

Exponents and powers

Exercise 12.1 : Solutions of Questions on Page Number : 197

Q1 :

Evaluate

$$(i) 3^{-2} \quad (ii) (-4)^{-2} \quad (iii) \left(\frac{1}{2}\right)^{-5}$$

Answer :

$$(i) 3^{-2} = \frac{1}{3^2} = \frac{1}{9} \quad \left(a^{-m} = \frac{1}{a^m}\right)$$

$$(ii) (-4)^{-2} = \frac{1}{(-4)^2} = \frac{1}{16} \quad \left(a^{-m} = \frac{1}{a^m}\right)$$

$$(iii) \left(\frac{1}{2}\right)^{-5} = \frac{1}{(2)^{-5}} = (2)^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$$

Q2 :

Simplify and express the result in power notation with positive exponent.

$$(i) (-4)^5 \div (-4)^8 \quad (ii) \left(\frac{1}{2^3}\right)^2$$

$$(iii) (-3)^4 \times \left(\frac{5}{3}\right)^4 \quad (iv) (3^{-7} \div 3^{-10}) \times 3^{-5}$$

$$(v) 2^{-3} \times (-7)^{-3}$$

Answer :

$$(i) (-4)^5 \div (-4)^8 = (-4)^{5-8} \quad (a^m \div a^n = a^{m-n})$$

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$$= (-4)^{-3}$$

$$= \frac{1}{(-4)^3} \quad \left(a^{-m} = \frac{1}{a^m} \right)$$

$$(ii) \quad \left(\frac{1}{2^3} \right)^2 = \frac{1}{(2^3)^2} = \frac{1}{2^6} \quad \left((a^m)^n = a^{mn} \right)$$

$$(iii) \quad (-3)^4 \times \left(\frac{5}{3} \right)^4 = (-1 \times 3)^4 \times \frac{5^4}{3^4}$$

$$= (-1)^4 \times 3^4 \times \frac{5^4}{3^4} \quad \left[(ab)^m = a^m \times b^m \right]$$

$$= (-1)^4 \times 5^4$$

$$= 5^4 \quad \left[(-1)^4 = 1 \right]$$

$$(iv) \quad (3^{-7} \div 3^{-10}) \times 3^{-5} = (3^{-7 - (-10)}) \times 3^{-5} \quad (a^m \div a^n = a^{m-n})$$

$$= 3^3 \times 3^{-5}$$

$$= 3^{3 + (-5)} \quad (a^m \times a^n = a^{m+n})$$

$$= 3^{-2}$$

$$= \frac{1}{3^2} \quad \left(a^{-m} = \frac{1}{a^m} \right)$$

$$(v) \quad 2^{-3} \times (-7)^{-3} = \frac{1}{2^3} \times \frac{1}{(-7)^3} \quad \left(a^{-m} = \frac{1}{a^m} \right)$$

$$= \frac{1}{[2 \times (-7)]^3} \quad \left[a^m \times b^m = (ab)^m \right]$$

$$= \frac{1}{(-14)^3}$$

Q3 :

Find the value of.

$$(i) (3^0 + 4^{-1}) \times 2^2 \quad (ii) (2^{-1} \times 4^{-1}) \div 2^{-2}$$

$$(iii) \left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2} \quad (iv) (3^{-1} + 4^{-1} + 5^{-1})^0$$

$$(v) \left\{ \left(\frac{-2}{3} \right)^{-2} \right\}^2$$

Answer :

$$(i) (3^0 + 4^{-1}) \times 2^2 = \left(1 + \frac{1}{4}\right) \times 2^2 \quad \left(a^0 = 1 \text{ and } a^{-m} = \frac{1}{a^m}\right)$$

$$= \frac{5}{4} \times 4 = 5$$

$$(ii) (2^{-1} \times 4^{-1}) \div 2^{-2} = [2^{-1} \times \{(2)^2\}^{-1}] \div 2^{-2}$$

$$= (2^{-1} \times 2^{-2}) \div 2^{-2} \quad \left((a^m)^n = a^{mn}\right)$$

$$= 2^{-1+(-2)} \div 2^{-2} \quad (a^m \times a^n = a^{m+n})$$

$$= 2^{-3} \div 2^{-2}$$

$$= 2^{-3-(-2)} \quad (a^m \div a^n = a^{m-n})$$

$$= 2^{-3+2} = 2^{-1}$$

$$= \frac{1}{2} \quad \left(a^{-m} = \frac{1}{a^m}\right)$$

$$(iii) \left(\frac{1}{2}\right)^{-2} + \left(\frac{1}{3}\right)^{-2} + \left(\frac{1}{4}\right)^{-2} = \left(\frac{2}{1}\right)^2 + \left(\frac{3}{1}\right)^2 + \left(\frac{4}{1}\right)^2 \quad \left(\therefore a^{-m} = \frac{1}{a^m}\right)$$

$$= 2^2 + 3^2 + 4^2 = 4 + 9 + 16 = 29$$

$$(iv) (3^{-1} + 4^{-1} + 5^{-1})^0 = \left(\frac{1}{3} + \frac{1}{4} + \frac{1}{5}\right)^0 \quad \left(a^{-m} = \frac{1}{a^m}\right)$$

$$= 1 (a^0 = 1)$$

$$(v) \left\{\left(\frac{-2}{3}\right)^{-2}\right\}^2 = \left\{\left(\frac{3}{-2}\right)^2\right\}^2 \quad \left(a^{-m} = \frac{1}{a^m}\right)$$

$$= \left\{\frac{3^2}{(-2)^2}\right\}^2 \quad \left[\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}\right]$$

$$= \left(\frac{9}{4}\right)^2 = \frac{81}{16}$$

Q4 :

$$\text{Evaluate (i) } \frac{8^{-1} \times 5^3}{2^{-4}} \quad (ii) (5^{-1} \times 2^{-1}) \times 6^{-1}$$

Answer :

$$(i) \frac{8^{-1} \times 5^3}{2^{-4}} = \frac{2^4 \times 5^3}{8^1} \quad \left(a^{-m} = \frac{1}{a^m}\right)$$

$$= \frac{2^4 \times 5^3}{2^3} = 2^{4-3} \times 5^3 \quad (a^m \div a^n = a^{m-n})$$

$$= 2 \times 125 = 250$$

$$(ii) (5^{-1} \times 2^{-1}) \times 6^{-1} = \left(\frac{1}{5} \times \frac{1}{2}\right) \times \frac{1}{6} \quad \left(a^{-m} = \frac{1}{a^m}\right)$$

$$= \frac{1}{10} \times \frac{1}{6} = \frac{1}{60}$$

Q5 :

Find the value of m for which $5^m \times 5^{-3} = 5^5$.

Answer :

$$5^m \times 5^{-3} = 5^5$$

$$5^{m - (-3)} = 5^5 \quad (a^m \times a^n = a^{m+n})$$

$$5^{m+3} = 5^5$$

Since the powers have same bases on both sides, their respective exponents must be equal.

$$m + 3 = 5$$

$$m = 5 - 3$$

$$m = 2$$

Q6 :

Evaluate (i) $\left\{ \left(\frac{1}{3} \right)^{-1} - \left(\frac{1}{4} \right)^{-1} \right\}^{-1}$ (ii) $\left(\frac{5}{8} \right)^{-7} \times \left(\frac{8}{5} \right)^{-4}$

Answer :

$$(i) \left\{ \left(\frac{1}{3} \right)^{-1} - \left(\frac{1}{4} \right)^{-1} \right\}^{-1} = \left\{ \left(\frac{3}{1} \right)^1 - \left(\frac{4}{1} \right)^1 \right\}^{-1} \quad \left(a^{-m} = \frac{1}{a^m} \right)$$

$$= \{3 - 4\}^{-1} = (-1)^{-1} = \frac{1}{-1} = -1$$

$$(ii) \left(\frac{5}{8} \right)^{-7} \times \left(\frac{8}{5} \right)^{-4} = \frac{5^{-7}}{8^{-7}} \times \frac{8^{-4}}{5^{-4}} \quad \left[\left(\frac{a}{b} \right)^m = \frac{a^m}{b^m} \right]$$

$$\begin{aligned}
&= \frac{8^7}{5^7} \times \frac{5^4}{8^4} & \left(a^{-m} = \frac{1}{a^m} \right) \\
&= \frac{8^{7-4}}{5^{7-4}} & \left(a^m \div a^n = a^{m-n} \right) \\
&= \frac{8^3}{5^3} = \frac{512}{125}
\end{aligned}$$

Q7 :

Simplify. (i) $\frac{25 \times t^{-4}}{5^{-3} \times 10 \times t^{-8}} (t \neq 0)$ (ii) $\frac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}}$

Answer :

$$\begin{aligned}
\text{(i)} \quad & \frac{25 \times t^{-4}}{5^{-3} \times 10 \times t^{-8}} = \frac{5^2 \times t^{-4}}{5^{-3} \times 5 \times 2 \times t^{-8}} \\
&= \frac{5^2 \times t^{-4}}{5^{-3+1} \times 2 \times t^{-8}} & \left(a^m \times a^n = a^{m+n} \right) \\
&= \frac{5^2 \times t^{-4}}{5^{-2} \times 2 \times t^{-8}} \\
&= \frac{5^{2-(-2)} t^{-4-(-8)}}{2} & \left(a^m \div a^n = a^{m-n} \right) \\
&= \frac{5^4 t^4}{2} = \frac{625 t^4}{2}
\end{aligned}$$

$$\begin{aligned}
\text{(ii)} \quad & \frac{3^{-5} \times 10^{-5} \times 125}{5^{-7} \times 6^{-5}} = \frac{3^{-5} \times (2 \times 5)^{-5} \times 5^3}{5^{-7} \times (2 \times 3)^{-5}} \\
&= \frac{3^{-5} \times 2^{-5} \times 5^{-5} \times 5^3}{5^{-7} \times 2^{-5} \times 3^{-5}} & \left[(a \times b)^m = a^m \times b^m \right] \\
&= 3^{-5-(-5)} \times 2^{-5-(-5)} \times 5^{-5+3-(-7)} & \left(a^m \div a^n = a^{m-n} \right) \\
&= 3^0 \times 2^0 \times 5^5 & \left(a^0 = 1 \right) \\
&= 5^5
\end{aligned}$$

Exercise 12.2 : Solutions of Questions on Page Number : 200

Q1 :

Express the following numbers in standard form.

(i) 0.00000000000085 (ii) 0.000000000000942

(iii) 6020000000000000 (iv) 0.00000000837

(v) 31860000000

Answer :

(i) $0.00000000000085 = 8.5 \times 10^{-12}$

(ii) $0.000000000000942 = 9.42 \times 10^{-12}$

(iii) $6020000000000000 = 6.02 \times 10^{15}$

(iv) $0.00000000837 = 8.37 \times 10^{-9}$

(v) $31860000000 = 3.186 \times 10^{10}$

Q2 :

Express the following numbers in usual form.

(i) 3.02×10^{-6} (ii) 4.5×10^4

(iii) 3×10^{-8} (iv) 1.0001×10^9

(v) 5.8×10^{12} (vi) 3.61492×10^6

Answer :

(i) $3.02 \times 10^{-6} = 0.00000302$

(ii) $4.5 \times 10^4 = 45000$

(iii) $3 \times 10^{-8} = 0.00000003$

(iv) $1.0001 \times 10^9 = 1000100000$

(v) $5.8 \times 10^{12} = 5800000000000$

(vi) $3.61492 \times 10^6 = 3614920$

Q3 :

Express the number appearing in the following statements in standard form.

(i) 1 micron is equal to $\frac{1}{1000000}$ m.

(ii) Charge of an electron is 0.000, 000, 000, 000, 000, 16 coulomb.

(iii) Size of a bacteria is 0.0000005 m

(iv) Size of a plant cell is 0.00001275 m

(v) Thickness of a thick paper is 0.07 mm

Answer :

(i) $\frac{1}{1000000} = 1 \times 10^{-6}$

(ii) $0.000, 000, 000, 000, 000, 16 = 1.6 \times 10^{-19}$

(iii) $0.0000005 = 5 \times 10^{-7}$

(iv) $0.00001275 = 1.275 \times 10^{-5}$

(v) $0.07 = 7 \times 10^{-2}$

Q4 :

In a stack there are 5 books each of thickness 20 mm and 5 paper sheets each of thickness 0.016 mm. What is the total thickness of the stack?

Answer :

Thickness of each book = 20 mm

Hence, thickness of 5 books = (5×20) mm = 100 mm

Thickness of each paper sheet = 0.016 mm

Hence, thickness of 5 paper sheets = (5×0.016) mm = 0.080 mm

Total thickness of the stack = Thickness of 5 books + Thickness of 5 paper sheets

= $(100 + 0.080)$ mm

= 100.08 mm

= 1.0008×10^2 mms