \sqrt{9+9} = 3\sqrt{2}$$

$$DA = \sqrt{(6-3)^2 + (1-4)^2} = \sqrt{9+9} = 3\sqrt{2}$$

$$\text{Diagonal } AC = \sqrt{(3-9)^2 + (4-4)^2} = \sqrt{(-6)^2 + 0^2} = 6$$

$$\text{Diagonal } BD = \sqrt{(6-6)^2 + (7-1)^2} = \sqrt{0^2 + (6)^2} = 6$$

$$AB = BC = CD = DA = 3\sqrt{2}$$

Allsidesareofequallength.Therefore,ABCDisasquare,andhence,Champa was correct.

6. Name the type ofquadrilateral formed, if any, bythe following points, and givereasons for your answer:

(i) (-1,–2),(1,0),(-1,2),(-3,0)

(ii) (-3,5),(3,1),(0,3),(-1,–4)

(iii) (4,5),(7,6),(4,3),(1,2)

Solution:

(i) Letthepoints(-1,−2),(1,0),(−1,2),and(−3,0)representtheverticesA, B, C, and D of the given quadrilateral, respectively.

$$AB = \sqrt{(1+1)^2 + (0+2)^2} = \sqrt{4+4} = 2\sqrt{2}$$
$$BC = \sqrt{(-1-1)^2 + (2-0)^2} = \sqrt{4+4} = 2\sqrt{2}$$
$$CD = \sqrt{(-3+1)^2 + (0-2)^2} = \sqrt{4+4} = 2\sqrt{2}$$
$$DA = \sqrt{(-3+1)^2 + (0-2)^2} = \sqrt{4+4} = 2\sqrt{2}$$
$$AC = \sqrt{(-1+1)^2 + (2+2)^2} = \sqrt{0+16} = 4$$
$$BD = \sqrt{(-3-1)^2 + (0-0)^2} = \sqrt{16+0} = 4$$

Sidelength=AB=BC=CD=DA=$2\sqrt{2}$

DiagonalMeasure=AC=BD=4

Therefore,thegivenpointsaretheverticesofasquare.

(ii) Letthepoints (-3, 5), (3, 1),(0, 3),and(-1, –4)representthevertices A,B, C, and D of the given quadrilateral, respectively.

$$AB = \sqrt{(-3-3)^2 + (1-5)^2} = \sqrt{36+16} = 2\sqrt{13}$$
$$BC = \sqrt{(0-3)^2 + (3-1)^2} = \sqrt{9+4} = \sqrt{13}$$
$$CD = \sqrt{(-1-0)^2 + (-4-3)^2} = \sqrt{1+49} = 5\sqrt{2}$$
$$AD = \sqrt{(-1+3)^2 + (-4-5)^2} = \sqrt{4+81} = \sqrt{85}$$

It's alsoseenthatpoints A, BandCarecollinear.
So, the given points can only form 3 sides, i.e. a triangle and not a quadrilateral which has 4 sides.
Therefore,thegivenpoints cannotforma generalquadrilateral.

(iii) Let the points (4, 5), (7, 6), (4, 3), and (1, 2) represent the vertices A, B, C, and D of the given quadrilateral, respectively.

$$AB = \sqrt{(7-4)^2 + (6-5)^2} = \sqrt{9+1} = \sqrt{10}$$
$$BC = \sqrt{(4-7)^2 + (3-6)^2} = \sqrt{9+9} = \sqrt{18}$$
$$CD = \sqrt{(1-4)^2 + (2-3)^2} = \sqrt{9+1} = \sqrt{10}$$
$$AD = \sqrt{(1-4)^2 + (2-5)^2} = \sqrt{9+9} = \sqrt{18}$$
$$AC\ (\text{diagonal}) = \sqrt{(4-4)^2 + (3-5)^2} = \sqrt{0+4} = 2$$
$$BD\ (\text{diagonal}) = \sqrt{(1-7)^2 + (2-6)^2} = \sqrt{36+16} = 13\sqrt{2}$$

Oppositesidesofthisquadrilateralareofthesamelength.However,thediagonals are of different lengths. Therefore, the given points are the vertices of a parallelogram.

7. Findthepointonthex-axiswhichisequidistantfrom(2,– 5)and(-2,9).

Solution:

Tofindapointonthex-axis.

Therefore,itsy-coordinatewillbe0.Letthepointonthex-axisbe(x,0).

ConsiderA=(x,0);B=(2,–5)andC=(-2,9).

$$AB = \sqrt{(2-x)^2 + (-5-0)^2} = \sqrt{(2-x)^2 + 25}$$
$$AC = \sqrt{(-2-x)^2 + (9-0)^2} = \sqrt{(-2-x)^2 + 81}$$

Since both the distance are equal in measure, so AB = AC

$$\sqrt{(2-x)^2 + 25} = \sqrt{(-2-x)^2 + 81}$$

Simplifytheaboveequation,

Removethesquarerootbytakingsquareonbothsides,weget $(2 -$

$x)^2 + 25 = [-(2 + x)]^2 + 81$

$(2-x)^2+25=(2+x)^2+81$

$x^2+4-4x+25=x^2+4+4x+81$ $8x = 25$

$- 81 = -56$

x=-7

Therefore,thepointis(-7,0).

8. Findthevaluesofyforwhichthedistancebetweenthepoints P(2,–3)andQ (10, y) is 10 units.

Solution:

Given: Distance between (2, – 3) and (10, y) is 10.

Using the distance formula,

$$PQ = \sqrt{(10-2)^2 + (y+3)^2} = \sqrt{(8)^2 + (y+3)^2}$$

Since PQ = 10

$$\sqrt{(8)^2 + (y+3)^2} = 10$$

Simplifytheaboveequationandfindthevalueofy. Squaring

both sides,

$64+(y+3)^2=100$

$(y+3)^2=36$

$y+3=\pm 6$

$y+3=+6$ or $y+3=-6$

$y=6-3=3$ or $y=-6-3=-9$

Therefore, $y=3$ or -9.

9. If Q (0, 1) is equidistant from P (5, –3) and R (x, 6), find the values of x. Also, find the distance QR and PR.

Solution:

Given: Q (0, 1) is equidistant from P (5,– 3) and R (x, 6), which means PQ = QR

Step 1: Find the distance between PQ and QR using the distance formula,

PQ = $\sqrt{(5-0)^2 + (-3-1)^2} = \sqrt{(-5)^2 + (-4)^2} = \sqrt{25+16} = \sqrt{41}$

QR = $\sqrt{(0-x)^2 + (1-6)^2} = \sqrt{(-x)^2 + (-5)^2} = \sqrt{x^2+25}$

Step 2: Use PQ = QR

$\sqrt{41} = \sqrt{x^2+25}$

Squaring both sides to omit square root $41 =$

$x^2 + 25$

$x^2=16$

$x=\pm 4$

$x=4$ or $x=-4$

Coordinates of Point R will be R(4,6) or R(-4,6),

If R(4,6), then QR

QR = $\sqrt{(0-4)^2 + (1-6)^2} = \sqrt{(4)^2 + (-5)^2} = \sqrt{16+25} = \sqrt{41}$

PR = $\sqrt{(5-4)^2 + (-3-6)^2} = \sqrt{(1)^2 + (9)^2} = \sqrt{1+81} = \sqrt{82}$

If R (-4, 6), then

QR = $\sqrt{(0+4)^2 + (1-6)^2} = \sqrt{(4)^2 + (-5)^2} = \sqrt{16+25} = \sqrt{41}$

PR = $\sqrt{(5+4)^2 + (-3-6)^2} = \sqrt{(9)^2 + (9)^2} = \sqrt{81+81} = 9\sqrt{2}$

10. Findarelationbetweenxandysuchthatthepoint(x,y)isequidistantfromthe point(3,6)and(-3,4).

Solution:

Point(x,y)isequidistantfrom(3,6)and(-3,4).

$$\sqrt{(x-3)^2 + (y-6)^2} = \sqrt{(x-(-3))^2 + (y-4)^2}$$
$$\sqrt{(x-3)^2 + (y-6)^2} = \sqrt{(x+3)^2 + (y-4)^2}$$

Squaringbothsides,$(x-3)_2+(y-6)_2 = (x+3)_2 +(y-4)_2$ $x_2 + 9$

$- 6x + y_2+ 36 - 12y = x_2 + 9 + 6x + y_2 +16 - 8y$ $36 -16$

$= 6x+ 6x+ 12y -8y$

$20=12x+ 4y$

$3x+y=5$

$3x+y-5=0$

Exercise7.2

1. Find the coordinates of the point which divides the join of (-1, 7) and (4, –3) in the ratio 2:3.

Solution:

Let P(x,y) be the required point. Using the section formula,we get x

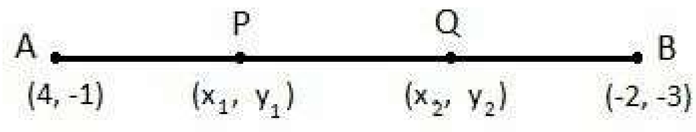
$= (2×4 + 3×(-1))/(2 + 3) = (8 - 3)/5 = 1$

$y=(2×-3+3×7)/(2+3)=(-6+21)/5=3$

Therefore,thepointis(1,3).

2. Findthecoordinatesofthepointsoftrisectionofthelinesegmentjoining(4,-1) and (-2, -3).

Solution:



Let P (x_1, y_1) and Q (x_2, y2) be the points of trisection of the line segment joining the given points, i.e. AP = PQ = QB

Therefore, point P divides AB internally in the ratio 1:2.

x_1 = (1×(-2) + 2×4)/3 = (-2 + 8)/3 = 6/3 = 2

y_1=(1×(-3)+2×(-1))/(1+2)=(-3−2)/3=-5/3

Therefore:P(x_1,y_1)=P(2,-5/3)

Point Q divides AB internally in the ratio 2:1.

x_2=(2×(-2)+1×4)/(2+1)=(-4+4)/3=0

y_2=(2×(-3)+1×(-1))/(2+1)=(-6−1)/3=-7/3

ThecoordinatesofthepointQare(0,-7/3)

3. To conduct sports day activities in your rectangular-shaped school ground ABCD, lines havebeendrawnwithchalk powder at a distanceof1 meach.100 flower pots have been placed at a distance of 1 m from each other along AD, as shown in the followingfigure.Niharika runs 1/4th thedistanceADonthe2ndlineandposts a greenflag. Preet runs 1/5th thedistanceAD ontheeighth lineandposts a redflag. What is the distance between both flags?If Rashmi has to post a blue flag exactly halfway between the line segment joining the two flags, where should she post her flag?

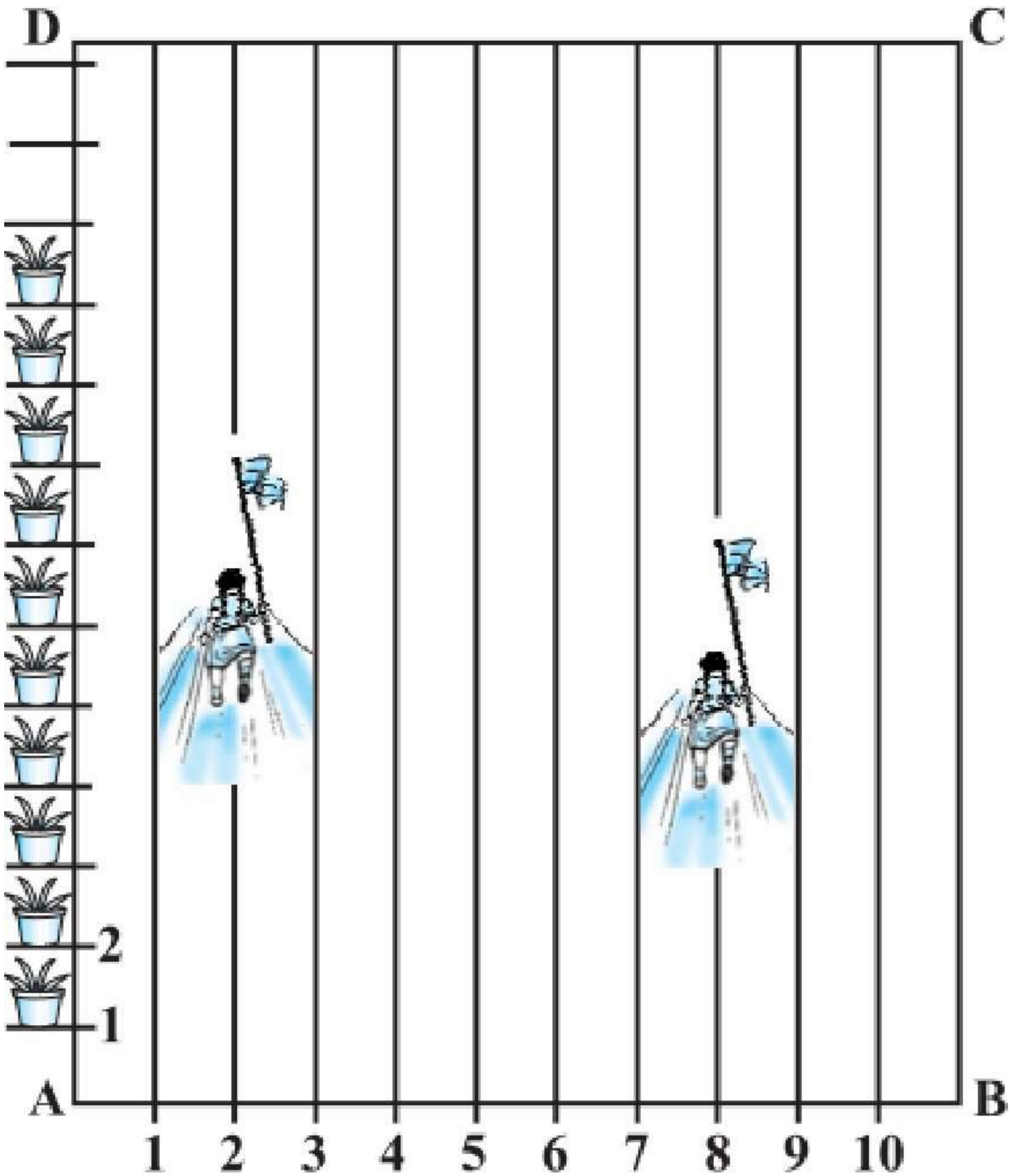


Fig. 7.12

Solution:

From the given instruction, we observed that Niharika posted the green flag at1/4thofthedistanceAD, i.e.(1/4×100)m =25m from thestartingpointofthe 2nd line. Therefore, the coordinates of this point are (2, 25).

Similarly, Preet posted ared flag at 1/5 of the distance AD, i.e. (1/5×100) m =20 mfromthestartingpointofthe8thline.Therefore,thecoordinatesofthispoint are (8, 20).

Distancebetweentheseflagscanbecalculatedbyusingthedistanceformula,

Distance between two flags = $\sqrt{(8-2)^2+(20-25)^2} = \sqrt{36+25} = \sqrt{61}$ m

ThepointatwhichRashmishouldpostherblueflagisthemid-pointoftheline joiningthesepoints.Let'ssaythispointisP(x,y).

x=(2+8)/2=10/2=5andy=(20+25)/2=45/2

Hence,P(x,y)=(5,45/2)

Therefore,Rashmishouldpostherblueflagat45/2=22.5monthe5thline.

4. Find the ratio in whichthe line segment joining the points (-3, 10) and (6,– 8) is divided by (-1, 6).

Solution:

Consider the ratio in which the line segment joining (-3, 10) and (6, -8) is divided by point (-1, 6) be k :1.

Therefore,-1=(6k-3)/(k+1)

−k−1=6k-3

7k=2

k=2/7

Therefore,the requiredratio is2:7.

5. Find the ratio in whichthe line segment joining A (1, – 5) and B (- 4, 5) is dividedbythex-axis.Also,findthecoordinatesofthepointofdivision.

Solution:

LettheratioinwhichthelinesegmentjoiningA(1,−5)andB(-4,5)isdivided bythex-axisbek:1. Therefore,thecoordinatesofthepointofdivision,sayP(x,y) is ((-4k+1)/(k+1), (5k-5)/(k+1)).

Or $P(x, y) = \frac{-4k+1}{k+1}, \frac{5k-5}{k+1}$

We know that the y-coordinate of any point on the x-axis is 0.

Therefore, (5k − 5)/(k + 1) = 0

5k=5

ork= 1

So,the x-axisdividesthe line segment in the ratio 1:1.

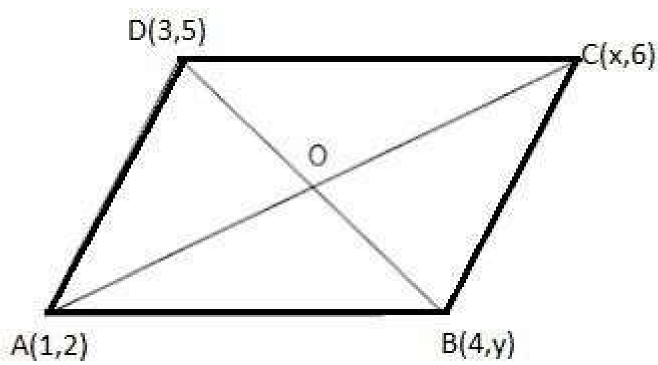
Now, find the coordinates of the point of division:

P (x,y)= ((-4(1)+1)/(1+1),(5(1)-5)/(1+1))=(-3/2 ,0)

6. If(1, 2), (4, y), (x,6) and(3, 5) arethevertices ofa parallelogram takenin order, find x and y.

Solution:

Let A, B, C and D be the points of a parallelogram: A(1, 2), B(4, y), C(x, 6) and D(3, 5).



Sincethediagonalsofaparallelogrambisect eachother,themidpoint isthesame. To find the value of x and y, solve for the midpoint first.

Midpoint of AC = ((1+x)/2 , (2+6)/2) = ((1+x)/2 , 4)

Midpoint of BD = ((4+3)/2 , (5+y)/2) = (7/2 , (5+y)/2)

The midpoint of AC and BD are the same, this implies

(1+x)/2 = 7/2 and 4 = (5+y)/2

x+1=7and5+y=8 x = 6

and y = 3

7. Find the coordinates of point A, whereAB is the diameter of a circle whose centreis(2,–3)andBis(1,4).

Solution:

LetthecoordinatesofpointAbe(x,y).

Mid-point of AB is (2, – 3), which is the centre of the

circle.Coordinate of B = (1, 4)

(2,-3)=((x+1)/2,(y+4)/2)

(x+1)/2=2and(y+4)/2=-3 x +

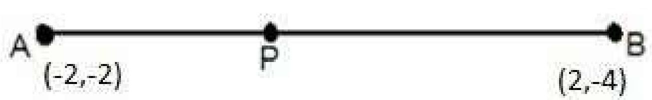
1 = 4 and y + 4 = -6

x=3andy=-10

ThecoordinatesofA(3,-10).

8. If A and B are (-2, -2) and (2, -4), respectively, find the coordinates of P such that AP = 3/7AB and P lies on the line segment AB.

Solution:



The coordinates of points A and B are (-2,-2) and (2,-4), respectively.

Since AP = 3/7 AB

Therefore,AP:PB=3:4

PointPdividesthelinesegmentABintheratio3:4.

Coordinate of P = $\left(\frac{3(2)+4(-2)}{3+4}, \frac{3(-4)+4(-2)}{3+4}\right) = \left(\frac{6-8}{7}, \frac{-12-8}{7}\right) = \left(-\frac{2}{7}, -\frac{20}{7}\right)$ which is required answer.

9. FindthecoordinatesofthepointswhichdividethelinesegmentjoiningA(-2,2) and B (2, 8) into four equal parts.

Solution:

Drawafigure,linedividingby4points.

From the figure, it can be observed that points X, Y, and Z are dividing the line segment in a ratio 1:3, 1:1, and 3:1, respectively.

Coordinates of X = $\left(\frac{1(2)+3(-2)}{1+3}, \frac{1(8)+3(2)}{1+3}\right)$ = (-1, 7/2)

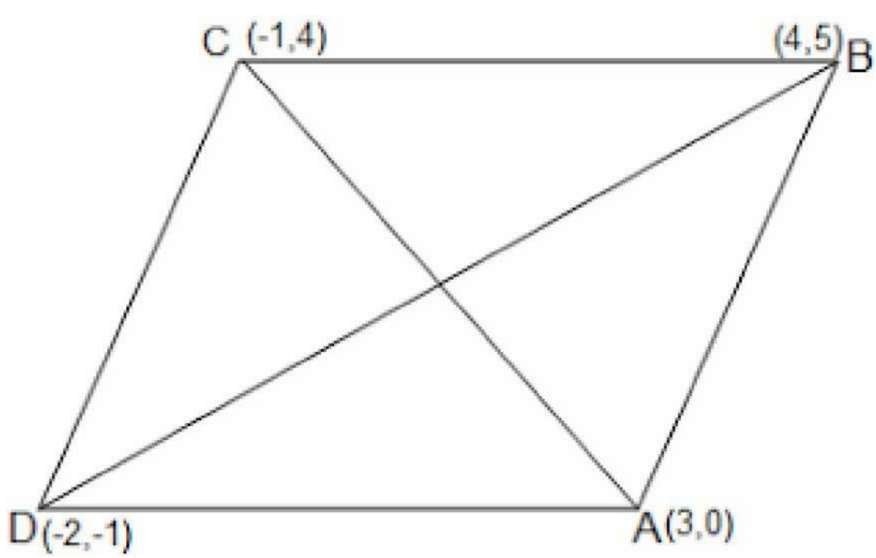
Coordinates of Y = $\left(\frac{2(1)-2(1)}{1+1}, \frac{2(1)+8(1)}{1+1}\right)$ = (0, 5)

Coordinates of Z = $\left(\frac{3(2)+1(-2)}{1+3}, \frac{3(8)+1(2)}{1+3}\right)$ = (1, 13/2)

10. Find the area of a rhombus if its vertices are (3,0), (4,5),(-1,4),and (-2,-1) taken in order.

[Hint: Area of a rhombus = 1/2 (product of its diagonals)Solution:

LetA(3,0),B(4,5),C(−1,4)andD (−2,−1)aretheverticesofa rhombus ABCD.



Length of diagonal AC = $\sqrt{(3-(-1)^2+(0-4)^2} = \sqrt{16+16} = 4\sqrt{2}$
Length of diagonal BD = $\sqrt{(4-(-2)^2+(5-(-1))^2} = \sqrt{36+36} = 6\sqrt{2}$
Therefore, area of rhombus ABCD = $\frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}$ = 24 square units

CIRCLES

Chapter10

Ex10.1

1. Howmanytangentscanacirclehave?

Solution:Asweknowthatif apoint liesona circle thenat thispointonlyonetangent canbedrawnto the circle. Since the circle has infinite points on it and at each point, we can draw one tangent, hence infinite tangents can be drawn to a circle.

2. Fillintheblanks:

(i) A tangentto acircleintersects itin ___________point (s).
(ii) A lineintersectingacircle intwopoints is calleda____________.
(iii) A circle can have_________paralleltangents atthemost.
(iv) Thecommon pointofatangentto acircleandthecircleis called _____________.

Solution:

(i) one
Ifalineis atangenttoacirclethenitmeets thecircleonlyatonepoint.

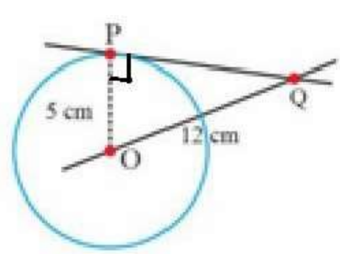
(ii) secant
Ifalineintersectsacircleattwodistinctpoints,thenthislineis calledasecantofthecircle.

(iii) two
tangentsdrawnattheendsofanydiameter areParallel.Adiameter containsonlytwoends,hence maximum two parallel tangents can be drawn to a circle.

(iv) point of contact
Alinemeetsa circleat exactlyonepointiscalledatangenttothecircleand thepointwhereline touches the circle is called point of contact.

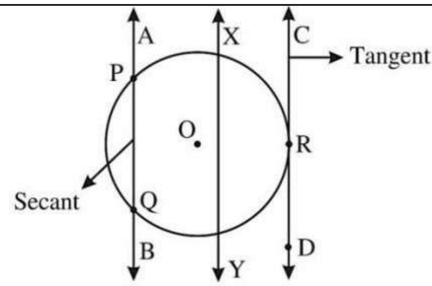
3. AtangentPQatapointPofacircleofradius5cmmeetsalinethroughthecentreOatapointQso thatOQ = 12 cm. Length PQ is:
A12cm (B)13cm (C)8.5cm (D)$\sqrt{119}$ cm.

Solution:(D)

Asweknowradiusisalwaysperpendicularto the tangent at the point of contact i.e.OP PQ. Now, applying Pythagoras theorem in ΔOPQ, $OP^2+PQ^2=OQ^2$ $\Rightarrow 5^2+PQ^2=12^2$ $\Rightarrow PQ^2=144-25$ $PQ = \sqrt{119}$ cm HencelengthofPQis $\sqrt{119}$ cm	

4. Draw a circle and two lines parallel to a given line such that one is a tangent and the other, a secantto the circle.

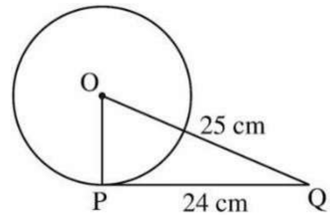
Solution:

We know how to draw a line paralleltosomeotherline.Yes,we can draw infinite such lines which are parallel to a given line. ConsiderthelineXY. Wealsoknowifalineintersectsa circle at two distinct points then this line is called secant to the circle. So,ifweshiftthelineXYparallel toitselfon left-handorright-hand side then these lines intersect the circle at two distinct points and represent secant to the circle for example line AB. Also,alinewhichmeetthecircleat exactly one point is called tangent. So, if we shift the line XY on right hand side such that it meets the circle at exactly one point like the line CD, will represent tangent to the circle.	

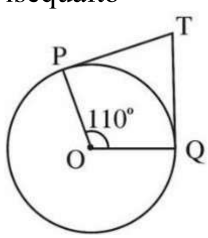
Ex10.2

InQ.1to3,choosethecorrect optionandgivejustification.

1. Froma pointQ,thelengthofthetangenttoacircleis24cmandthedistanceofQfromthecentreis 25cm. Theradiusofthecircleis
   - (A) 7cm
   - (B) 12cm
   - (C) 15cm
   - (D) 24.5cm**Solution:(A)**

LetObethecenterofthecircleandthetangent from Q meet the circle at P. Hence the length PQ will represent length ofthetangentfromQwhichisgiven24cm i.e.PQ=24cm (i)AlsoOQ=25 cm (ii) Asweknowradiusisperpendiculartotangent at the point of contact i.e. OP PQ. Hence applying PythagorastheoreminΔOPQ,weget $OP^2+PQ^2=OQ^2$	
$OP^2+24^2=25^2$ $OP^2=625-576$ $OP^2=49$ $OP=7$ Thus, theradiusof thecircleis7cm.	

2. InFig.,ifTPandTQarethetwotangentstoacirclewithcentreOsothat $POQ=110^\circ$,then$\angle PTQ$ isequalto



(A) 60°
(B) 70°
(C) 80°
(D) 90°

Solution:(B)

Asweknowradiusisperpendiculartotangentatthepointofcontact,henceOP TPand

OQ TQ.

$OPT=90^\circ$and $OQT=90^\circ$ (i)

Also, in aquadrilateralsumof interior angles is 360°.Hencefor the quadrilateral POQT, we can write

$OPT+POQ+OQT+PTQ=360^\circ$ (ii)⇒ $90^\circ+110^\circ+90^\circ+PTQ=360^\circ$

$PTQ=70^\circ$.

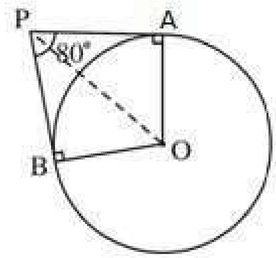
3. IftangentsPAandPBfromapointPtoacirclewithcentreOareinclinedtoeachotheratangleof 80°,then POAisequalto

(A) 50°
(B) 60°
(C) 70°
(D) 80°

Solution:(A)

Asweknowradiusisperpendiculartotangentatthepointofcontact,henceOA PAandOB PB.

$OBP=90^{\circ}$and $OAP=90^{\circ}$ (i)



Also, in aquadrilateralsumof interior angles is 360°, hencefor thequadrilateral , wecan write

$OAP+ APB+ PBO+ BOA=360^{\circ}$ (ii)

$\Rightarrow 90^{\circ}+80^{\circ}+90^{\circ}+ BOA=360^{\circ}$

$BOA=100^{\circ}$

InΔOPAandΔOPB

AP = BP (lengthoftangentsfromanexternalpointtoacircleisequal) OA =

OB (radius of circle)

OP=OP (commonside)

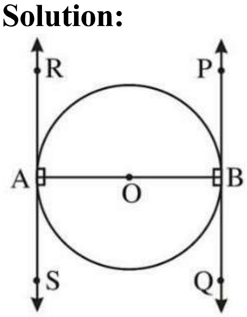
Hence,ΔOPB ΔOPA (bySSScongruency)

Hence, $\angle POA = \angle POB = \frac{1}{2}\angle BOA$

$POA = \frac{100^{\circ}}{2} = 50^{\circ}$

4. Provethatthetangentsdrawnattheendsofadiameterof acircleareparallel.

Solution:



ConsiderABasadiameterofthecircle.PQandRSaretwotangentsdrawnattheendpointsofthe diameter AB.

Asweknowthattheradiusisperpendiculartotangentatthepointof contact. Hence

$OAR = 90^{\circ}$and $OBQ = 90^{\circ}$ (i)

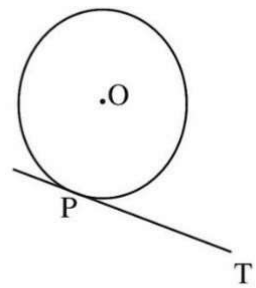
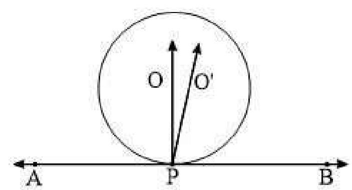
Fromalternate interioranglestheorem,we cansay

$OAR= OBQ$ (ii)

Since,alternateinterioranglesareequal,hencelinesPQandRSmustbeparallel.

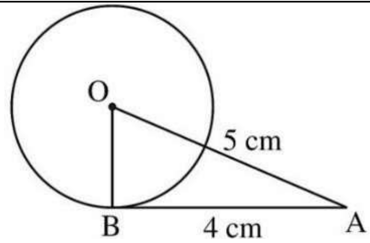
5. Provethattheperpendicularatthepointofcontacttothetangenttoacirclepassesthroughthe centre.

Solution:

LetPbethepointofcontactandPTbethetangentat the point P on the circle with centre O. SinceOPisradiusofthecircleandPTisatangentat P,OP PT. Thus,theperpendicularatthepointofcontacttothe tangent passes through the centre. LetusconsideracirclewithcentreO.LetABbea tangent which touches the circle at point P. WehavetoprovethatalineperpendiculartoAB at pointPpasses throughthecentre.	
LetusassumethattheperpendiculartoABatPdoes not pass through centre O. Let it pass through another point O'. AsperpendiculartoABatPpassesthroughO', therefore, ∠ ' =90 ……………(1) ButOisthecentreofthecircle.Asweknowtheline joining the centre and the point of contact to the tangent of the circle are perpendicular to each other. ∴ ∠ = 90 (2) Fromequation(1)&equation(2) ∠ ' = ∠ Fromthefigure, ∠ ' < ∠ ∴ ∠ ' < ∠ isnotpossible. Itisonlypossible,whenthelineO'PcoincideswithOP. Therefore, the perpendicular to AB through P passes through centre O.	

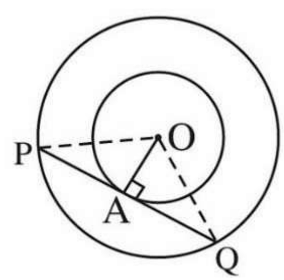
6. Thelengthof a tangentfrom apointAatdistance5cmfromthe centreof thecircleis4cm.Find the radius of the circle.

Solution:

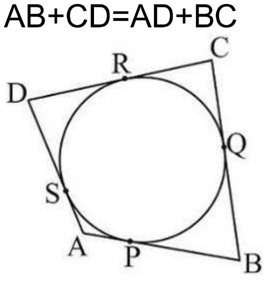
Let tangent to the circle from A meetthecircleatBandObethe centre of the circle. Given$OA=5cm$and$AB=4$ cm (i) As we know radius is perpendiculartotangentatthe point of contact i.e. $OB \perp AB$ (ii) ApplyingPythagorastheoremin ΔABO, $AB^2+OB^2=OA^2$ $\Rightarrow 4^2+OB^2=5^2$ $OB^2=9$ $OB=3$ Hence,theradius ofthecircleis 3cm.	

7. Two concentric circles are of radii5cm and3cm. Find the length of the chord of the larger circle which touches the smaller circle.

Solution:

Letthecentreofthetwoconcentriccircles be O and chord PQ of the larger circle touches the smaller circle at A. SincePQistangenttothesmallercircle, hence OA PQ. ApplyingPythagorastheoreminΔOAP,we get $OA^2+AP^2=OP^2$ $\Rightarrow 3^2+AP^2=5^2$ $AP^2=16$ $AP=4cm$ Since$AP=AQ$(Perpendicularfrom center of circle bisects the chord) $PQ=2AP=2\times4cm=8cm$ Lengthof chord oflarger circleis8cm.	

8. AquadrilateralABCDisdrawntocircumscribeacircle(seeFig.).Provethat $AB+CD=AD+BC$



Solution:

Asweknow,

Lengthofthetangentsfromanexternalpoint toacircleisalwaysequal.Hencefromthegiven diagram we can easily say

LengthofthetangentsfromthepointA:AP=AS (i)

LengthofthetangentsfromthepointB:BP=BQ (ii)

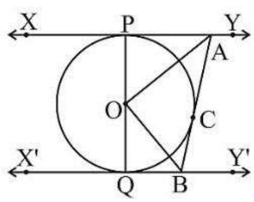
LengthofthetangentsfromthepointC:CR=CQ (iii)

LengthofthetangentsfromthepointD:DR=DS (iv)

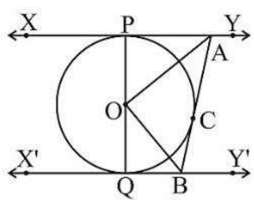
Addingtheabovefourequations,weget

AP+BP+CR+DR=AS+BQ+CQ+DS
(DR+CR)+(BP+AP)=(DS+AS)+(CQ+BQ)
CD+AB=AD+BC

9. InFig.,XYandX′Y′aretwoparalleltangentstoacirclewithcentreOandanothertangentABwithpoint ofcontactCintersectingXYatAandX′Y′atB.Provethat AOB=90_o.



Solution:



First,wewillshowthat POA= COAand QOB= COB.JoinOwiththe point of contact C.

NowinΔOPAandΔOCA,wecaneasilyobserve

OP=OC (radiusofthesamecircle)

AP=AC (lengthoftangentsfromexternalpoint)

AO=AO (commonside)

HenceΔOPA ΔOCA(bySSScongruency)

POA= COA(byCPCT)… (i)

Similarly,ΔOQB ΔOCB

QOB= COB (ii)

Since,POQisadiameterofcircle,wecansay POQ=180_o

POA+ AOC+ COB+ BOQ=180_o

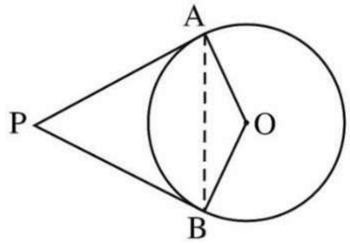
Nowfromequations(i)and(ii),

2 AOC+2 COB=180_o
(AOC+ COB)=90_o

$\angle AOB = 90^\circ$

10. Provethattheanglebetween thetwotangentsdrawnfromanexternalpointtoa circleissupplementary to the angle subtended by the line-segment joining the points of contact at the centre.

Solution:



Herewehavetoshowthat $\angle APB + \angle BOA = 180^\circ$.

LetthecentreoftheCirclebeO.TangentsfromPAandPBdrawntothecirclefromPmeet thecircle at A and B.

ABisthelinesegmentjoiningpointofcontactsAandBtogethersuchthatitsubtends $\angle AOB$atcenterOofthecircle.

Astheradius isperpendiculartothetangentatthepointofcontact.Hence,wecansay

$\angle OAP = 90^\circ$ and $\angle OBP = 90^\circ$ (i)

Asweknowsum ofinterioranglesinaquadrilateralis360°,hence inquadrilateral

OAPB

$\angle OAP + \angle APB + \angle PBO + \angle BOA = 360^\circ$

$\Rightarrow 90^\circ + \angle APB + 90^\circ + \angle BOA = 360^\circ$

$\angle APB + \angle BOA = 180^\circ$

Hence,the anglebetweenthetwotangentsdrawnfromanexternalpointto acircleissupplementary to the angle subtended by the line-segment joining the points of contact at the centre.

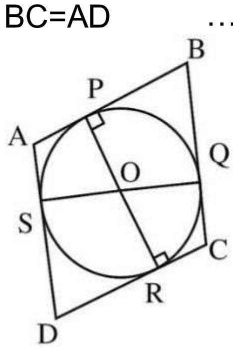
11. Provethattheparallelogramcircumscribingacircleisarhombus.

Solution:

Let ABCD be the parallelogram circumscribed about a circle with centre O and the sides AB, BC, CDandDAtouchesthecircleatpointsP,Q,RandSrespectively.Since,ABCDisaparallelogram.

AB=CD …(i)

BC=AD …(ii)



Asweknowlengthofthetangentsfromanexternal pointtoa circleisalwaysequal. Hence from

the given diagram we can easily say

LengthofthetangentsfromthepointA:AP=AS (iii)

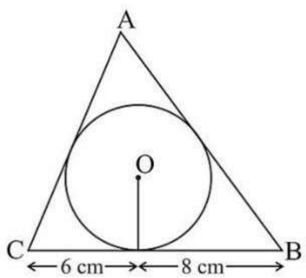
LengthofthetangentsfromthepointB:BP=BQ (iv)

LengthofthetangentsfromthepointC:CR=CQ (v)

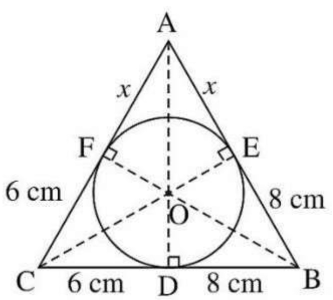
LengthofthetangentsfromthepointD:DR=DS (vi)Adding(iii),(iv),(v) and(vi),weget

DR+CR+BP+AP=DS+CQ+BQ+AS
(DR+CR)+(BP+AP)=(DS+AS)+(CQ+BQ)
CD+AB=AD+BC …(vii)Fromequation(i),
(ii)and(vii):
2AB=2BC
AB=BC AB=BC=CD=DA
Hence,ABCDisarhombus.

12. AtriangleABCisdrawntocircumscribeacircleofradius4cmsuchthatthesegmentsBDandDCinto whichBCisdividedby thepointofcontactDareoflengths8cmand6cmrespectively(see Fig.). Find the sides AB and AC.



Solution:



LetthesidesABandACofthetriangleABC touchesthecircleatEand Frespectively. Also side

BC touches the circle at D.

ConsiderthelengthofthelinesegmentAFbe .

Asweknowlengthofthetangentsfromanexternal pointtoa circleisalwaysequal.Hencefromthe given diagram we can easily say

AE = AF = x (Lengthofthetangents fromthepointA) BE

= BD = 8 cm (LengthofthetangentsfromthepointB) CF

= CD = 6 cm (Length ofthetangents fromthepointC)

Letthesemi-perimeterofthetrianglebes.

Perimeter=AB+BC+CA= +8+14+6+ =28+2
s=14+

Area of $\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)}$ (Heron's formula)

$= \quad (14+x)\big((14+x)-14\big)\big((14+x)-(6+x)\big)\big((14+x)-(8+x)\big)$ $\sqrt{}$

$= \sqrt{(14+x)(x)(8)(6)}$

Area of $\Delta ABC = 4\sqrt{3(14x+x^2)}$……………………..(i)

Area of $\Delta OBC = \frac{1}{2} \times OD \times BC = \frac{1}{2} \times 4 \times 14 = 28$…….(ii)

Area of $\times \Delta OCA = \frac{1}{2} \times OF \times AC = \frac{1}{2} \times 4 \times (16+x) = 12+2$ …….(iii)

Area of $\Delta OAB = \frac{1}{2} \times OE \times AB = \frac{1}{2} \times 4 \times (8+x) = 16+2$ (iv)

Area of ΔABC = Area of ΔOBC + Area of ΔOCA + Area of ΔOAB

$\Rightarrow 4\sqrt{3(14x+x^2)} = 28+12+2x+16+2$

$\Rightarrow 4\sqrt{3(14x+x^2)} = 56+4$

$\Rightarrow \sqrt{3(14x+x^2)} = 14+x$

$\Rightarrow 3(14 \quad + \quad ^2) = (14+ \quad)^2$

$\Rightarrow 42 \quad +3 \quad ^2 = 196 + x^2 + 28$

$\Rightarrow 2 \quad ^2 + 14 \quad -196 = 0$

$\Rightarrow \quad ^2 + 7 \quad -98 = 0$

$\Rightarrow \quad ^2 + 14 \quad -7 \quad -98 = 0$

$\Rightarrow (\quad +14) - 7(\quad +14) = 0$

$\Rightarrow (\quad +14)(\quad -7) = 0$

$\Rightarrow \quad = -14$ or 7

But $\quad = -14$ is not possible as length of sides will be negative. So,

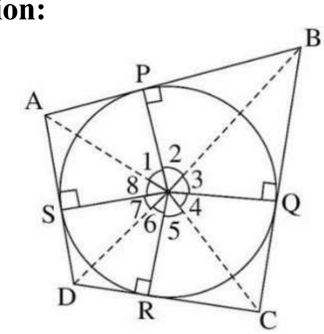
$= 7$

Hence $AB = \quad +8 = 7+8 = 15$cm

$CA = 6+ \quad = 6+7 = 13$cm

8. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Solution:



Let ABCD be a quadrilateral circumscribing a circle with centre O. Also, the sides AB, BC, CD and DA touches the circle at point P, Q, R, and respectively.

Now join the vertices of the quadrilateral ABCD to the center O of the circle.

In triangles ΔOAP and ΔOAS

AP = AS (length of tangents from same point)

OP = OS (radius of the same circle)

OA=OA (commonside)

HenceΔOAP$\cong\Delta$OAS(bySSScongruency)

POA= SOA(byCPCT)
$\Rightarrow\angle 1=\angle 8$ (i)

Similarly,wecanprove

$\angle 2=\angle 3$ (ii)

$\angle 4=\angle 5$ (iii)and

$\angle 6=\angle 7$ (iv)

Onadding(i),(ii),(iii)and(iv),weget

$\angle 1+\angle 2+\angle 3+\angle 4+\angle 5+\angle 6+\angle 7+\angle 8=360^o$
$\Rightarrow(\angle 1+\angle 8)+(\angle 2+\angle 3)+(\angle 4+\angle 5)+(\angle 6+\angle 7)=360^o$
$\Rightarrow 2\angle 1+2\angle 2+2\angle 5+2\angle 6=360^o$
$\Rightarrow 2(\angle 1+\angle 2)+2(\angle 5+\angle 6)=360^o$
$\Rightarrow(\angle 1+\angle 2)+(\angle 5+\angle 6)=180^o$
AOB+ COD=180^o
Similarly, BOC+ DOA=180^o
Henceoppositesidesofaquadrilateralcircumscribingacirclesubtendsupplementaryanglesatthe centre of the circle.

Chapter 11
Construction

Ineachofthefollowing,givethejustificationoftheconstructionalso:

1. Drawalinesegmentof length$7.6\,cm$and divideitin theratio$5:8$.Measurethetwo parts.

Solution:

StepsofConstruction:

(1) DrawlinesegmentABof$7.6\,cm$anddrawanyrayAXmakinganacuteanglewithAB.

(2) Locate$13(=5+8)$points A_1, A_2, A_3, A_4 A_{13}onAXsothat

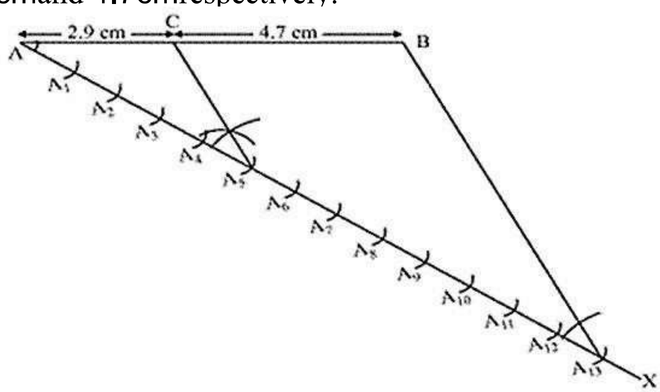
$AA_1 = A_1A_2 = A_2A_3 =$ $= A_{12}A_{13}$

(3) JoinBA_{13}.

(4) ThroughthepointA_5,drawalineparallelto$A_{13}B$(bymakingananangleequalto $AA_{13}B$)atA_5 intersectingABatthepointC.

NowCisthepointdividinglinesegmentABof$7.6\,cm$intherequiredratioof$5:8$.

WecanmeasuretheapproximatelengthsofACandCB.ThelengthofACandCBcomesto $2.9\,cm$and $4.7\,cm$respectively.



2. Constructa triangleofsides $4\,cm$,$5\,cm$and$6\,cm$andthen atrianglesimilarto itwhosesides are $\frac{2}{3}$of the corresponding sides of the first triangle.

Solution:

Thestepsofconstructionareasfollows:

(1) Draw a line segment $AB=4\,cm$. Taking point A as centre draw an arc of $5\,cm$ radius. Similarly, taking point B as its centre, draw an arc of $6\,cm$ radius. These arcs will intersect eachotheratpointC.Now$AC=5\,cm$and$BC=6\,cm$andΔABCistherequiredtriangle.

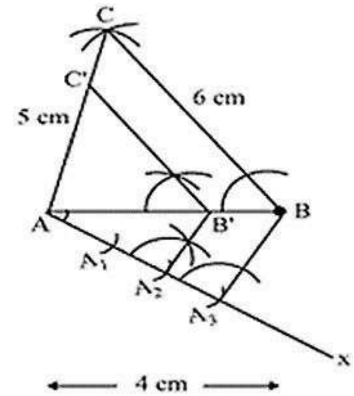
(2) DrawanyrayAXmakinganacuteanglewithABonoppositesideofvertexC.

(3) Locate3pointsA_1, A_2, A_3(as3isgreaterbetween2and3)onAXsuchthat$AA_1 = A_1A_2 = A_2A_3$

(4) JoinBA_3anddrawalinethroughA_2paralleltoBA_3tointersectABatpointB'.

(5) Draw a line through B' parallel to the line BC to intersect AC at C'. $\Delta AB'C'$ is the required triangle.

Diagram:



3. Construct a triangle with sides $5cm$, $6cm$ and $7cm$ and then another triangle whose sides are $\frac{7}{5}$ of the corresponding sides of the first triangle.

Solution:

Thestepsofconstructionareasfollows:

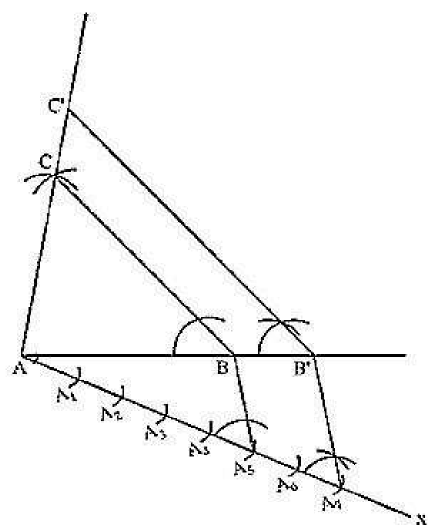
(1) Draw a line segment AB of 5cm. Taking A and B as centre, draw arcs of 6cm and 7cm radius respectively. Let these arcs intersect each other at point C. ΔABC is the required triangle having length of sides as 5 cm, 6 cm and 7 cm respectively.

(2) DrawanyrayAXmakinganacuteanglewithlineABonoppositesideofvertexC.

(3) Locate7points$A_1, A_2, A_3, A_4, A_5, A_6, A_7$(as7isgreaterbetween5and7)onAXsuchthat $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5 = A_5A_6 = A_6A_7$.

(4) Join BA_5 and draw a line through A_7 parallel to BA_5 to intersect extended line segment AB at point B′.

(5) DrawalinethroughB′paralleloBCintersectingtheextendedlinesegmentACatC′. $\Delta AB'C'$ isrequiredtriangle.



4. Constructanisoscelestrianglewhosebaseis 8cmandaltitude4cmandthenanothertrianglewhose sides are $1\frac{1}{2}$times the corresponding sides of the isosceles triangle.

Solution:

LetΔABCbeanisoscelestrianglehavingCAandCBofequallengths,baseABis8cmandADisthe altitude of length 4 cm.

Now,thestepsofconstructionareasfollows:

(1) Draw a line segment AB of 8cm. Draw arcs of same radius on both sides of line segment whiletakingpoint and asitscentre.LetthesearcsintersecteachotheratOandO′. JoinOO′.LetOO′intersectABatD

(2) TakeDascentreanddrawan arcof4cmradiuswhichcuts theextended linesegmentOO′at pointC.NowanisoscelesΔABCisformed,havingCD(attitude)as4cmandAB(base)as8 cm.

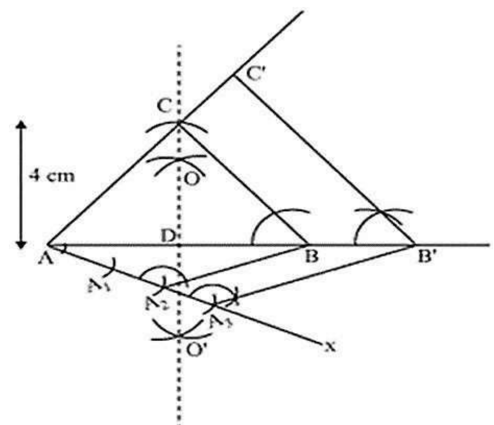
(3) DrawanyrayAXmakinganacuteanglewithlinesegmentABonoppositesideofvertexC.

(4) Locate3points(as3isgreaterbetween3and2)on AXsuchthat

$AA_1=A_1A_2=A_2A_3$.

(5) Join $BA_2$and draw a line through A_3parallel to $BA_2$to intersect extended line segment AB at point B′.

(6) DrawalinethroughB′paralleltoBCintersectingtheextendedlinesegmentACatC′.ΔAB′C′ istherequiredtriangle.



5. DrawatriangleABCwithsideBC=6cm,AB=5cmand ABC=60o.Thenconstructatriangle whose sides are $\frac{3}{4}$of the corresponding sides of the triangle ABC.

Solution:

Thestepsofconstructionareasfollows:

(1) Draw a line segment BC of length 6cm. Draw an arc of any radius while taking B as centre. Letitintersectline BCatpointO.Nowtaking Oascentredrawanotherarctocuttheprevious arc at point O′. Join BO′ which is the ray making $60_o$with line BC.

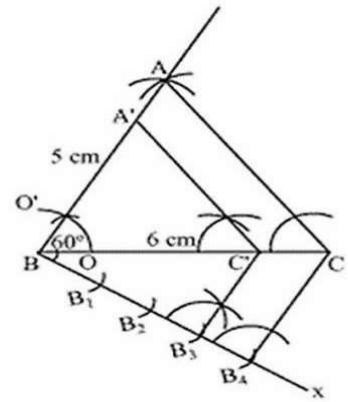
(2) Nowdrawanarcof5cmradiuswhile takingBascentre,intersectingextendedlinesegment BO′atpointA.JoinAC.Δ ABCishavingAB=5cm.BC=6cmand ABC=60_o.

(3) DrawanyrayBXmakinganacuteanglewithBConoppositesideofvertexA.

(4) Locate4points(as4 isgreaterin3 and4).B_1,B_2,B_3,B_4onlinesegmentBX.

(5) JoinB_4CanddrawalinethroughB_3,paralleltoB_4CintersectingBCatC′.

(6) DrawalinethroughC′paralleltoACintersectingABatA′.ΔA′BC′istherequiredtriangle.



6. Drawatriangle ABC withside $BC=7cm$, $B=45^\circ$, $A=105^\circ$.Then,constructatrianglewhose sides are $\frac{4}{3}$ times the corresponding sides of ΔABC.

Solution:

$$B=45^\circ, \quad A= 105^\circ$$

Itis known thatthesumofallinterior angles inatriangleis 180°

$$A+ \quad B+ \quad C=180^\circ$$

$$\Rightarrow 105^\circ+45^\circ+\angle \qquad =180^\circ$$

$$\Rightarrow \angle \qquad =180^\circ-150^\circ=30^\circ$$

Now,thestepsofconstructionareasfollows:

(1) Draw a line segment $BC=7cm$. Draw an arc of any radius while taking B as centre. Let it intersects BC at P. Draw an arc from P, of same radius as before, to intersect this arc at Q. From Q, again draw an arc, of same radius as before, to cut the arc at R. Now from points Q and R draw arcs of same radius as before, to intersect each other at S. Join BS.

LetBSintersectthearcatT.FromTandPdrawarcsofsameradiusasbeforetointersect each other at U. Join BU which is making $45^\circ$with BC.

(2) Drawanarc ofany radius taking Cas itscentre. Let itintersects BC at O.Taking Oas centre, draw an arc of same radius intersecting the previous arc at O′. Now taking O and O′ s centre, drawarcsofsameradiusasbefore,tointersecteachatY.JoinCYwhichismaking30°toBC.

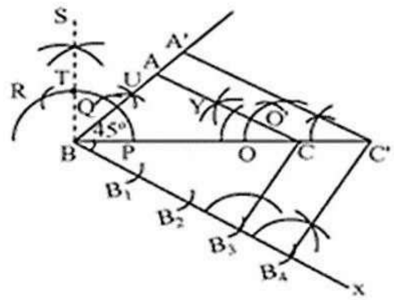
(3) Extend line segment CY and BU. Let they intersect each other at A. ΔABC is the triangle having $A = 105^\circ$, $B = 45^\circ$ and $BC = 7$ cm.

(4) DrawanyrayBXmakinganacuteanglewithBConoppositesideofvertexA.

(5) Locate4points (as 4is greaterin 4and3)B_1,B_2,B_3andB_4onBX.

(6) JoinB_3C.DrawalinethroughB_4paralleltoB_3CintersectingextendedBCatC′.

(7) ThroughC′drawalineparalleltoACintersectingextendedlinesegmentBAatA′.$\Delta A'BC'$is required triangle.



7. Draw a right triangle in which the sides (other than hypotenuse) are of lengths 4 cm and 3 cm. Then construct another triangle whose sides are $\frac{5}{3}$ times the corresponding sides of the given triangle.

Solution:

The steps of construction are as follows:

(1) Draw a line segment AB = 4 cm and draw a ray SA making 90^{o} with it.

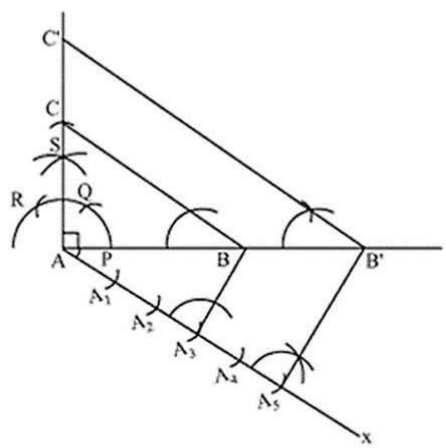
(2) Draw an arc of 3 cm radius while taking as its centre to intersect SA at C. Join BC. ΔABC is required triangle.

(3) Draw any ray AX making an acute angle with AB on the side opposite to vertex C.

(4) Locate 5 points (as 5 is greater in 5 and 3) A_1, A_2, A_3, A_4, A_5 on line segment AX.

(5) Join A_3B. Draw a line through A_5 parallel to A_3B intersecting extended line segment AB at B′.

(6) Through B′, draw a line parallel to BC intersecting extended line segment AC at C′. $\Delta AB'C'$ is required triangle.



EXERCISE 11.2

Ineachofthefollowing,givealsothejustificationoftheconstruction:

1. Draw a circle of radius . From a point away from its centre, construct the pair of tangents to the circle and measure their lengths.

Solution:

Thestepsofconstructionareasfollows:

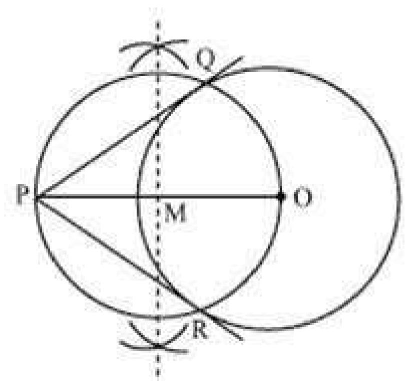
(1) Takinganypoint ofthegivenplaneascentre.Drawacircleof radius.Locateapoint , awayfrom .Join .

(2) Bisect .Let bethemidpointof .

(3) Taking ascentreand asradius,drawacircle.

(4) Letthiscircleintersectourfirstcircleatpoint and .

(5) Join and . and aretherequiredtangents.Thelengthoftangents and are each.



2. Constructatangenttoacircleofradius fromapointontheconcentriccircleofradius andmeasureitslength.Alsoverifythemeasurementbyactual calculation.

Solution:

Thestepsofconstructionareasfollows:

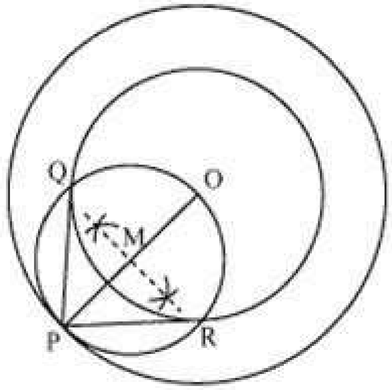
(1) Drawacircleof radiuswithcentreas onthegivenplane.

(2) Draw a circle of radius taking as its centre. Locate a point on this circle and join .

(3) Bisect .Let bethemidpointof .

(4) Taking asitscentreand asitsradiusdrawacircle.Letitintersectthegiven circle at the points and .

(5) Join and . and aretherequiredtangents.



Now,PQandPRareoflength4.47cmeach.

InΔPQO,sincePQistangent, $PQO=90^{\circ}$.

$PO = 6$ cm

$QO=4$cm

ApplyingPythagorastheoreminΔPQO,

$PQ^2+QO^2=PO^2$

$PQ^2+(4)^2=(6)^2$

$PQ^2=20$

$PQ=2\sqrt{5}=4.47$cm

3. Draw a circle of radius . Take two points and on one of its extended diameter each at a distance of from its centre. Draw tangents to the circle from these two points and .

Solution:

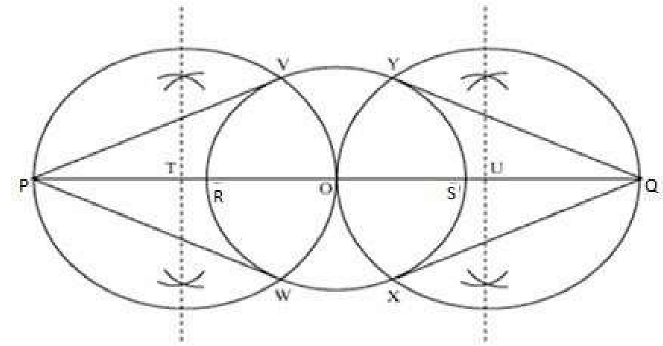
Thestepsofconstructionareasfollows:

(1) Takinganypoint ongivenplaneascentre,drawacircleof radius.

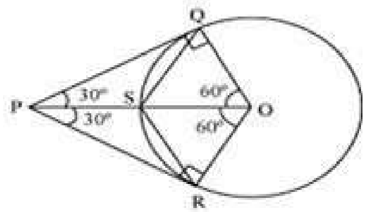
(2) Takeoneofitsdiameters, ,extendeditonbothsides.Locatetwopointsonthis diameter such that = = .

(3) Bisect and .Let and bethemidpointsof and respectively.

(4) Taking and as its centre, with and as radius, draw two circles. Thesetwocircleswillintersectourcircleatpoint , , respectively.Join , , and .Thesearerequiredtangents.



4. Drawapairoftangents toacircleofradius5cm whichareinclinedtoeachotheratanangleof60°.



Considertheabovefigure.PQandPRarethetangentstothegivencircle. If they

are inclined at 60°, then QPO= OPR=30°

Hence, POQ= POR=60°

ConsiderΔQSO,

QOS=60°

OQ = OS (radius)

So, OQS= OSQ=60°

ΔQSOisanequilateraltriangle

So, QS = SO= QO = radius

PQS=90°− OQS=$90^\circ-60^\circ=30^\circ$

QPS=30°

PS = SQ (Isoscelestriangle)

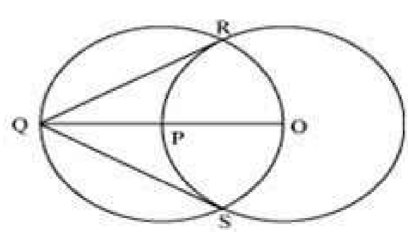
Hence, PS = SQ = OS (radius)

Now,thestepsofconstructionareasfollows:

(1) Drawacircleof 5cm radiusandwithcentreO.

(2) TakeapointPoncircumferenceofthiscircle.ExtendOPtoQsuchthat OP=PQ.

(3) MidpointofOQisP.DrawacirclewithradiusOPwithcentreasP.LetitIntersectourcircle atRandS.JoinQRandQSQRandQS arerequiredtangents.



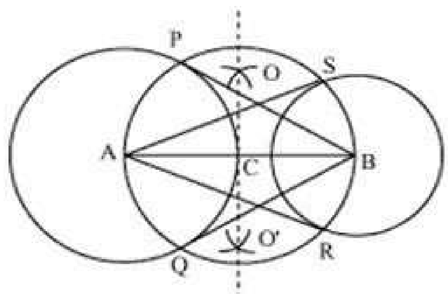
5. DrawalinesegmentABoflength8cm.TakingA ascentre,drawacircleofradius 4cmandtaking B as centre, draw another circle of radius 3cm. Construct tangents to each circle from the centre of the other circle.

Solution:

Thestepsofconstructionareasfollows:

(1) Draw alinesegment ABof 8cm.Taking A and Bas centre,draw twocircles of 4cm and 3 cm radius.

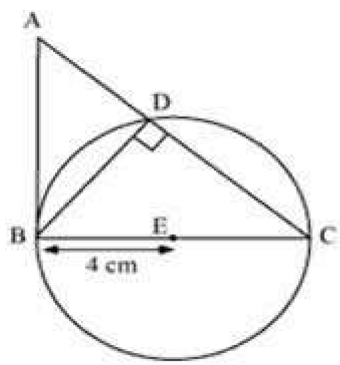
(2) BisectthelineAB.LetmidpointofABisC.TakingCascentredrawacircleofradiusAC whichwillintersectourcirclesatpointP,Q,RandS. JoinBP,BQ,ASandAR.Theseareourrequiredtangents.



6. LetABCbearighttriangleinwhichAB=6cm,BC=8cmand B =90o.BDistheperpendicular fromBonAC. Thecirclethrough B,C,D is drawn.Constructthe tangents from Atothis circle.

Solution:

Inthefollowingfigure,itcanbeseenthatifacircleisdrawnthrough B,DandC,thenBCwillbeits diameter as BDC is 90 . The centre of this circle will be the midpoint of BC.



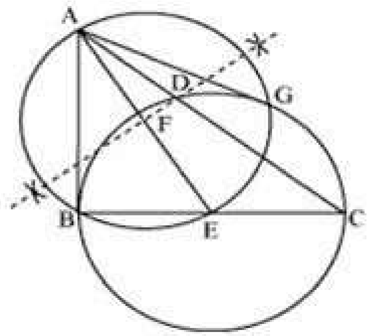
Thestepsofconstructionareasfollows:

(1) JoinAEandbisectit.LetFbethemidpointofAE.

(2) NowwithFascentreandradiusFE,drawacircleintersectingthefirstcircleatpointBandG.

(3) JoinAG.

Thus,ABandAGaretherequiredtangents.



7. Drawacircle with thehelpof a bangle. Takea point outside the circle.Construct the pair oftangents from this point to the circle.

Solution:

Thestepsofconstructionareasfollows:

(1) Drawacirclewithbangle.

(2) Takeapoint outsidethiscircleandtaketwonon-parallelchords and .

(3) Drawperpendicularbisectorsofthesechordsintersectingeachotheratpoint whichis centre of the given circle.

(4) Join andbisectit.Let bethemidpointof .With ascentreandradius ,drawacircle,intersectingourfirstcircleat and .Join and .

, and aretherequiredtangents.