Introduction to Trigonometry Class 10

Unit 8 Introduction to Trigonometry Exercise 8.1, 8.2, 8.3, 8.4 Solutions

Exercise 8.1 : Solutions of Questions on Page Number : **181**

Q1 :

In ΔABC right angled at B, AB = 24 cm, BC = 7 m. Determine

(i) sin A, cos A

(ii) sin C, cos C

Answer :

Applying Pythagoras theorem for ΔABC, we obtain

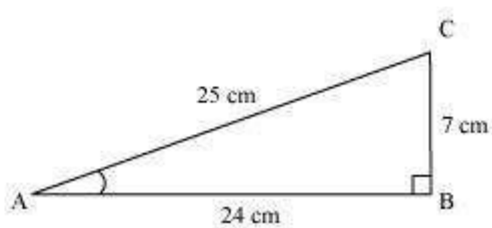
$AC^2 = AB^2 + BC^2$

$= (24 \text{ cm})^2 + (7 \text{ cm})^2$

$= (576 + 49) \text{ cm}^2$

$= 625 \text{ cm}^2$

$\therefore AC = \sqrt{625}$ cm = 25 cm

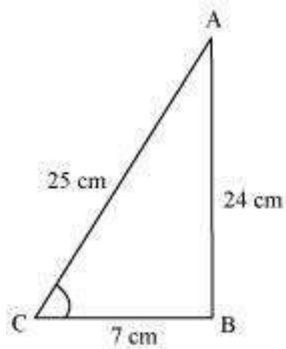


(i) sin A = $\frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC}$

$= \frac{7}{25}$

cos A = $\frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{24}{25}$

(ii)



sin C = $\dfrac{\text{Side opposite to } \angle C}{\text{Hypotenuse}} = \dfrac{AB}{AC}$

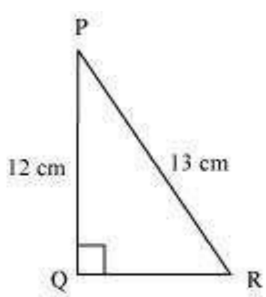
$= \dfrac{24}{25}$

cos C = $\dfrac{\text{Side adjacent to } \angle C}{\text{Hypotenuse}} = \dfrac{BC}{AC}$

$= \dfrac{7}{25}$

Q2 :

In the given figure find tan P - cot R



Answer :

Applying Pythagoras theorem for ΔPQR, we obtain

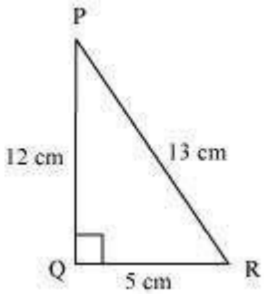
$PR^2 = PQ^2 + QR^2$

$(13\text{ cm})^2 = (12\text{ cm})^2 + QR^2$

$169\text{ cm}^2 = 144\text{ cm}^2 + QR^2$

$25\text{ cm}^2 = QR^2$

QR = 5 cm



$$\tan P = \frac{\text{Side opposite to } \angle P}{\text{Side adjacent to } \angle P} = \frac{QR}{PQ}$$

$$= \frac{5}{12}$$

$$\cot R = \frac{\text{Side adjacent to } \angle R}{\text{Side opposite to } \angle R} = \frac{QR}{PQ}$$

$$= \frac{5}{12}$$

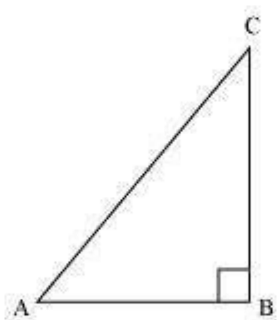
tan P - cot R = $\frac{5}{12} - \frac{5}{12} = 0$

Q3 :

If sin A = $\frac{3}{4}$, calculate cos A and tan A.

Answer :

Let ΔABC be a right-angled triangle, right-angled at point B.



Given that,

$$\sin A = \frac{3}{4}$$

$$\frac{BC}{AC} = \frac{3}{4}$$

Let BC be $3k$. Therefore, AC will be $4k$, where k is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

$AC^2 = AB^2 + BC^2$

$(4k)^2 = AB^2 + (3k)^2$

$16k^2 - 9k^2 = AB^2$

$7k^2 = AB^2$

AB = $\sqrt{7}k$

$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}}$$

$$= \frac{AB}{AC} = \frac{\sqrt{7}k}{4k} = \frac{\sqrt{7}}{4}$$

$$\tan A = \frac{\text{Side opposite to } \angle A}{\text{Side adjacent to } \angle A}$$

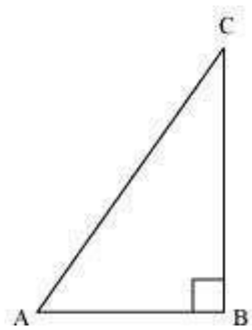
$$= \frac{BC}{AB} = \frac{3k}{\sqrt{7}k} = \frac{3}{\sqrt{7}}$$

Q4 :

Given 15 cot A = 8. Find sin A and sec A

Answer :

Consider a right-angled triangle, right-angled at B.



$$\cot A = \frac{\text{Side adjacent to } \angle A}{\text{Side opposite to } \angle A}$$

$$= \frac{AB}{BC}$$

It is given that,

$\cot A = \frac{8}{15}$

$$\frac{AB}{BC} = \frac{8}{15}$$

Let AB be 8*k*.Therefore, BC will be 15*k*, where *k* is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

$AC^2 = AB^2 + BC^2$

$= (8k)^2 + (15k)^2$

$= 64k^2 + 225k^2$

$= 289k^2$

AC = 17*k*

$$\sin A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$= \frac{15k}{17k} = \frac{15}{17}$$

$$\sec A = \frac{\text{Hypotenuse}}{\text{Side adjacent to } \angle A}$$

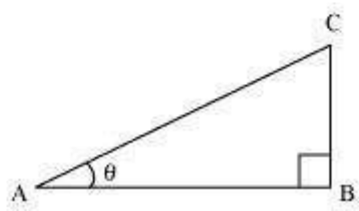
$$= \frac{AC}{AB} = \frac{17}{8}$$

Q5 :

Given sec θ = $\frac{13}{12}$, calculate all other trigonometric ratios.

Answer :

Consider a right-angle triangle ΔABC, right-angled at point B.



$$\sec\theta = \frac{\text{Hypotenuse}}{\text{Side adjacent to } \angle\theta}$$

$$\frac{13}{12} = \frac{AC}{AB}$$

If AC is $13k$, AB will be $12k$, where k is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

$(AC)^2 = (AB)^2 + (BC)^2$

$(13k)^2 = (12k)^2 + (BC)^2$

$169k^2 = 144k^2 + BC^2$

$25k^2 = BC^2$

$BC = 5k$

$$\sin\theta = \frac{\text{Side opposite to } \angle\theta}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{5k}{13k} = \frac{5}{13}$$

$$\cos\theta = \frac{\text{Side adjacent to } \angle\theta}{\text{Hypotenuse}} = \frac{\text{AB}}{\text{AC}} = \frac{12k}{13k} = \frac{12}{13}$$

$$\tan\theta = \frac{\text{Side opposite to } \angle\theta}{\text{Side adjacent to } \angle\theta} = \frac{\text{BC}}{\text{AB}} = \frac{5k}{12k} = \frac{5}{12}$$

$$\cot\theta = \frac{\text{Side adjacent to } \angle\theta}{\text{Side opposite to } \angle\theta} = \frac{\text{AB}}{\text{BC}} = \frac{12k}{5k} = \frac{12}{5}$$

$$\operatorname{cosec}\theta = \frac{\text{Hypotenuse}}{\text{Side opposite to } \angle\theta} = \frac{\text{AC}}{\text{BC}} = \frac{13k}{5k} = \frac{13}{5}$$

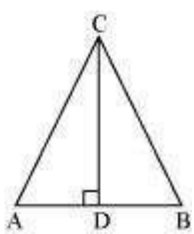
Q6 :

If ∠ A and ∠ B are acute angles such that cos A = cos B, then show that

∠ A = ∠ B.

Answer :

Let us consider a triangle ABC in which CD ⊥ AB.

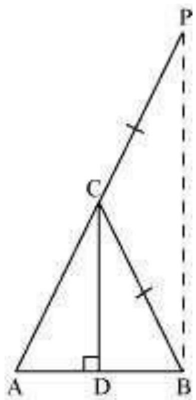


It is given that

cos A = cos B

$$\Rightarrow \frac{AD}{AC} = \frac{BD}{BC}$$ ... (1)

We have to prove ∠A = ∠B. To prove this, let us extend AC to P such that BC = CP.



From equation (1), we obtain

$$\frac{AD}{BD} = \frac{AC}{BC}$$

$$\Rightarrow \frac{AD}{BD} = \frac{AC}{CP} \qquad \left(\text{By construction, we have } BC = CP\right) \qquad \ldots (2)$$

By using the converse of B.P.T,

CD||BP

⇒∠ACD = ∠CPB (Corresponding angles) … (3)

And, ∠BCD = ∠CBP (Alternate interior angles) … (4)

By construction, we have BC = CP.

∴ ∠CBP **=** ∠CPB (Angle opposite to equal sides of a triangle) … (5)

From equations (3), (4), and (5), we obtain

∠ACD = ∠BCD … (6)

In ΔCAD and ΔCBD,

∠ACD = ∠BCD [Using equation (6)]

∠CDA = ∠CDB [Both 90°]

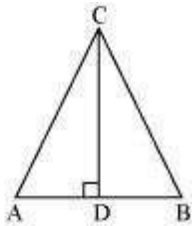
Therefore, the remaining angles should be equal.

∴∠CAD = ∠CBD

⇒ ∠A = ∠B

Alternatively,

Let us consider a triangle ABC in which CD ⊥ AB.



It is given that,

cos A = cos B

$$\Rightarrow \frac{AD}{AC} = \frac{BD}{BC}$$

$$\Rightarrow \frac{AD}{BD} = \frac{AC}{BC}$$

Let $\frac{AD}{BD} = \frac{AC}{BC} = k$

⇒ AD = k BD … (1)

And, AC = k BC … (2)

Using Pythagoras theorem for triangles CAD and CBD, we obtain

$CD^2 = AC^2 - AD^2$ … (3)

And, $CD^2 = BC^2 - BD^2$ … (4)

From equations (3) and (4), we obtain

$AC^2 - AD^2 = BC^2 - BD^2$

$\Rightarrow (k\ BC)^2 - (k\ BD)^2 = BC^2 - BD^2$

$\Rightarrow k^2 (BC^2 - BD^2) = BC^2 - BD^2$

$\Rightarrow k^2 = 1$

$\Rightarrow k = 1$

Putting this value in equation (2), we obtain

AC = BC

⇒ ∠A = ∠B(Angles opposite to equal sides of a triangle)

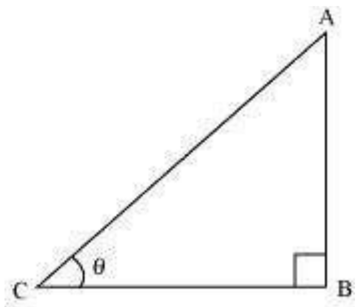
Q7 :

If cot θ = $\frac{7}{8}$, evaluate

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$ **(ii) $\cot^2\theta$**

Answer :

Let us consider a right triangle ABC, right-angled at point B.



$$\cot\theta = \frac{\text{Side adjacent to } \angle\theta}{\text{Side opposite to } \angle\theta} = \frac{BC}{AB}$$

$$= \frac{7}{8}$$

If BC is 7*k*, then AB will be 8*k*, where *k* is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

$AC^2 = AB^2 + BC^2$

$= (8k)^2 + (7k)^2$

$= 64k^2 + 49k^2$

$= 113k^2$

AC = $\sqrt{113}k$

$$\sin\theta = \frac{\text{Side opposite to } \angle\theta}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$= \frac{8k}{\sqrt{113}k} = \frac{8}{\sqrt{113}}$$

$$\cos\theta = \frac{\text{Side adjacent to } \angle\theta}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$= \frac{7k}{\sqrt{113}k} = \frac{7}{\sqrt{113}}$$

(i) $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{(1-\sin^2\theta)}{(1-\cos^2\theta)}$

$$= \frac{1-\left(\frac{8}{\sqrt{113}}\right)^2}{1-\left(\frac{7}{\sqrt{113}}\right)^2} = \frac{1-\frac{64}{113}}{1-\frac{49}{113}}$$

$$= \frac{\frac{49}{113}}{\frac{64}{113}} = \frac{49}{64}$$

(ii) cot² θ = (cot θ)² = $\left(\frac{7}{8}\right)^2$ = $\frac{49}{64}$

Q8 :

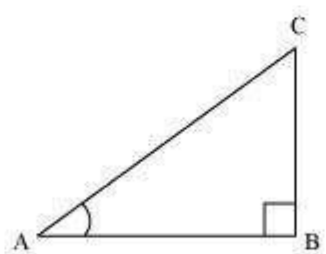
If 3 cot A = 4, Check whether $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$ or not.

Answer :

It is given that 3cot A = 4

Or, cot A = $\frac{4}{3}$

Consider a right triangle ABC, right-angled at point B.



$$\cot A = \frac{\text{Side adjacent to } \angle A}{\text{Side opposite to } \angle A}$$

$$\frac{AB}{BC} = \frac{4}{3}$$

If AB is 4*k*, then BC will be 3*k*, where *k* is a positive integer.

In ΔABC,

$(AC)^2 = (AB)^2 + (BC)^2$

$= (4k)^2 + (3k)^2$

$= 16k^2 + 9k^2$

$= 25k^2$

AC = $5k$

$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}} = \frac{AB}{AC}$$

$$= \frac{4k}{5k} = \frac{4}{5}$$

$$\sin A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC}$$

$$= \frac{3k}{5k} = \frac{3}{5}$$

$$\tan A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AB}$$

$$= \frac{3k}{4k} = \frac{3}{4}$$

$$\frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2} = \frac{1-\frac{9}{16}}{1+\frac{9}{16}}$$

$$= \frac{\frac{7}{16}}{\frac{25}{16}} = \frac{7}{25}$$

$\cos^2 A - \sin^2 A =$ $\left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

$$= \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$$

$\therefore$ $\dfrac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$

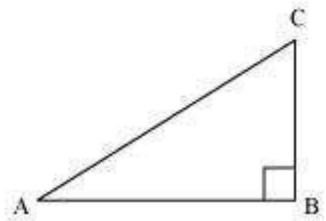
Q9 :

In ΔABC, right angled at B. If $\tan A = \frac{1}{\sqrt{3}}$ **, find the value of**

(i) sin A cos C + cos A sin C

(ii) cos A cos C - sin A sin C

Answer :



$$\tan A = \frac{1}{\sqrt{3}}$$

$$\frac{BC}{AB} = \frac{1}{\sqrt{3}}$$

If BC is k, then AB will be $\sqrt{3}k$, where k is a positive integer.

In ΔABC,

$AC^2 = AB^2 + BC^2$

$= \left(\sqrt{3}k\right)^2 + (k)^2$

$= 3k^2 + k^2 = 4k^2$

∴ AC = $2k$

$$\sin A = \frac{\text{Side opposite to } \angle A}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$$

$$\cos A = \frac{\text{Side adjacent to } \angle A}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

$$\sin C = \frac{\text{Side opposite to } \angle C}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

$$\cos C = \frac{\text{Side adjacent to } \angle C}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{k}{2k} = \frac{1}{2}$$

(i) sin A cos C + cos A sin C

$$= \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{1}{4} + \frac{3}{4}$$

$$= \frac{4}{4} = 1$$

(ii) cos A cos C - sin A sin C

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$$

Q10 :

In ΔPQR, right angled at Q, PR + QR = 25 cm and PQ = 5 cm. Determine the values of sin P, cos P and tan P.

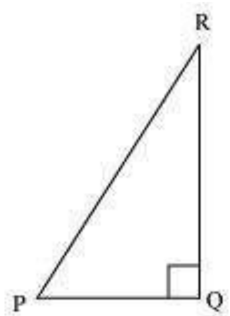
Answer :

Given that, PR + QR = 25

PQ = 5

Let PR be x.

Therefore, QR = 25 - x



Applying Pythagoras theorem in ΔPQR, we obtain

$PR^2 = PQ^2 + QR^2$

$x^2 = (5)^2 + (25 - x)^2$

$x^2 = 25 + 625 + x^2 - 50x$

$50x = 650$

$x = 13$

Therefore, PR = 13 cm

QR = (25 - 13) cm = 12 cm

$$\sin P = \frac{\text{Side opposite to } \angle P}{\text{Hypotenuse}} = \frac{QR}{PR} = \frac{12}{13}$$

$$\cos P = \frac{\text{Side adjacent to } \angle P}{\text{Hypotenuse}} = \frac{PQ}{PR} = \frac{5}{13}$$

$$\tan P = \frac{\text{Side opposite to } \angle P}{\text{Side adjacent to } \angle P} = \frac{QR}{PQ} = \frac{12}{5}$$

Q11 :

State whether the following are true or false. Justify your answer.

(i) The value of tan A is always less than 1.

(ii) sec A = $\frac{12}{5}$ **for some value of angle A.**

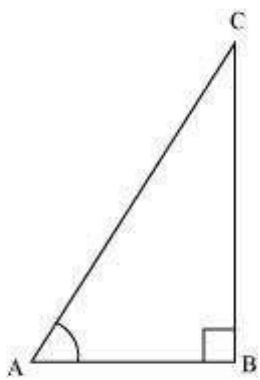
(iii) cos A is the abbreviation used for the cosecant of angle A.

(iv) cot A is the product of cot and A

(v) sin θ = $\frac{4}{3}$ **, for some angle θ**

Answer :

(i) Consider a ΔABC, right-angled at B.



$$\tan A = \frac{\text{Side opposite to } \angle A}{\text{Side adjacent to } \angle A}$$

$$= \frac{12}{5}$$

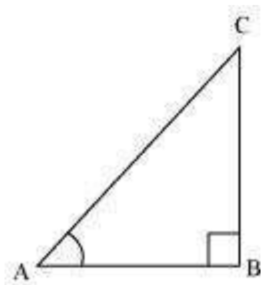
But $\frac{12}{5}$ > 1

∴tan A > 1

So, tan A < 1 is not always true.

Hence, the given statement is false.

(ii) $\sec A = \frac{12}{5}$



$$\frac{\text{Hypotenuse}}{\text{Side adjacent to } \angle A} = \frac{12}{5}$$

$$\frac{AC}{AB} = \frac{12}{5}$$

Let AC be $12k$, AB will be $5k$, where k is a positive integer.

Applying Pythagoras theorem in ΔABC, we obtain

$AC^2 = AB^2 + BC^2$

$(12k)^2 = (5k)^2 + BC^2$

$144k^2 = 25k^2 + BC^2$

$BC^2 = 119k^2$

$BC = 10.9k$

It can be observed that for given two sides $AC = 12k$ and $AB = 5k$,

BC should be such that,

$AC - AB < BC < AC + AB$

$12k - 5k < BC < 12k + 5k$

$7k < BC < 17\,k$

However, $BC = 10.9k$. Clearly, such a triangle is possible and hence, such value of sec A is possible.

Hence, the given statement is true.

(iii) Abbreviation used for cosecant of angle A is cosec A. And cos A is the abbreviation used for cosine of angle A.

Hence, the given statement is false.

(iv) cot A is not the product of cot and A. It is the cotangent of ∠A.

Hence, the given statement is false.

(v) $\sin\theta = \frac{4}{3}$

We know that in a right-angled triangle,

$$\sin\theta = \frac{\text{Side opposite to } \angle\theta}{\text{Hypotenuse}}$$

In a right-angled triangle, hypotenuse is always greater than the remaining two sides. Therefore, such value of sin θ is not possible.

Hence, the given statement is false

Exercise 8.2 : Solutions of Questions on Page Number : **187**

Q1 :

Evaluate the following

(i) sin60° cos30° + sin30° cos 60°

(ii) 2tan²45° + cos²30° - sin²60°

(iii) $\dfrac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$

(iv) $\dfrac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$

(v) $\dfrac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$

Answer :

(i) sin60° cos30° + sin30° cos 60°

$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

$$= \frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$$

(ii) $2\tan^2 45^\circ + \cos^2 30^\circ - \sin^2 60^\circ$

$$= 2(1)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2$$

$$= 2 + \frac{3}{4} - \frac{3}{4} = 2$$

(iii) $\dfrac{\cos 45^\circ}{\sec 30^\circ + \operatorname{cosec} 30^\circ}$

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}}+2} = \frac{\frac{1}{\sqrt{2}}}{\frac{2+2\sqrt{3}}{\sqrt{3}}}$$

$$= \frac{\sqrt{3}}{\sqrt{2}\left(2+2\sqrt{3}\right)} = \frac{\sqrt{3}}{2\sqrt{2}+2\sqrt{6}}$$

$$= \frac{\sqrt{3}\left(2\sqrt{6}-2\sqrt{2}\right)}{\left(2\sqrt{6}+2\sqrt{2}\right)\left(2\sqrt{6}-2\sqrt{2}\right)}$$

$$= \frac{2\sqrt{3}\left(\sqrt{6}-\sqrt{2}\right)}{\left(2\sqrt{6}\right)^2-\left(2\sqrt{2}\right)^2} = \frac{2\sqrt{3}\left(\sqrt{6}-\sqrt{2}\right)}{24-8} = \frac{2\sqrt{3}\left(\sqrt{6}-\sqrt{2}\right)}{16}$$

$$= \frac{\sqrt{18}-\sqrt{6}}{8} = \frac{3\sqrt{2}-\sqrt{6}}{8}$$

(iv) $\dfrac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$

$$= \frac{\frac{1}{2}+1-\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}+\frac{1}{2}+1} = \frac{\frac{3}{2}-\frac{2}{\sqrt{3}}}{\frac{3}{2}+\frac{2}{\sqrt{3}}}$$

$$= \frac{\frac{3\sqrt{3}-4}{2\sqrt{3}}}{\frac{3\sqrt{3}+4}{2\sqrt{3}}} = \frac{\left(3\sqrt{3}-4\right)}{\left(3\sqrt{3}+4\right)}$$

$$= \frac{\left(3\sqrt{3}-4\right)\left(3\sqrt{3}-4\right)}{\left(3\sqrt{3}+4\right)\left(3\sqrt{3}-4\right)} = \frac{\left(3\sqrt{3}-4\right)^2}{\left(3\sqrt{3}\right)^2-\left(4\right)^2}$$

$$= \frac{27+16-24\sqrt{3}}{27-16} = \frac{43-24\sqrt{3}}{11}$$

(v) $\dfrac{5\cos^2 60^\circ + 4\sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$

$$= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{5\left(\frac{1}{4}\right) + \left(\frac{16}{3}\right) - 1}{\frac{1}{4} + \frac{3}{4}}$$

$$= \frac{\frac{15+64-12}{12}}{\frac{4}{4}} = \frac{67}{12}$$

Q2 :

Choose the correct option and justify your choice.

(i) $\frac{2\tan 30°}{1+\tan^2 30°} =$

(A). sin60°

(B). cos60°

(C). tan60°

(D). sin30°

(ii) $\frac{1-\tan^2 45°}{1+\tan^2 45°} =$

(A). tan90°

(B). 1

(C). sin45°

(D). 0

(iii) sin2A = 2sinA is true when A =

(A). 0°

(B). 30°

(C). 45°

(D). 60°

(iv) $\frac{2\tan 30°}{1-\tan^2 30°} =$

(A). cos60°

(B). sin60°

(C). tan60°

(D). sin30°

Answer :

(i) $\frac{2\tan 30°}{1+\tan^2 30°}$

$$= \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1+\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}}$$

$$= \frac{6}{4\sqrt{3}} = \frac{\sqrt{3}}{2}$$

Out of the given alternatives, only $\sin 60° = \frac{\sqrt{3}}{2}$

Hence, (A) is correct.

(ii) $\frac{1-\tan^2 45°}{1+\tan^2 45°}$

$$= \frac{1-(1)^2}{1+(1)^2} = \frac{1-1}{1+1} = \frac{0}{2} = 0$$

Hence, (D) is correct.

(iii)Out of the given alternatives, only A = 0° is correct.

As sin 2A = sin 0° = 0

2 sinA = 2sin 0° = 2(0) = 0

Hence, (A) is correct.

(iv) $\frac{2\tan 30°}{1-\tan^2 30°}$

$$= \frac{2\left(\frac{1}{\sqrt{3}}\right)}{1-\left(\frac{1}{\sqrt{3}}\right)^2} = \frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}}$$

$$= \sqrt{3}$$

Out of the given alternatives, only tan 60° $= \sqrt{3}$

Hence, (C) is correct.

Q3 :

If $\tan(A+B) = \sqrt{3}$ **and** $\tan(A-B) = \frac{1}{\sqrt{3}}$ **;**

0° < A + B ≤90°, A > B find A and B.

Answer :

$\tan(A+B) = \sqrt{3}$

⇒ $\tan(A+B) = \tan 60$

⇒ A + B = 60 ... (1)

$\tan(A-B) = \frac{1}{\sqrt{3}}$

⇒ tan (A - B) = tan30

⇒ A - B = 30 ... (2)

On adding both equations, we obtain

2A = 90

⇒ A = 45

From equation (1), we obtain

45 + B = 60

B = 15

Therefore, ∠A = 45° and ∠B = 15°

Q4 :

State whether the following are true or false. Justify your answer.

(i) sin(A + B) = sin A + sin B

(ii) The value of sinÃŽÂ¸increases as ÃŽÂ¸increases

(iii) The value of cos ÃŽÂ¸increases as ÃŽÂ¸increases

(iv) sinÃŽÂ¸= cos ÃŽÂ¸ for all values of ÃŽÂ¸

(v) cot A is not defined for A = 0°

Answer :

(i) sin(A + B) = sin A + sin B

Let A = 30° and B = 60°

sin (A + B) = sin (30° + 60°)

= sin 90°

= 1

sin A + sin B = sin 30° + sin 60°

$$=\frac{1}{2}+\frac{\sqrt{3}}{2}=\frac{1+\sqrt{3}}{2}$$

Clearly, sin (A + B) ≠sin A + sin B

Hence, the given statement is false.

(ii) The value of sin θ increases as θ increases in the interval of 0° < θ < 90° as

sin 0° = 0

$$\sin 30° = \frac{1}{2} = 0.5$$

$$\sin 45° = \frac{1}{\sqrt{2}} = 0.707$$

$$\sin 60° = \frac{\sqrt{3}}{2} = 0.866$$

sin 90° = 1

Hence, the given statement is true.

(iii) cos 0° = 1

$$\cos 30° = \frac{\sqrt{3}}{2} = 0.866$$

$$\cos 45° = \frac{1}{\sqrt{2}} = 0.707$$

$$\cos 60° = \frac{1}{2} = 0.5$$

cos90° = 0

It can be observed that the value of cos θ does not increase in the interval of 0° < θ < 90°.

Hence, the given statement is false.

(iv) sin θ = cos θ for all values of θ.

This is true when θ = 45°

As $\sin 45° = \frac{1}{\sqrt{2}}$

$\cos 45° = \frac{1}{\sqrt{2}}$

It is not true for all other values of θ.

As $\sin 30° = \frac{1}{2}$ and $\cos 30° = \frac{\sqrt{3}}{2}$,

Hence, the given statement is false.

(v) cot A is not defined for A = 0°

As $\cot A = \frac{\cos A}{\sin A}$,

$\cot 0° = \frac{\cos 0°}{\sin 0°} = \frac{1}{0}$ = undefined

Hence, the given statement is true.

Exercise 8.3 : Solutions of Questions on Page Number : **189**

Q1 :

Evaluate

(I) $\frac{\sin 18°}{\cos 72°}$

(II) $\frac{\tan 26°}{\cot 64°}$

(III) cos 48° - sin 42°

(IV)cosec 31° - sec 59°

Answer :

(I) $\frac{\sin 18°}{\cos 72°} = \frac{\sin(90° - 72°)}{\cos 72°}$

$= \frac{\cos 72°}{\cos 72°} = 1$

(II) $\frac{\tan 26°}{\cot 64°} = \frac{\tan(90° - 64°)}{\cot 64°}$

$$= \frac{\cot 64°}{\cot 64°} = 1$$

(III)cos 48° - sin 42° = cos (90° - 42°) - sin 42°

= sin 42° - sin 42°

= 0

(IV) cosec 31° - sec 59° = cosec (90° - 59°) - sec 59°

= sec 59° - sec 59°

= 0

Q2 :

Show that

(I) tan 48° tan 23° tan 42° tan 67° = 1

(II)cos 38° cos 52° - sin 38° sin 52° = 0

Answer :

(I) tan 48° tan 23° tan 42° tan 67°

= tan (90° - 42°) tan (90° - 67°) tan 42° tan 67°

= cot 42° cot 67° tan 42° tan 67°

= (cot 42° tan 42°) (cot 67° tan 67°)

= (1) (1)

= 1

(II) cos 38° cos 52° - sin 38° sin 52°

= cos (90° - 52°) cos (90°-38°) - sin 38° sin 52°

= sin 52° sin 38° - sin 38° sin 52°

= 0

Q3 :

If tan 2A = cot (A- 18°), where 2A is an acute angle, find the value of A.

Answer :

Given that,

tan 2A = cot (A- 18°)

cot (90° - 2A) = cot (A -18°)

$90° - 2A = A - 18°$

$108° = 3A$

$A = 36°$

Q4 :

If tan A = cot B, prove that A + B = 90°

Answer :

Given that,

$\tan A = \cot B$

$\tan A = \tan (90° - B)$

$A = 90° - B$

$A + B = 90°$

Q5 :

If sec 4A = cosec (A- 20°), where 4A is an acute angle, find the value of A.

Answer :

Given that,

$\sec 4A = \operatorname{cosec} (A - 20°)$

$\operatorname{cosec} (90° - 4A) = \operatorname{cosec} (A - 20°)$

$90° - 4A = A - 20°$

$110° = 5A$

$A = 22°$

Q6 :

If A, Band C are interior angles of a triangle ABC then show that

$$\sin\left(\frac{B+C}{2}\right) = \cos\frac{A}{2}$$

Answer :

We know that for a triangle ABC,

$\angle A + \angle B + \angle C = 180°$

- ∠B + ∠C= 180° - ∠A

$$\frac{\angle B+\angle C}{2}=90°-\frac{\angle A}{2}$$
$$\sin\left(\frac{B+C}{2}\right)=\sin\left(90°-\frac{A}{2}\right)$$
$$=\cos\left(\frac{A}{2}\right)$$

Q7 :

Express sin 67° + cos 75° in terms of trigonometric ratios of angles between 0° and 45°.

Answer :

sin 67° + cos 75°

= sin (90° - 23°) + cos (90° - 15°)

= cos 23° + sin 15°

Exercise 8.4 : Solutions of Questions on Page Number : **193**

Q1 :

Express the trigonometric ratios sin A, sec A and tan A in terms of cot A.

Answer :

We know that,

$$\operatorname{cosec}^2 A=1+\cot^2 A$$
$$\frac{1}{\operatorname{cosec}^2 A}=\frac{1}{1+\cot^2 A}$$
$$\sin^2 A=\frac{1}{1+\cot^2 A}$$
$$\sin A=\pm\frac{1}{\sqrt{1+\cot^2 A}}$$

$\sqrt{1+\cot^2 A}$ will always be positive as we are adding two positive quantities.

Therefore, $\sin A=\frac{1}{\sqrt{1+\cot^2 A}}$

We know that, $\tan A=\frac{\sin A}{\cos A}$

However, $\cot A = \dfrac{\cos A}{\sin A}$

Therefore, $\tan A = \dfrac{1}{\cot A}$

Also, $\sec^2 A = 1 + \tan^2 A$

$$= 1 + \frac{1}{\cot^2 A}$$

$$= \frac{\cot^2 A + 1}{\cot^2 A}$$

$$\sec A = \frac{\sqrt{\cot^2 A + 1}}{\cot A}$$

Q2 :

Write all the other trigonometric ratios of ∠ A in terms of sec A.

Answer :

We know that,

$$\cos A = \frac{1}{\sec A}$$

Also, $\sin^2 A + \cos^2 A = 1$

$\sin^2 A = 1 - \cos^2 A$

$$\sin A = \sqrt{1 - \left(\frac{1}{\sec A}\right)^2}$$

$$= \sqrt{\frac{\sec^2 A - 1}{\sec^2 A}} = \frac{\sqrt{\sec^2 A - 1}}{\sec A}$$

$\tan^2 A + 1 = \sec^2 A$

$\tan^2 A = \sec^2 A - 1$

$$\tan A = \sqrt{\sec^2 A - 1}$$

$$\cot A = \frac{\cos A}{\sin A} = \frac{\frac{1}{\sec A}}{\frac{\sqrt{\sec^2 A - 1}}{\sec A}}$$

$$= \frac{1}{\sqrt{\sec^2 A - 1}}$$

$$\operatorname{cosec} A = \frac{1}{\sin A} = \frac{\sec A}{\sqrt{\sec^2 A - 1}}$$

Q3 :

Evaluate

(i) $\frac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°}$

(ii) sin25° cos65° + cos25° sin65°

Answer :

(i) $\frac{\sin^2 63° + \sin^2 27°}{\cos^2 17° + \cos^2 73°}$

$$= \frac{\left[\sin\left(90° - 27°\right)\right]^2 + \sin^2 27°}{\left[\cos\left(90° - 73°\right)\right]^2 + \cos^2 73°}$$

$$= \frac{\left[\cos 27°\right]^2 + \sin^2 27°}{\left[\sin 73°\right]^2 + \cos^2 73°}$$

$$= \frac{\cos^2 27° + \sin^2 27°}{\sin^2 73° + \cos^2 73°}$$

$= \frac{1}{1}$ (As $\sin^2 A + \cos^2 A = 1$)

= 1

(ii) sin25° cos65° + cos25° sin65°

$$= \left(\sin 25°\right)\left\{\cos\left(90° - 25°\right)\right\} + \cos 25°\left\{\sin\left(90° - 25°\right)\right\}$$

$$= \left(\sin 25°\right)\left(\sin 25°\right) + \left(\cos 25°\right)\left(\cos 25°\right)$$

= $\sin^2 25° + \cos^2 25°$

= 1 (As $\sin^2 A + \cos^2 A = 1$)

Q4 :

Choose the correct option. Justify your choice.

(i) $9 \sec^2 A - 9 \tan^2 A$ =

(A) 1

(B) 9

(C) 8

(D) 0

(ii) $(1 + \tan\theta + \sec\theta)(1 + \cot\theta - \operatorname{cosec}\theta)$

(A) 0

(B) 1

(C) 2

(D) - 1

(iii) $(\sec A + \tan A)(1 - \sin A)$ =

(A) secA

(B) sinA

(C) cosecA

(D) cosA

(iv) $\dfrac{1+\tan^2 A}{1+\cot^2 A}$

(A) $\sec^2 A$

(B) - 1

(C) $\cot^2 A$

(D) $\tan^2 A$

Answer :

(i) $9 \sec^2 A - 9 \tan^2 A$

$= 9 (\sec^2 A - \tan^2 A)$

$= 9 (1)$ [As $\sec^2 A - \tan^2 A = 1$]

= 9

Hence, alternative (B) is correct.

(ii)

(1 + tan θ + sec θ) (1 + cot θ - cosec θ)

$$= \left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)\left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)$$

$$= \left(\frac{\cos\theta + \sin\theta + 1}{\cos\theta}\right)\left(\frac{\sin\theta + \cos\ \theta - 1}{\sin\theta}\right)$$

$$= \frac{(\sin\theta + \cos\ \theta)^2 - (1)^2}{\sin\theta\cos\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta + 2\sin\theta\ \cos\ \theta - 1}{\sin\theta\cos\ \theta}$$

$$= \frac{1 + 2\sin\theta\cos\ \theta - 1}{\sin\theta\cos\ \theta}$$

$$= \frac{2\sin\theta\ \cos\ \theta}{\sin\ \theta\cos\ \theta} = 2$$

Hence, alternative (C) is correct.

(iii) (secA + tanA) (1 - sinA)

$$= \left(\frac{1}{\cos A} + \frac{\sin A}{\cos A}\right)(1 - \sin A)$$

$$= \left(\frac{1 + \sin A}{\cos A}\right)(1 - \sin A)$$

$$= \frac{1 - \sin^2 A}{\cos A} = \frac{\cos^2 A}{\cos A}$$

= cosA

Hence, alternative (D) is correct.

(iv) $$\frac{1 + \tan^2 A}{1 + \cot^2 A} = \frac{1 + \frac{\sin^2 A}{\cos^2 A}}{1 + \frac{\cos^2 A}{\sin^2 A}}$$

$$= \frac{\frac{\cos^2 A + \sin^2 A}{\cos^2 A}}{\frac{\sin^2 A + \cos^2 A}{\sin^2 A}} = \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}}$$

$$= \frac{\sin^2 A}{\cos^2 A} = \tan^2 A$$

Hence, alternative (D) is correct.

Q5 :

Prove the following identities, where the angles involved are acute angles for which the expressions are defined.

Answer :

(i) $(\operatorname{cosec}\theta - \cot\theta)^2 = \dfrac{1-\cos\theta}{1+\cos\theta}$

$$\text{L.H.S.} = (\operatorname{cosec}\theta - \cot\theta)^2$$

$$= \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2$$

$$= \frac{(1-\cos\theta)^2}{(\sin\theta)^2} = \frac{(1-\cos\theta)^2}{\sin^2\theta}$$

$$= \frac{(1-\cos\theta)^2}{1-\cos^2\theta} = \frac{(1-\cos\theta)^2}{(1-\cos\theta)(1+\cos\theta)} = \frac{1-\cos\theta}{1+\cos\theta}$$

$$= \text{R.H.S.}$$

(ii) $\dfrac{\cos A}{1+\sin A} + \dfrac{1+\sin A}{\cos A} = 2\sec A$

$$\text{L.H.S.} = \frac{\cos A}{1+\sin A} + \frac{1+\sin A}{\cos A}$$

$$= \frac{\cos^2 A + (1+\sin A)^2}{(1+\sin A)(\cos A)}$$

$$= \frac{\cos^2 A + 1 + \sin^2 A + 2\sin A}{(1+\sin A)(\cos A)}$$

$$= \frac{\sin^2 A + \cos^2 A + 1 + 2\sin A}{(1+\sin A)(\cos A)}$$

$$= \frac{1+1+2\sin A}{(1+\sin A)(\cos A)} = \frac{2+2\sin A}{(1+\sin A)(\cos A)}$$

$$= \frac{2(1+\sin A)}{(1+\sin A)(\cos A)} = \frac{2}{\cos A} = 2\sec A$$

$$= \text{R.H.S.}$$

(iii) $\dfrac{\tan\theta}{1-\cot\theta} + \dfrac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta\operatorname{cosec}\theta$

$$\text{L.H.S.} = \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{1-\frac{\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{1-\frac{\sin\theta}{\cos\theta}}$$

$$= \frac{\frac{\sin\theta}{\cos\theta}}{\frac{\sin\theta-\cos\theta}{\sin\theta}} + \frac{\frac{\cos\theta}{\sin\theta}}{\frac{\cos\theta-\sin\theta}{\cos\theta}}$$

$$= \frac{\sin^2\theta}{\cos\theta(\sin\theta-\cos\theta)} - \frac{\cos^2\theta}{\sin\theta(\sin\theta-\cos\theta)}$$