Constants : A symbol having a fixed numerical value is called a constant.

Example : 7, 3, -2, 3/7, etc. are all constants.

Variables : A symbol which may be assigned different numerical values is known avariable.

Example : $C = 2\pi r$ cumference of circle

r - radius of circle Where 2 & are constants.

while C and r are variable

Algebraic expressions : A combination of constants and variables. Connected by some or all of the operations +, -, X and is known as algebraic expression.

Example : $4 + 9x - 5x^2y + \frac{3}{8}xy$ etc.

Terms : The several parts of an algebraic expression separated by '+' or '-' operations are called the terms of the expression.

Example : $x^3 + 2x^2y + 4xy^2 + y^3 + 7$ is an algebraic expression containing 5 terms $x^3, 2x^3y, -4xy^2, y^3$ & 7

Polynomials : An algebraic expression in which the variables involved have only nonnegative integral powers is called a polynomial.

(i) $5x^3 - 4x^2 - 6x - 3$ is a polynomial in variable x.

Polynomials are denoted by $p(x)$, $q(x) and\ r(x) etc.$

Coefficients : In the polynomial

$x^3 + 3x^2 + 3x + 1$.coefficient of

$x^3, x^2, x\ are\ 1, 3, 3$

respectively and we also say that +1 is the constant term in it. Degree of a polynomial in one variable : In case of a polynomial in one variable the highest power of the variable is called the degree of the polynomial.

Classification of polynomials on the basis of degree.

	degree	Polynomial	Example
(a)	1	Linear	$x+1,\ 2x+3\ etc$
(b)	2	Quadratic	ax^2+bx+c etc
(c)	3	Cubic	x^3-3x^2+1 etc.
(d)	4	Biquadratic	x^4-1

Classification of polynomials on the basis of no. of terms

	No. of terms	Polynomial & Examples.
(i)	1	Monomial - $5, 3x, \frac{1}{3}y$ etc.
(ii)	2	Binomial - $(3+6x),\ (x-5y)$ etc.

(iii) 3 Trinomial- $2x^2 + 4x + 2$ etc.

Constant polynomial : A polynomial containing one term only, consisting a constanterm is called a constant polynomial the degree of non-zero constant polynomial is zero.

Zero polynomial : A polynomial consisting of one term, namely zero only is called a zero polynomial.The degree of zero polynomial is not defined.

Zeroes of a polynomial : Let p(x).be a polynomial. If $p(\alpha) = 0$, then we say that is zero of the polynomial of p(x).

Remark : Finding the zeroes of polynomial p(x) means solving the equation p(x)=0.

Remainder theorem : Let $f(x)$ be a polynomial of degree ≥ 1 and let a be any real number. When $f(x)$ is divided $(x - a)$ by then the remainder is $f(a)$

Factor theorem : Let $f(x)$ be a polynomial of degree $n > 1$and let a be any real number.

(i) If $f(a) = 0$ *then* $(x-a)$*is factor of* $f(x)$

(ii) *If* $(x-a)$*is a factor of* $f(x)$ *then* $f(a) = 0$

Factor : A polynomial $p(x)$ is called factor of $q(x)$, *if* $p(x)$ divides $q(x)$ exactly.

Factorization : To express a given polynomial as the product of polynomials each of degree less than that of the given polynomial such that no such a factor has a factor of lower degree, is called factorization.

Example : $x^2 - 16 = (x+4)(x-4)$

Methods of Factorization : Factorization by taking out the common factore.g.

$$36q^3b - 60a^2bc = 12a^2b\,(3a - 5c)$$

Factorizing by grouping

$$\begin{aligned} ab + bc + ax + cx &= (ab+bc) + (ax+cx) \\ &= b\,(a+c) + x\,(a+c) \\ &= (a+c)\,(b+x) \end{aligned}$$

Factorization of quadratic trinomials by middle term splitting method.

$$\begin{aligned} x^2 + bc + c &= x^2 + (p+q)x + pq \\ &= (x+p)(x+q) \end{aligned}$$

Identity : Identity is a equation (trigonometric, algebraic) which is true for every value of variable.

Some algebraic identities useful in factorization:

(i) $(x+y)^2 = x^2 + 2xy + y^2$

(ii) $(x-y)^2 = x^2 - 2xy + y^2$

(iii) $x^2 - y^2 = (x-y)(x+y)$

(iv) $(x+a)(x+b) = x^2 + (a+b)x + ab$

(v) $(x+y+z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$

(vi) $(x+y)^3 = x^3 + y^3 + 3xy(x+y)$

(vii) $(x-y)^3 = x^3 - y^3 - 3xy(x-y)$

(viii) $x^3 + y^3 + z^3 - 3xyz = (x+y+z)(x^2 + y^2 + z^2 - xy - yz - zx)$

$x^3 + y^3 + z^3 = 3xyz \quad if \;\; x+y+z = 0$

Class IX Math
Notes for Linear Equations in Two Variables

An equation of the form $ax + by + c = 0$ where a, b, c are real numbers, $a \neq 0$, $b \neq 0$ and x, y are variables is called a linear equation in two variable.

Anys pair of values of x and y which satisfies the equation $ax + by + c = 0$, where a, b, c are non-zero real numbers, is called a solution of the equation

- A linear equation in two variables has infinitely many solutions.
- Every point on the graph of a linear equation in two variables is a solution of the equation.

Geometric representation of $ax + c = 0$ as an equation

- in one variable is $x = -\frac{c}{a}$.
- in two variable is
  $a.x + 0.y = -c$.
- If $a \neq 0$, $c \neq 0$ and $b = 0$.
  The equation
  $ax + by + c$ �� 0 reduces to $ax + c = 0$ or $x = -\frac{c}{a}$.

The graph is a straight line. Parallel to y-axis and passing through the point $\left(-\frac{c}{a}, 0\right)$.

- If $b \neq 0$, $c \neq 0$ and $a \neq 0$. The equation $ax + by + c = 0$ reduces to $by + c = 0$ or $y = -\frac{c}{b}$. The graph is a straight line parallel to x axis and passing through the point $\left(0, -\frac{c}{b}\right)$.
- If $a \neq 0$, $b \neq 0$, $c = 0$. The equation $ax + by + c = 0$ reduces to $ax = 0$ i.e. $x = 0$. The graph is y-axis itself
- If $a = 0$, $b \neq 0$, $c = 0$. The equation $ax + by + c = 0$ reduces to $by = 0$ i.e. $y = 0$. The graph is x-axis itself
- If $c = 0$, the equation $ax + by + c = 0$ reduces to $ax + by = 0$. The graph is a line passing through origin.

TERM - 1

Polynomials

Exercise 2.1

1 Which of the following expressions are polynomials in one variable and which are not? State reasons for your answer

i $4x^2 - 3x + 7$

ii $y^2 + \sqrt{2}$

iii $3\sqrt{t} + t\sqrt{2}$

iv $y + 2/y$

v $x^{10} + y^3 + t^{50}$

Sol. *i* Given expression is a polynomial in one variable, as it degree is whole

ii Given expression is a polynomial in one variable, as it degree is whole

iii Given expression is not a polynomial , as it degree is not whole

iv Given expression is not a polynomial , as it degree is not whole

v Given expression is a polynomial in three variable, as it degree is whole

2. Write the coefficients of x^2 in each of the following:

(i) $2 + x^2 + x$

(ii) $2 - x^2 + x^3$

(iii) $\frac{\pi}{2}x^2 + x$

(iv) $\sqrt{2}x - 1$

Sol. **i** Given polynomial is $2 + x^2 + x$.

Hence, the coefficient of x^2 in given polynomial is equal to 1.

ii Given polynomial is $2 - x^2 + x^3$.

Hence, the coefficient of x^2 in given polynomial is equal to -1.

iii Given polynomial is $\frac{\pi}{2}x^2 + x$

Hence, the coefficient of x^2 in given polynomial is equal to $\frac{\pi}{2}$

iv Given polynomial is $\sqrt{2}x - 1$.

In the given polynomial, there is no x^2 term.

Hence, the coefficient of x^2 in given polynomial is equal to 0.

3 Give one example each of a binomial of degree 35 and of a monomial ofdegree 100.

Sol. A binomial of degree 35 can be $x^{35} + 7$

A monomial of degree 100 can be $2x^{100}$

4 . Write the degree of each of the following polynomials

i $5x^3 + 4x^2 + 7x$

ii $4 - y^2$

iii $5t - \sqrt{7}$

iv 3

Solution: i) Given polynomial is $5x^3 + 4x^2 + 7x$

Hence, the degree of given polynomial is equal to 3.

ii)Given polynomial is $4 - y^2$

Hence, the degree of given polynomial is equal to 2.

iii) Given polynomial is $5t - \sqrt{7}$

Hence, the degree of given polynomial is 1.

iv) Given polynomial is 3.

Hence, the degree of given polynomial is0

5. Classify the following as linear, quadratic and cubic polynomials

i $x^2 + x$

ii $x - x^3$

iii $y + y^2 + 4$

iv $1 + x$

v $3t$

vi r^2

vii $7x^3$

Solution:

i) Given polynomial is x^2+x

It is a quadratic polynomial as its degree is 2.

ii) Given polynomial is $x - x^3$.

It is a cubic polynomial as its degree is 3.

iii) $y + y^2 + 4$

It is a quadratic polynomial as its degree is 2.

iv) Given polynomial $1 + x$

It is a linear polynomial as its degree is 1

v) Given polynomial $3t$

It is a linear polynomial as its degree is 1

vi) Given polynomial is r^2

It is a quadratic polynomial as its degree is 2.

vii) Given polynomial is x^3

It is a cubic polynomial as its degree is 3.

EXERCISE 2.2

1. Find the value of the polynomial $5x - 4x^2 + 3$ at

i $x = 0$

ii $x = -1$

iii $x = 2$

Sol.

i) Given polynomial is $5x - 4x^2 + 3$

Value of polynomial at $x = 0$ is $5(0) - 4(0)^2 + 3 = 0 - 0 + 3 = 3$

(ii) Given polynomial is $5x - 4x^2 + 3$

Value of given polynomial at $x = -1$ is $5(-1) - 4(-1)^2 + 3$

$= -5 - 4 + 3 = -6$

iii) Given polynomial is $5x - 4x^2 + 3$

Value of given polynomial at $x = 2$ is $5(2) - 4(2)^2 + 3$

$= 10 - 16 + 3 = -3$

2 Find $P(0)$, $P(1)$ and $P(2)$ for each of the following polynomials.

(i) $P(y) = y^2 - y + 1$

(ii) $P(t) = 2 + t + 2t^2 - t^3$

(iii) $P(x) = x^3$

(iv) $P(x) = (x - 1)(x + 1)$

Sol.

i Given polynomial is $P(y) = y^2 - y + 1$

$P(0) = (0)^2 - 0 + 1 = 1$

$P(1) = (1)^2 - 1 + 1 = 1$

$P(2) = (2)^2 - 2 + 1 = 4 - 2 + 1 = 3$

ii Given polynomial is $P(t) = 2 + t + 2t^2 - t^3$

$P(0) = 2 + 0 + 2.(0)^2 - (0)^3 = 2$

$P(1) = 2 + 1 + 2(1)^2 - (1)^3 = 4$

$P(2) = 2 + 2 + 2.(2)^2 - (2)^3 = 4$

iii Given polynomial is $P(x) = x^3$

$P(0) = (0)^3 = 0$

$P(1) = (1)^3 = 1$

$P(2) = (2)^3 = 8$

iv Given polynomial is $p(x) = (x - 1)(x + 1)$

$P(0) = (0 - 1)(0 + 1) = (-1)(1) = -1$

$P(1) = (1 - 1)(1 + 1) = (0)(2) = 0$

$P(2) = (2 - 1)(2 + 1) = (1)(3) = 3$

3. Verify whether the following are zeroes of the polynomial, indicated against them.

Sol i $P(x) = 3x + 1$

$X = -1/3$

$P(-1/3) = 3(-1/3) + 1 = -1 + 1 = 0$

Hence $^{-1/3}$ is a zero of polynomial $P(x) = 3x + 1$.

ii Given polynomial is $P(x) = 5x - \pi$

$X = 4/5$

$P(4/5) = 5(4/5) - \pi$

$= 4 - \pi$

Hence $x = 4/5$ is not a zero of polynomial $5x - \pi$

iii Given polynomial is $P(x) = x^2 - 1$

At $x = 1$

$P(1) = (1)^2 - 1 = 0$

And $x = -1$

$P(-1) = (-1)^2 - 1 = 1 - 1 = 0$

Hence $x = 1, -1$ are zeroes of polynomial $x^2 - 1$.

iv . Given polynomial is $P(x) = (x + 1)(x - 2)$

At $x = -1$,

$P(-1) = (-1 + 1)(-1 - 2)$

$= (0)(-3) = 0$

And $x = 2$,

$P(2) = (2 + 1)(2 - 2)$

$= (0)(3) = 0$

Hence $x = -1, 2$ are zeroes of polynomial $(x + 1)(x - 2)$

v. Given polynomial is $P(x) = x^2$

AT $X = 0$

$P(0) = (0)^2 = 0$

Hence $x = 0$ is zero of polynomial x^2

vi) Given polynomial is $P(x) = lx + m$

At $x = -\frac{l}{m}$

$P(-\frac{l}{m}) = l(-\frac{l}{m}) + m$

$= -m + m = 0$

Hence $x = -\frac{l}{m}$ is zero of polynomial $lx + m$ { vii and viii same as previous parts}

4. Find the zero of the polynomials in each of the following cases.

Solution:

i. Given polynomial is $P(x) = x + 5$

Now, $P(x) = 0$

$\Rightarrow x + 5 = 0$

$\Rightarrow x = -5$

Hence $x = -5$ is zero of polynomial $P(x) = x + 5$

ii. Given polynomial is $P(x) = x - 5$

Now, $P(x) = 0$

$\Rightarrow x - 5 = 0$

$\Rightarrow x = 5$

Hence $x = 5$ is zero of polynomial $P(x) = x - 5$.

iii. Given polynomial is $P(x) = 2x + 5$

Now, $P(x) = 0$

$\Rightarrow 2x + 5 = 0$

$\Rightarrow X = -5/2$

Hence $x = -5/2$ is zero of polynomial $P(n) = 2x + 5$.

iv. Given polynomial is $P(x) = 3x - 2$

Now, $P(x) = 0$

$\Rightarrow 3x - 2 = 0$

$\Rightarrow X = 2/3$

Hence $x = 2/3$ is zero of polynomial $P(x) = 3x - 2$

vi) Given polynomial is $P(x) = ax$

Now, $P(x) = 0$

$\Rightarrow ax = 0$

$\Rightarrow a = 0$ or $x = 0$

But given that $a \neq 0$

Hence $x = 0$ is zero of polynomial $P(x) = ax$. [V and vii same as previous parts]

Exercise: 2.3

1. Find the remainder when $x^3 + 3x^2 + 3x + 1$ is divided by
2. **Solution:**

i) Given polynomial is $P(x) = x^3 + 3x^2 + 3x + 1$

When $P(x)$ is divided by $x + 1$, then the remainder is $P(-1)$

Hence, remainder $= P(-1) = (-1)^3 + 3.(-1)^2 + 3(-1) + 1$

$= -1 + 3 - 3 + 1 = 0$

Remainder when polynomial $x^3 + 3x^2 + 3x + 1$ is divided by $x + 1$ is equal to 0

ii) Given polynomial is $P(x) = x^3 + 3x^2 + 3x + 1$

When $P(x)$ is divided by $x - 1/2$, then the remainder is $P(1/2)$

Hence, remainder $= P(1/2) = (1/2)^3 + 3.(1/2)^2 + 3(1/2) + 1$

$= 1/8 + 3/4 + 3/2 + 1$

$= 1/8 + 6/8 + 12/8 + 8/8$

$= 27/8$

(iii) Given polynomial is $P(x) = x^3 + 3x^2 + 3x + 1$

When $P(x)$ is divided by x, then the remainder is $P(0)$

Hence, remainder $= P(0) = (0)^3 + 3.(0)^2 + 3(0) + 1 = 1$

The remainder when polynomial $x^3 + 3x^2 + 3x + 1$ is divided by x is equal to 1.

iv. Given polynomial is $P(x) = x^3 + 3x^2 + 3x + 1$

When $P(x)$ is divided by $x + \pi$, then the remainder is $P(-\pi)$

Hence, remainder $= P(-\pi) = (-\pi)^3 + 3.(-\pi)^2 + 3(-\pi) + 1$

$= -\pi^3 + 3\pi^2 - 3\pi + 1$

The remainder when polynomial $P(n) = x^3 + 3x^2 + 3x + 1$ is divided by $x + \pi$ is equal to $-\pi^3 + 3\pi^2 - 3\pi + 1$

2. Find the remainder when $x^3 - ax^2 + 6x - a$ is divided by $(x-a)$

Let $P(x) = x^3 - ax^2 + 6x - a$ and $g(x) = x - a$

Put $g(x) = 0$

$\Rightarrow X - a = 0$

$\Rightarrow x = a$

Hence, remainder $= P(a) = a^3 - a.(a)^2 + 6(a) - a$

$= a^3 - a^3 + 6a - a$

$= 5a$

The remainder when polynomial $P(x) = x^3 - ax^2 + 6x - a$ is divided by $x - a$ is equal to 5a

3. Check whether $7 + 3x$ is factor of $3x^3+7x$.

Solution:

Let $P(x) = 3x^3 + 7x$ and $g(x)= 7+3x$

$7 + 3x$ to be a factor of $3x^3 + 7x$, when remainder of polynomial $3x^3 + 7x$ divided by $7 + 3x$ must be zero.

Put $g(x)=0$

$\Rightarrow$ $7+3x=0$

$\Rightarrow$ $X=-\frac{7}{3}$

$P(-7/3)= 3(-7/3)^3 + 7(-7/3)$

$=3(-343/27)- 49/3$

$= -343/9- 49/3$

$= -490/9$

As remainder is not equal to zero

Hence $7 + 3x$ is not a factor of $3x^2 + 7x$

Exercise: 2.4

1. Determine which of the following polynomials has $(x + 1)$ as factor:

Solution:

i) Given polynomial is $p(x) = x^3 + x^2 + x + 1$

Let $g(x)= x+1$

Put $g(x) = 0$

$\Rightarrow$ $X+1= 0$

$\Rightarrow$ $x= -1$

Hence, remainder $= p(-1) = (-1)^3+ (-1)^2+ (-1) + 1$

$= -1 + 1 - 1 + 1$

$= 0.$

Hence $x + 1$ is a factor of polynomial $x^3 + x^2 + x + 1$

ii. Given polynomial is $p(x) = x^4 + x^3 + x^2 + x + 1$

Let $g(x)= x+1$

Put $g(x) = 0$

$\Rightarrow x+1= 0$

$\Rightarrow x = -1$

Hence, remainder $= p(-1) = (-1)^4+ (-1)^3+ (-1)^2+ (-1) +1$

$= 1 - 1 + 1 - 1 + 1$

$= 1$

As remainder $\neq 0$,

Hence $x + 1$ is not a factor of polynomial $x^4 + x^3 + x^2 + x + 1$.

iii. Given polynomial is $p(x) = x^4 + 3x^3 + 3x^2 + x + 1$.

Let $g(x)= x+1$

Put $g(x) = 0$

$\Rightarrow X+1= 0$

$\Rightarrow X = -1$

Hence, remainder $= p(-1) = (-1)^4 + 3(-1)^3 + 3(-1)^2 + (-1) + 1$

$= 1 - 3 + 3 - 1 + 1 = 1$

As remainder $\neq 0$.

Hence $(x + 1)$ is not a factor of polynomial $x^4 + 3x^3 + 3x^2 + x + 1$

iv. Given polynomial is $p(x) = x^3 - x^2 - (2 + \sqrt{2})x + \sqrt{2}$

Let $g(x)= x+1$

Put $g(x) = 0$

$\Rightarrow X+1= 0$

$\Rightarrow X = -1$

$p(-1) = (-1)^3 - (-1)^2 - (2 + \sqrt{2})(-1) + \sqrt{2} = 2\sqrt{2}$

As remainder $\neq 0$,

Hence $(x + 1)$ is not a factor of polynomial $- x^2 - (2 + \sqrt{2})x + \sqrt{2}$.

2. Use the factor theorem to determine whether $g(x)$ is a factor of $p(x)$ in each of the following cases:

Solution: **i)** $p(x) = 2x^3 + x^2 - 2x - 1$, $g(x) = x + 1$

put $g(x) = 0$

$\Rightarrow x+1= 0$

$\Rightarrow x = -1$

Hence, remainder $= p(-1) = 2(-1)^2 + (-1)^3 - 2(-1) - 1$

$= -2 + 1 + 2 - 1 = 0$.

As remainder when polynomial $p(x)$ is divided by polynomial $g(x)$ is equal to zero, then polynomial $g(x) = x + 1$ is a factor of polynomial

$p(x) = 2x^3 + x^2 - 2x - 1$

ii. $p(x) = x^3 + 3x^2 + 3x + 1$, $g(x) = x + 2$

put $g(x)= 0$

$\Rightarrow x + 2= 0$

$\Rightarrow x = -2$

Hence, remainder $= p(-2) = (-2)^3 + 3(2)^2 + 3(-2) + 1$

$= -8 + 12 - 6 + 1$

$= -1$

Since remainder $\neq 0$, the polynomial $g(x) = x + 2$ is not a factor of polynomial $p(x) = x^3 + 3x^2 + 3x + 1$.

iii. $p(x) = x^3 - 4x^2 + x + 6$, $g(x) = x - 3$

put g(x)= 0

$\Rightarrow$ x-3 = 0

$\Rightarrow$ x = 3

Hence, remainder $= p(3) = (3)^3 - 4(3)^2 + 3 + 6$

$= 27 - 36 + 9$

$= 0$

Since reminder $= 0$, the polynomial $g(x) = x - 3$ is factor of polynomial $p(x) = x^3 - 4x^2 + x + 6$.

3. Find the value of k if x-1 is the Factor of p(x) in each of the following cases:

Solution: **(i)** $(x) = x^2 + x + k$ and $g(x) = x - 1$

put g(x)= 0

$\Rightarrow$ x-1= 0

$\Rightarrow$x= 1

p(1)= 0 since, x-1 is the factor of $x^2 + x + k$

$\Rightarrow (1)^2 + 1 + k = 0$

$\Rightarrow$ 1+1+k= 0

$\Rightarrow$ 2+k= 0

$\Rightarrow$K= -2

K= -2- √2=-(2+√2) {part ii same as i}

(iii) $p(x) = kx^2 - \sqrt{2}x + 1$and $g(x) = x - 1$

put g(x)= 0

$\Rightarrow$ x-1= 0

$\Rightarrow$ x= 1

p(1)= 0 since, x-1 is the factor of p(x)

$\Rightarrow k(1)^2 - \sqrt{2}(1) + 1 = 0$

$\Rightarrow k - \sqrt{2} = -1$

$\Rightarrow$ $k = -1+\sqrt{2}$

(iv) $p(x) = kx^2 - 3x + k$ and $g(x) = x-1$

put $g(x) = 0$

$\Rightarrow x-1 = 0$

$\Rightarrow x = 1$

$p(1) = 0$ since, $x-1$ is the factor of $p(x)$

$\Rightarrow k(1)^2 - 3(1) + k = 0$

$\Rightarrow 2k = 3$

$\Rightarrow k = 3/2$

4. Factorise;

Sol. (i) Given polynomial is $12x^2 - 7x + 1$

$= 12x^2 - 7x + 1$

$= 12x^2 - 4x - 3x + 1$

$= 4x(3x - 1) - 1(3x - 1)$

$= (4x - 1)(3x - 1)$

$12x^2 - 7x + 1 = (3x - 1)(4x - 1)$

(ii) Given polynomial is $2x^2 + 7x + 3$

$= 2x^2 + 7x + 3$

$= 2x^2 + 6x + x + 3$

$= 2x(x + 3) + 1(x + 3)$

$= (2x + 1)(x + 3)$

$2x^2 + 7x + 3 = (2x + 1)(x + 3)$

(iii) Given polynomial is $6x^2 + 5x - 6$

$= 6x^2 + 9x - 4x - 6$

$= 3x(2x + 3) - 2(2x + 3)$

$= (2x + 3)(3x - 2)$

$6x^2 + 5x - 6 = (3x - 2)(2x + 3)$

(iv) Given polynomial is $3x^2 - x - 4$

On splitting middle term

$= 3x^2 - 4x + 3x - 4$

$= x(3x - 4) + 1(3x - 4)$

$= (x + 1)(3x - 4)$

$3x^2 - x - 4 = (x + 1)(3x - 4)$

5Factorise

i) $x^3 - 2x^2 - x + 2$

sol. $x^3 - 2x^2 - x + 2$

$= x^2 (x-2) - 1(x-2)$

$= (x-2)(x^2 - 1)$

$= (x-2)(x+1)(x-1)$

ii) $x^3 - 3x^2 - 9x - 5$

sol. $x^3 - 3x^2 - 9x - 5$

$= x^3 + x^2 - 4x^2 - 4x - 5x - 5$

$= x^2 (x+1) - 4x(x+1) - 5(x+1)$

$= (x+1)\{(x^2 - 4x - 5)\}$

$= (x+1)\{(x^2 - 5x + x - 5)\}$
$= (x+1)\{x(x-5) + 1(x-5)\}$
$= (x+1)(x-5)(x+1)$

iii. $x^3 + 13x^2 + 32x + 20$

sol. $x^3 + 13x^2 + 32x + 20$

. $= x^{3+} x^2 + 12x^2 + 12x + 20x + 20$

$= x^2 (x+1) + 12x(x+1) + 20(x+1)$

$= (x+1)\{(x^2 + 12x + 20)\}$

$= (x+1)\{(x^2 + 10x + 2x + 20)\}$
$= (x+1)\{x(x+10) + 2(x+10)\}$
$= (x+1)(x+10)(x+2)$

iv. $2y^3 + y^2 - 2y - 1$

Sol. $2y^3 + y^2 - 2y - 1$

$= y^2 (2y+1) - 1(2y + 1)$

$= (y^2 - 1)(2y+1)$

$= (y+1)(y-1)(2y+1)$

Exercise: 2.5

1. Use suitable identities to find the following products

Solution: (i) $(x + 4)(x + 10)$

We know that

$(x + a)(x + b) = x^2 + (a + b)x + ab$

Given polynomial is $(x + 4)(x + 10)$

Here, $a = 4, b = 10$

$(x + 4)(x + 10) = x^2 + (4 + 10)x + 40$

$= x^2 + 14x + 40$

(ii) $(x + 8)(x - 10)$

We know that

$(x + a)(x + b) = x^2 + (a + b)x + ab$

Given polynomial is $(x + 8)(x - 10)$

Here $a = 8, b = -10$

$(x + 8)(x - 10) = x^2 + (8 - 10)x - 80$

$= x^2 - 2x - 80.$

Rest parts are same as part (iii)(iv)(v)

2. Evaluate the following products without multiplyingdirectly

Solution:

(i) $103 \times 107 = (100 + 3) \times (100 + 7)$

We know that $(x + a)(x + b) = x^2 + (a + b)x + ab$

Here $x = 100, a = 3, b = 7$

$(100 + 3)(100 + 7) = (100)^2 + 10 \times 100 + 3 \times 7$

$= 10000 + 1000 + 21$

$= 11021$

(ii) $95 \times 96 = (100 - 5)(100 - 4)$

We know that $(x + a)(x + b) = x^2 + (a + b)x + ab$

Here $x = 100, a = -5, b = -4$ $95 \times 96 = (100 - 5)(100 - 4)$

$= (100)^2+[(-5 + (-4))]100 + (-5)(-4)$

$= 10000 - 900 + 20$

$= 9120.$

(iii) $104 \times 96 = (100 + 4)(100 - 4)$

We know that $(x + a)(x - a) = x^2 - a^2$

Here $x = 100$, $a = 4$

$$(100 + 4)(100 - 4) = (100)^2 - (4)^2$$
$$= 10000 - 16$$
$$= 9984$$

3. Factorise the following using appropriate identities

(i) $9x^2 + 6xy + y^2 = (3x)^2 + 2 \cdot 3x \cdot y + (y)^2$

We know that $x^2 + 2xy + y^2 = (x + y)^2$

$9x^2 + 6xy + y^2 = (3x^2) + 2 \cdot 3x \cdot y + (y)^2$

$= (3x + y)^2$

$= (3x + y)(3x + y)$

(ii) $4y^2 - 4y + 1 = (2y)^2 - 2 \cdot 2y(1) + 1$

We know that $x^2 - 2xy + y^2 = (x - y)^2$

$(2y)^2 - 2 \cdot 2y)(1) + 1 = (2y - 1)^2$

$= (2y - 1)(2y - 1)$

(iii) $x^2 - y^2/100$

We know that $(x + a)(x - a) = x^2 - a^2$

$= x^2 - (y/10)^2$

$=(x+y/10)(x-y/10)$

4. Expand each of the following, using suitable Identities

(i) $(x + 2y + 4z)^2$

We know that $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$(x + 2y + 4z)^2 = x^2 + (2y)^2 + (4z)^2 + 2 \cdot x \cdot 2y + 2 \cdot 2y \cdot 4z + 2 \cdot x \cdot 4z$

$= x^2 + 4y^2 + 16z^2 + 4xy + 16yz + 8xz$

(ii) $(2x - y + z)^2$

We know that $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$(2x - y + z)^2 = (2x)^2 + (-y)^2 + (z)^2 + 2 \cdot 2x(-y) + 2(-y)(z) + 2 \cdot 2x \cdot z$

$= 4x^2 + y^2 + z^2 - 4xy - 2yz + 4xz$

(iii) $(-2x + 3y + 2z)^2$

We know that $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$(-2x + 3y + 2z)^2 = (-2x)^2 + (3y)^2 + (2z)^2 + 2 \cdot (-2x) \cdot 3y + 2 \cdot 3y \cdot 2z + 2 \cdot (-2x) \cdot 2z$

$= 4x^2 + 9y^2 + 4z^2 - 12xy + 12yz - 8xz$

(iv)$(3a - 7b - c)^2$

We know that $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$(3a - 7b - c)^2 = (3a)^2 + (-7b)^2 + (-c)^2 + 2 \cdot 3a \cdot (-7b) + 2 \cdot (-7b)(-c) + 2. (3a). (-c)$

$= 9a^2 + 49b^2 + c^2 - 42ab + 14bc - 6ac.$

(v) $(-2x + 5y - 3z)^2$

We know that $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$(-2x + 5y - 3z)^2 = (-2x)^2 + (5y)^2 + (-3z)^2 + 2 \cdot (-2x)(5y) + 2 \cdot (5y)(-3z) + 2 \cdot (-2x)(-3z)$

$= 4x^2 + 25y^2 + 9z^2 - 20xy - 30yz + 12xz$ {part vi same as i,ii,iii,iv and v}

5. Factorise

Sol.
i $4x^2 + 9y^2 + 16z^2 + 12xy - 24yz - 16xz$

$= (2x)^2 + (3y)^2 + (-4z)^2 + 2 \cdot 2x \cdot 3y + 2 \cdot (3y)(-4z) + 2 \cdot (2x)(-4z)$

We know that $x^2 + y^2 + z^2 + 2xy + 2yz + 2xz = (x + y + z)^2$

$= (2x)^2 + (3y)^2 + (-4z)^2 + 2 \cdot 2x \cdot 3y + 2 \cdot (3y)(-4z) + 2 \cdot (2x)(-4z)$

$= (2x + 3y - 4z)^2$

$= (2x + 3y - 4z)(2x + 3y - 4z)$

ii $2x^2 + y^2 + 8z^2 - 2\sqrt{2}xy + 4\sqrt{2}yz - 8xz$

We know that $x^2 + y^2 + z^2 + 2xy + 2yz + 2xz = (x + y + z)^2$
$(-\sqrt{2}x)^2 + y^2 + (2\sqrt{2}z)^2 + 2(-\sqrt{2}x)(y) + 2(y)(2\sqrt{2}z) + 2(2\sqrt{2}z)(-\sqrt{2}x)$

$= (-\sqrt{2}x + y + 2\sqrt{2}z)^2$

$= (-\sqrt{2}x + y + 2\sqrt{2}z)(-\sqrt{2}x + y + 2\sqrt{2}z)$

6. Write the following cubes in expanded form

Sol.i)We know that $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

Given polynomial is $(2x + 1)^3$ $a = 2x, b = 1$

$$(2x + 1)^3 = (2x)^3 + (1)^3 + 3 \cdot (2x) \cdot (1)(2x + 1)$$

$$= 8x^3 + 1 + 6x(2x + 1)$$

$$= 8x^3 + 1 + 12x^2 + 6x$$

$$= 8x^3 + 12x^2 + 6x + 1$$

ii) We know that $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

$$(2a - 3b)^3 = (2a)^3 - (3b)^3 - 3(2a)(3b)(2a - 3b)$$

$$= 8a^3 - 27b^3 - 18ab(2a - 3b)$$

$$= 8a^3 - 27b^3 - 36a^2b + 54ab^2$$

$$= 8a^3 - 36a^2b + 54ab^2 - 27b^3$$

Rest parts are same as part (i)(ii)

7. Evaluate the following using suitable identities

Solution: i. $(99)^3 = (100 - 1)^3$

We know that $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$ a

$= 100, b = 1$

$(99)^3 = (100 - 1)^3 = (100)^3 - (1)^3 - 3(100)(1)(100\text{-}1)$
$= (100)^3 - (1)^3 - 3(100)(1)(99)$

CHEMICAL $= 1000000 - 1 - 29,700$

$= 9\ 70\ 299.$

ii. $(102)^3 = (100 + 2)^3$

We know that $(a + b)^3 = a^3 + b^3 + 3ab(a + b)$

$a = 100, b = 2$

$$(102)^3 = (100 + 2)^3 = (100)^3 + (2)^3 + 3 \cdot 100 \cdot 2(100 + 2)$$

$= 1000000 + 8 + 600 \times 102$

$= 1000008 + 61{,}200$

$= 1061208$

iii. $(998)^3 = (1000 - 2)^3$

We know that $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

Here $a = 1000$, $b = 2$

$(998)^3 = (1000 - 2)^3 = (1000)^3 - 8 - 3(1000)(2)(200-2)$

$= (1000)^3 - 8 - 3(1000)(2)(998)$

$= 1000000000 - 8 - 6000 \times 998$

$= 994011992$

8. Factorise each of the following

Solution:

i. $8a^3 + b^3 + 12a^2b + 6ab^2$

$= (2a)^3 + (b)^3 + 3 \cdot (2a)(b)(2a + b)$

We know that $a^3 + b^3 + 3ab(a + b) = (a + b)^3$

$(2a)^3 + b^3 + 3(2a) \cdot b(2a + b) = (2a + b)^3$

$= (2a + b)(2a + b)(2a + b)$

ii. $8a^3 - b^3 - 12a^2b + 6ab^2$

We know that $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

$8a^3 - b^3 - 12a^2b + 6ab^2 = (2a)^3 - (b)^3 - 3 \cdot (2a)(b)(2a - b)$

$= (2a - b)^3$

$= (2a - b)(2a - b)(2a - b)$

iii. $27 - 125a^3 - 135a + 225a^2$

we know that $(a - b)^3 = a^3 - b^3 - 3ab(a - b)$

$27 - 125a^3 - 135a + 225a^2 = -(125a^3 - 27 - 225a^2 + 135a)$

$= [(3)^3 - (5a)^3 - 3 \cdot (3)(5a)(3-5a)]$

$= [3 - 5a]^3$

$= (3 - 5a)^3$

$= (3 - 5a)(3 - 5a)(3 - 5a)$

iv. $64a^3 - 27b^3 - 144a^2b + 108ab^2$

we know that $a^3 - b^3 - 3ab(a - b) = (a - b)^3$

$64a^3 - 27b^3 - 144a^2b + 108ab^2$

$= (4a)^3 - (3b)^3 - 3 \cdot (4a) \cdot (3b)(4a - 3b)$

$= (4a - 3b)^3$

$= (4a - 3b)(4a - 3b)(4a - 3b)$

v. $27p^3 - \frac{1}{216} - \frac{9}{2}p^2 + \frac{1}{4}p$

we know that $a^3 - b^3 - 3ab(a - b) = (a - b)^3$

$= (3p)^3 - (-\frac{1}{6})^3 - \frac{9}{2}p^2 + \frac{1}{4}p$

$= (3p)^3 - (-\frac{1}{6})^3 - 3(3p)(\frac{1}{6})[3p - \frac{1}{6}]$

$= (3p - \frac{1}{6})^3$

$= (3p - \frac{1}{6})(3p - \frac{1}{6})(3p - \frac{1}{6})$

9. Verify

I. $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

ii. $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

Solution:

i $x^3 + y^3 = (x + y)(x^2 - xy + y^2)$

Take R.H.S $(x + y)(x^2 - xy + y^2)$

$= x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3$

$= x^3 + y^3$

Therefore R.H.S = L.H.S

Hence verified.

ii. $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

Take R.H.S $(x - y)(x^2 + xy + y^2)$

$= x^3 + x^2y + xy^2 - x^2y - xy^2 - y^3$

$= x^3 - y^3$

Therefore R.H.S = L.H.S

Hence verified.

10. Factorise each of the following

SOL. I. $27y^3 + 125z^3 = (3y)^3 + (5z)^3$

We know that $x^3 + y^3 = (x + y)(x^2 + y^2 - xy)$

$(3y)^3 + (5z)^3 = (3y + 5z)\{(3y)^2 + (5z)^2 - (3y)(5z)\} = 27y^3 + 125z^3 = (3y + 5z)(9y^2 + 25z^2 - 15yz)$

II. $64m^3 - 343n^3 = (4m)^3 - (7n)^3$

We know that $(x)^3 - (y)^3 = (x - y)(x^2 + xy + y^2)$

$(4m)^3 - (7n)^3 = (4m - 7n)\{(4m)^2 + (4m)7n + (7n)^2\} = (4m - 7n)(16m^2 + 28mn + 49n)^2\}$

11. Factorise $27x^2 + y^3 + z^3 - 9xyz$

Solution: $27x^2 + y^3 + z^3 - 9xyz$

$= (3x)^3 + (y)^3 + (z)^3 - 3(3x)(y)(z)$

We know that

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - xz)$

Now, $27x^3 + y^3 + z^3 - 9xyz = (3x)^3 + (y)^3 + (z)^3 - 3(3x)(y)(z)$

$= (3x + y + z)((3x)^2 + (y)^2 + (z)^2 - 3xy - yz - 3xz)$

$= (3x + y + z)(9x^2 + y^2 + z^2 - 3xy - yz - 3xz)$

12 Verify that $x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)[(x - y)^2 + (y - z)^2 + (x - z)^2]$

Solution: We know that

$$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - xz)$$

$= (x + y + z)\frac{1}{2}(2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2xz)$

$(x + y + z) \quad \frac{1}{2}(x^2 + y^2 - 2xy + y^2 + z^2 - 2yz + x^2 + z^2 - 2xz)$

We know that $a^2 + b^2 - 2ab = (a - b)^2$

$\frac{1}{2}(x + y + z)((x + y)^2 + (y - z)^2 + (x - z)^2)$

$x^3 + y^3 + z^3 - 3xyz = \frac{1}{2}(x + y + z)((x - y)^2 + (y - z)^2 + (x - z)^2)$

Hence verified

13. If $x + y + z = 0$, show that $x^3 + y^3 + z^3 = 3xyz$

Solution: We know that,

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)(x^2 + y^2 + z^2 - xy - yz - xz)$

Given that $x + y + z = 0$

$x^3 + y^3 + z^3 - 3xyz = (0)(x^2 + y^2 + z^2 - xy - yz - xz)$

$\Rightarrow x^3 + y^3 + z^3 - 3xyz = 0$

$\Rightarrow x^3 + y^3 + z^3 = 3xyz$

Hence proved

14 Without actually calculating the cubes, find the value of each of the following**Solution:** I). $(-12)^3 + (7)^3 + (5)^3$

Let $x = -12, y = 7, z = 5$

now $x + y + z = -12 + 7 + 5 = 0$

we know if $x + y + z = 0$

then, $x^3 + y^3 + z^3 = 3xyz$

Therefore, $(-12)^3 + (7)^3 + (5)^3 = 3(12)(7)(5)$

$= -1260$

ii. $(28)^3 + (-15)^3 + (-13)^3$

Let $x = -28, y = -15, z = -13$ = $x + y + z = -28 - 15 - 13 = 0$

We know if $x + y + z = 0$

Then , $x^3 + y^3 + z^3 = 3xyz$

Therefore, $(28)^3 + (-15)^3 + (-13)^3 = 3(28)(-15)(-13)$

$= 16380$

15 Give possible expressions for the length and breadth of each of the following rectangles, in which the areas are given

Solution:i. Given area $= 25a^2 - 35a + 12$

$= 25a^2 - 15a - 20a + 12$

$= 5a(5a - 3) - 4(5a - 3)$

$= (5a - 3)(5a - 4)$

We know that area = length × breadth

So possible expression forlength= $5a - 3$

possible expression for breadth = 5a – 4.

ii. Given area = $35^2 + 13y^2 - 12$

$= 35y^2 + 28y - 15y - 12$

$= 7y(5y + 4) - 3(5y + 4)$

$= (5y + 4)(7y - 3)$

We know that area = length × breadth

So possible expression for length $= 5y + 4$

possible expression for breadth $= 7y - 3$.

16. What are the possible expressions for the dimension of the cuboids whose volume are given below?

(i) Volume$= 3x^2 - 12x$

(ii) Volume$= 12ky^2 + 8ky - 20k$

Solution:

(i) Given volume $= 3x^2 - 12x$

$= 3(x^2 - 4x)$

$= 3x(x - 4)$

We know that Volume of cuboid = length × breadth × height

Possibleexpressionfor length of cuboid$= 3$

Possible expression for breadth $= x$

Possible expression for height $= x - 4$.

(ii) Given Volume $= 12ky^2 + 8ky - 20k$

$= 4k(3y^2 + 2y - 5)$

$= 4k(3y^2 + 5y - 3y - 5)$

$= 4k(y(3y + 5) - 1(3y + 5)$

$= 4k(3y + 5)(y - 1)$

Possible value of length of cuboid$= 4k$

Possible expression for breadth $= 3y + 5$

Possible expression for breadth $= y - 1$.

Linear equations in two variables

Exercise: 3.1

1. The cost of a notebook is twice the cost of pen write a linear equation in two variables to represent this statement.

Solution:

Let the cost of a notebook be x and cost of pen be y

Given that cost of a notebook $=2\times$ cost of a pen

$$\Rightarrow x = 2y$$

$$\Rightarrow x - 2y = 0$$

Hence, $x - 2y = 0$ is the representation of the given statement.

2. Express the following linear equations in the form $ax + by + c = 0$ and indicate the values of a, b and c in each case:
   i). $2x+3y=9.35$

   sol. $2x+3y=9.35$

   $\Rightarrow 2x+3y-9.35=0$

   Comparing above equation with $ax+by+c=0$.

   We get ,$a=2, b=3, c=-9.35$

iii) $-2x+3y=6$

sol $\Rightarrow -2x+3y-6=0$

Comparing above equation with $ax+by+c=0$.

We get, $a=-2$, $b=3$, $c=-6$

Vi) $3x+2=0$

Sol $\Rightarrow 3x+0.y+2=0$

Comparing above equation with $ax+by+c=0$.

We get, $a=3$, $b=0$, $c=2$

Exercise: 3.2

1. Which one of the following options is true, and why

$y = 3x + 5$ has

(I) a unique solution (ii)only two solutions (iii) infinitely many solutions

sol. $y = 3x + 5$ has infinitely many solutions

2.Write four solutions for each of the following equations

(i) $2x + y = 7$

(ii) $\pi x + y = 9$

(iii) $x = 4y$

Solution: I) $2x + y = 7$

For $x = 0$

$y = 7 - 2x$

$\Rightarrow y = 7 - 2(0) = 7$

$\therefore (0, 7)$ is a solution.

For $x = 1$

$\Rightarrow y = 7 - 2(1) = 5$

$\therefore (1, 5)$ is a solution.

For $x = 2$

$\Rightarrow y = 7 - 2(2) = 3$

$\therefore (2, 3)$ is a solution.

For $x = 3$

$\Rightarrow y = 7 - 2(3) = 1$

$\therefore (2, 1)$ is a solution

ii) $\pi x + y = 9$

sol. For $x = 0$

$y = 9 - \pi x$

$\Rightarrow y = 9 - \pi(0) = 9$

$\therefore (0, 9)$ is a solution.

For $x = 1$

$\Rightarrow y = 9 - \pi(1) = 9 - \pi$

$\therefore (1, 9 - \pi)$ is a solution.

For $x = 2$

$\Rightarrow y = 9 - \pi(2) = 9 - 2\pi$

$\therefore$ (2, 9-2 π) is a solution.

For $x = 3$

$\Rightarrow y = 9 - (\pi)(3) = 9 - 3\pi$

$\therefore$ (2,9-3 π) is a solution

iii) $x = 4y$

For $y = 0$

$x = 4(0)$

$\Rightarrow x = 0$

$\therefore$ (0, 0) is a solution.

For $y = 1$

$\Rightarrow x = 4(1) = 4$

$\therefore$ (4, 1) is a solution.

For $y = 2$

$\Rightarrow x = 4(2) = 8$

$\therefore$ (8, 2) is a solution.

For $y = 3$

$\Rightarrow x = 4(3) = 12$

$\therefore$ (12, 3) is a solution

3. Check which of the following are solutions of the equation $x - 2y = 4$ and which arenot

(i) (0,2)

Sol. Put x=0 and y=2 in L.H.S of the equation $x - 2y = 4$

$0 - 2(2) = -4$

$\Rightarrow -4 \neq 4$

$\therefore$ L.H.S $\neq$ R.H.S

Hence, (0, 2) is not a solution of $x - 2y = 4$.

(ii) (2,0)

Sol. Put x=2 and y=0 in L.H.S of the equation $x - 2y = 4$

$2 - 2(0) = 2$

$\Rightarrow 2 \neq 4$

$\therefore$ L.H.S $\neq$ R.H.S

Hence, (2, 0) is not a solution of $x - 2y = 4$

(iii) (4,0)

Put x=4 and y=0 in L.H.S of the equation $x - 2y = 4$

$4 - 2(0) = 4$

$\Rightarrow 4 = 4$

$\therefore$ L.H.S = R.H.S

Hence, (4,0) is a solution of $x - 2y = 4$.

(iv) $(\sqrt{2}, 4\sqrt{2})$

. Put $x = \sqrt{2}$, and $y = 4\sqrt{2}$ in L.H.S of the equation $x - 2y = 4$

$\Rightarrow x - 2y = \sqrt{2} - 2 \cdot 4\sqrt{2} = -7\sqrt{2}$

$\Rightarrow -7\sqrt{2} \neq 4$

$\therefore$ L.H.S $\neq$ R.H.S

Hence, $(\sqrt{2}, 4\sqrt{2})$ is a solution of $x - 2y = 4$.

(v) (1,1)

Put x=1 and y=1 in L.H.S of the equation $x - 2y = 4$

1-1(0)= 1

$\Rightarrow 1 \neq 4$

$\therefore$ L.H.S $\neq$ R.H.S

Hence, (1, 1) is not a solution of $x - 2y = 4$

4. Find the value of k, if $x = 2$, $y = 1$ is a solution of the equation $2x + 3y = k$.

Solution:

Given that (2, 1) is a solution of the equation $2x + 3y = k$

$\Rightarrow 2(2) + 3(1) = k$

$\Rightarrow k = 4 + 3 = 7$

Therefore, if $x = 2$, $y = 1$ is a solution of equation $2x + 3y = k$, then $k = 7$.

Exercise: 3.3

1. Draw the graph of each of the following linear equations in twovariables:

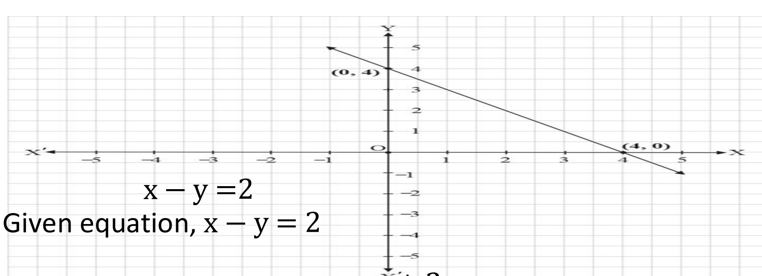
(i) $x + y = 4$

sol. Given equation, $x + y = 4$

$\Rightarrow y = 4 - x$

At $x = 0$ and $x = 4$ we get $y = 4$ and $y = 0$ respectively.

$\therefore$ (0, 4) and (4, 0) are solutions of $x + y = 4$

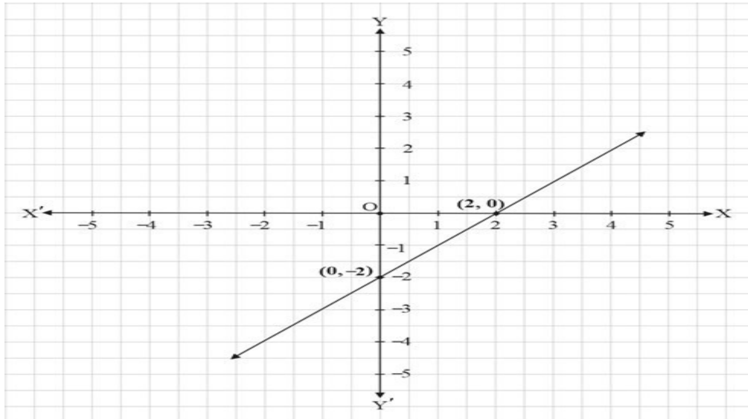


$x - y = 2$

Solution:. Given equation, $x - y = 2$

$\Rightarrow x = y + 2$

At $y = 0$ and $y = 2$ we get $x = 2$ and $x = 0$ respectively



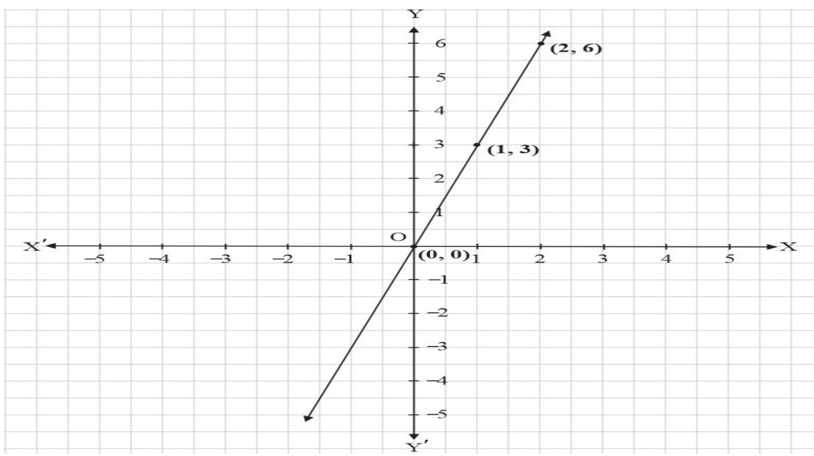
(ii) $y = 3x$

sol. Given equation, $y = 3x$.

At $x = 0$ we get $y = 0$

Similarly, at $x = 1$ and $x = 2$ we get $y = 3$ and $y = 6$ respectively.

$\therefore$ (0, 0), (1, 3) and (2, 6) are the solutions of $y = 3x$.

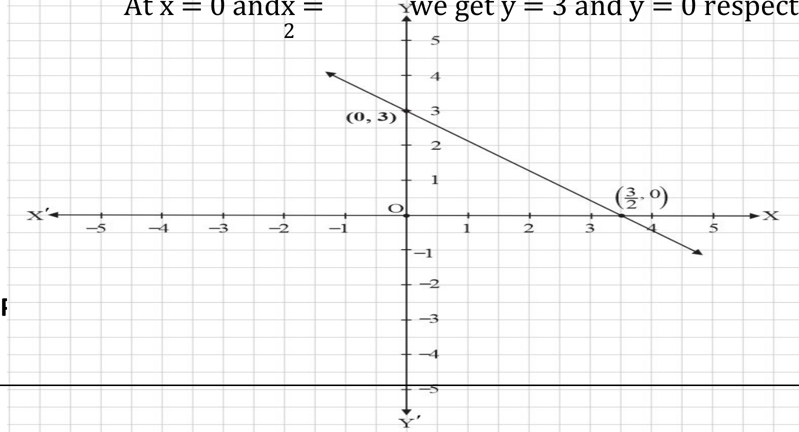


Iv $3 = 2x + y$

Sol. Given equation, $3 = 2x + y$.

$\Rightarrow y = 3 - 2x$

At $x = 0$ and $x = \frac{3}{2}$ we get $y = 3$ and $y = 0$ respectively.



2 Give the equations of two lines passing through (2, 14). How many more such lines are there, and why?

Solution

Given point (2, 14)

Let $x = 2$ and $y = 14$

We can write $14 = 7 \times 2$

$\Rightarrow y = 7x$ is a line passing through (2, 12)

Similarly, $14 = 2 + 12$

$\Rightarrow y = x + 12$ is a line passing through (2, 14).

$\therefore y = 7x$ and $y = x + 12$ are two lines passing through (2, 14).

From above process we can say that there are different possible combinations of lines which passing through (2, 14).

Therefore, from a given point (2, 14), there are infinite lines passing through it.

3. If the point (3, 4) lies on the graph of the equation $3y = ax + 7$, find the value of a.
Solution; Given that point (3, 4) lies on graph of the equation $3y = ax + 7$

$\Rightarrow 3(4) = a(3) + 7 \Rightarrow 12\text{-}7=3a$

$\Rightarrow 5 = 3a$

$\therefore a = \frac{5}{3}$

Therefore, if (3, 4) is the solution of equation $3y = ax + 7$ then $a = 5/3$

4. The taxi fare in a city is as follows: For the first kilometer, the fare is rs. 8 and for the subsequent distance it is rs. 5 per km. Taking the distance covered as x km and total fare as rs. y. write a linear equation for this information and draw its graph.

Solution:

Given

Total distance covered = x km

Total fare = Rs y

Fare for first km = Rs 8

Fare for subsequent distance = Rs 5

Total fare = Fare for first kilometer + Fare for rest of the distance.

$\Rightarrow y = 8 + (x - 1)5$

$\Rightarrow y = 8 + 5x - 5$

$\Rightarrow y = 5x + 3$

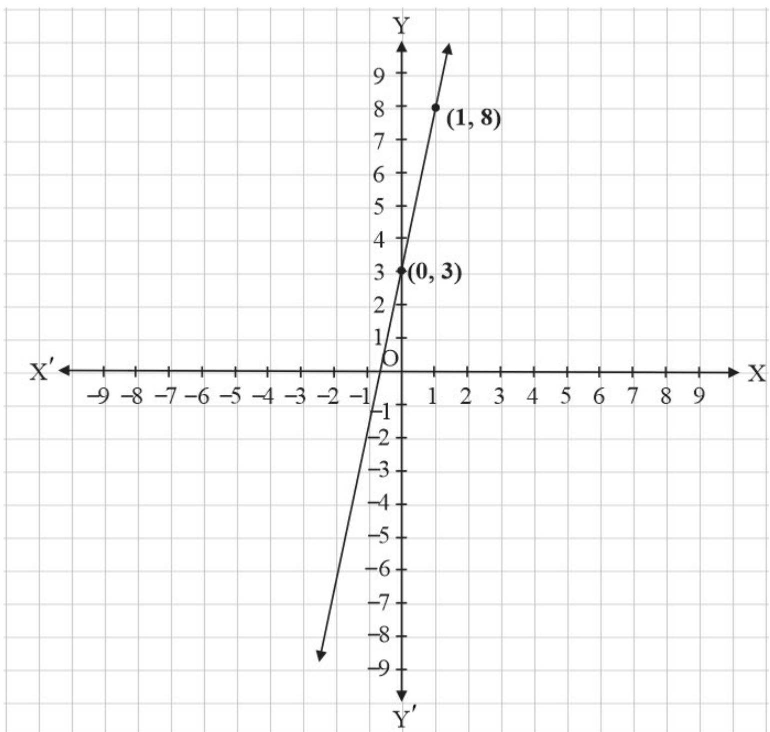
$\Rightarrow 5x - y + 3 = 0$

Therefore, $5x - y + 3 = 0$ is the linear equation for the given information. Graph of $5x - y + 3 = 0$

$\Rightarrow y = 3 + 5x$

At $x = 0$ and $x = 1$ we get $y = 3$ and $y = 8$ respectively.

$\therefore (0, 3)$ and $(1, 8)$ are the solutions of $5x - y + 3 = 0$.



QUESTION 5 IS BOOK WORK

6 If the work done by a body on application of constant force is directly proportional to the distance travelled by the body, express this in the form of an equation in two variables and draw the graph of the same by taking

the constant force as 5 units. Also read from the graph the work done when the distance travelled by the body is

(i) 2units

(ii) 0units

Solution:

Let distance travelled be x km and work

done be y units given that y directly

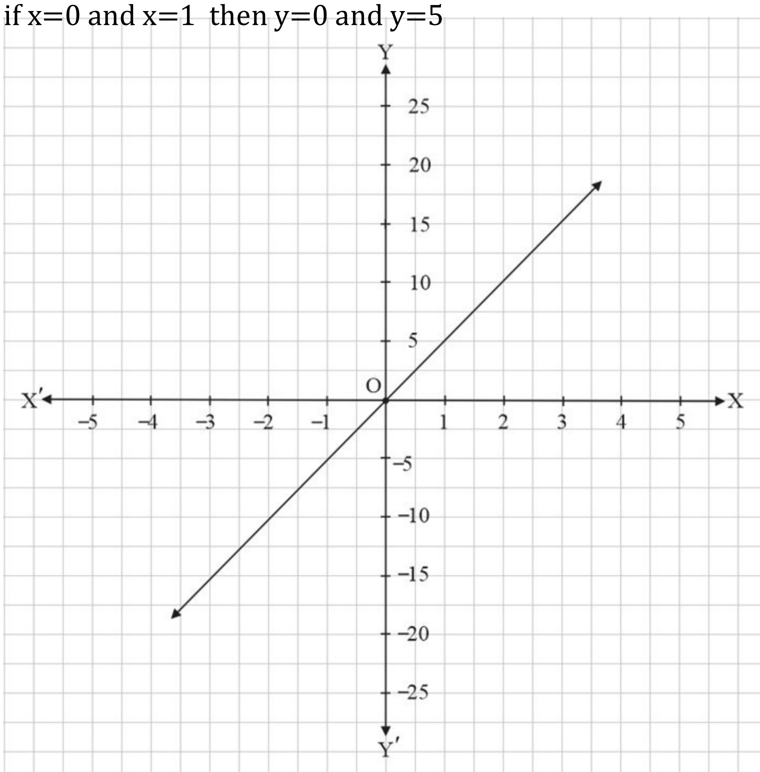
proportional to x

$y = kx$, where k is an arbitrary

constant Given$k = 5$ units

$y = 5x$

if x=0 and x=1 then y=0 and y=5



(i) From the graph, work done by the body, when distance travelled by it is2
units, is 10 units.

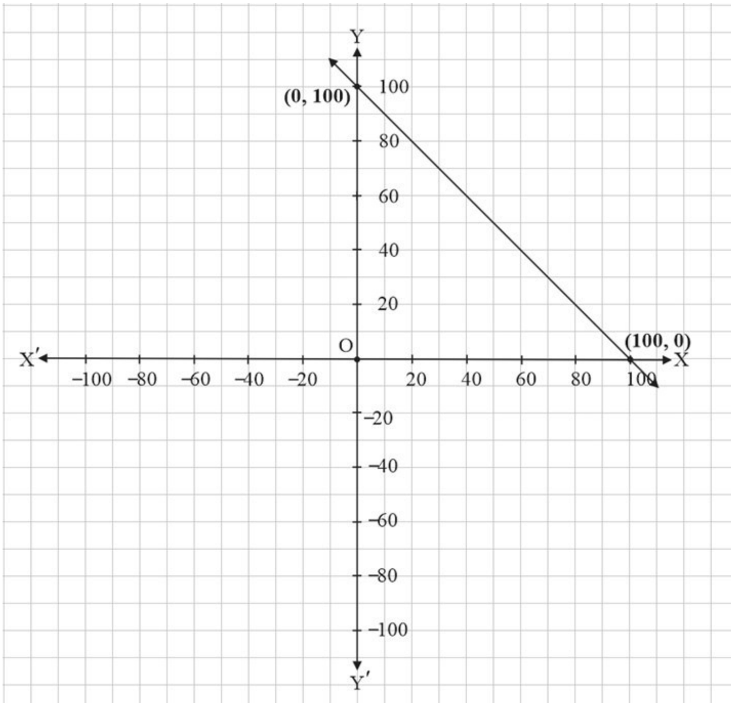
(ii) From the graph, work done by the body, is 0 units when distance travelled by it is 0units.

7. Yemini and Fatima, two students of class IX of school, together contributed Rs 100 towards the prime minister's relief fund to help the earthquake victims. Write a linear equation which satisfies this data.

Solution: Let Yamini contributed be x and Fatima contributed be y

$x + y = 100 \Rightarrow x = 100 - y$

if y =0 and y=100 then x=100 and x=0



8. In countries like USA and Canada, temperature is measured in Fahrenheit, where In countries like India, it is measures in Celsius.Here is a linear equation that concerts Fahrenheit to Celsius

$F=(9/5)C+32$

I) Draw the graph of the linear equation above using Celsius for $x-$axis ad Fahrenheit for $y-$axis.

ii) If the temperature is $30^{o}C$, what is the temperature in Fahrenheit?

iii) If the temperature is $95^{o}F$, what is the temperature in Celsius?

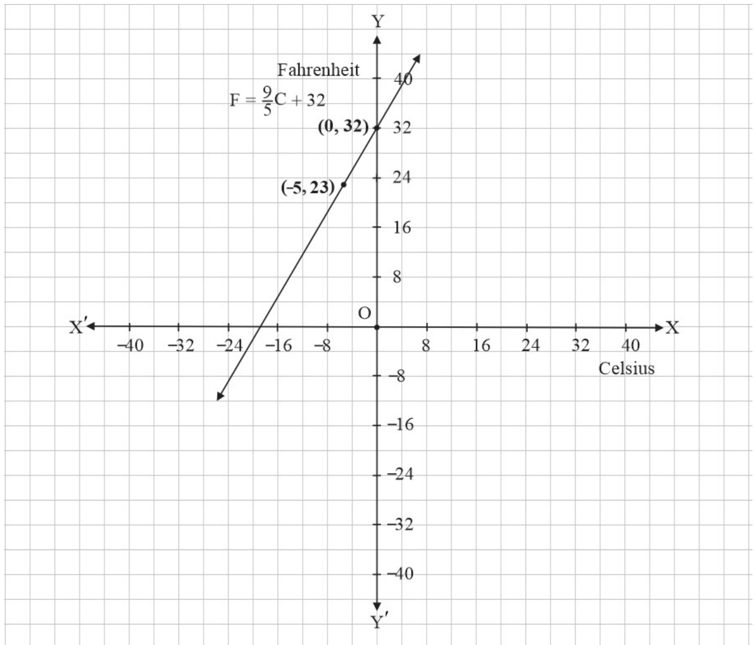
iv) If the temperature is $0^{o}C$, what is the temperature in Fahrenheit and if the temperature is $0^{o}F$, what is the temperature in Celsius?

v) Is there a temperature which is numerically the same in both Fahrenheit and Celsius? If yes, find it

Solution: i. $F=(9/5)C+32$

At $C = 0$ and $C = -5$ we get $F = 32$ and $F = 23$

So, $(0,32)$ and $(-5,23)$ are the solution of equation



II. Given temperature $= 30^{o}c$

$F = (9/5)\ 30 + 32$

$\Rightarrow F = 54 + 32 = 86^{o}F$

III. Given temperature $= 95^{o}F$

$\Rightarrow 95 = (9/5)c + 32$

$\Rightarrow 95 - 32 = (9/5)c$

$\Rightarrow 63(5/9) = c$

$\Rightarrow C = 35$ { part iv same as ii and iii }

v. Yes, there is a temperature which is numerically same in both Fahrenheit and Celsius.

Let $F = x$ and $C = x$

$\Rightarrow F = (9/5)C + 32$

$\Rightarrow x = (9/5)x + 32 \Rightarrow x - \frac{9x}{5} = 32$

$\Rightarrow \frac{5x - 9x}{5} = 32 \Rightarrow -5x = 160 \Rightarrow x = \frac{160}{-5}$

$\Rightarrow x = -40$

Exercise: 3.4

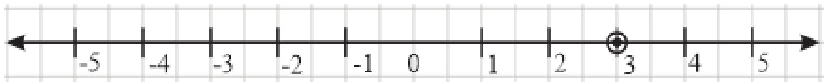
1. Give the geometric representation of $y = 3$ as an equation.

(i) in one variable

(ii) in twovariables.

Solution:

(i) In one variable $y = 3$ represents as a point.



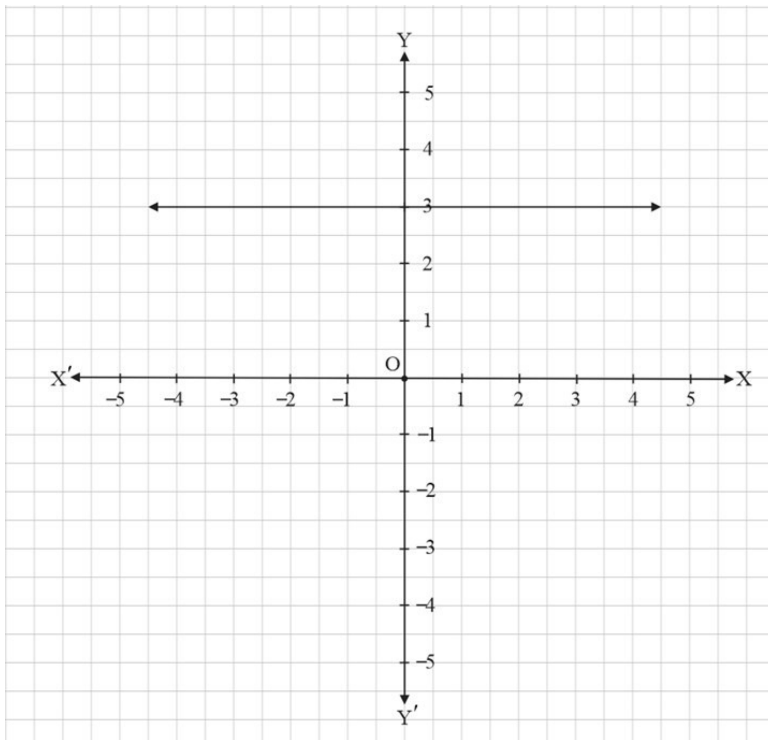
(ii) In two variables, $y = 3$ represents as a line.

$\Rightarrow$ $0x + y = 3$

$\Rightarrow$ $y = 3 - 0x$

If $x = 1$ and $x = 2$

Then $y = 3$ and $y = 3$



2. Give the geometric representations of $2x + 9 = 0$ as an

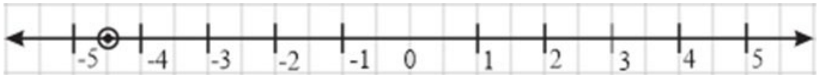
(i) in one variable

(ii) in two variables

Solution:

(i) $2x + 9 = 0$

$X = -9/2 = -4.5$



ii. $2x+0y+9=0$

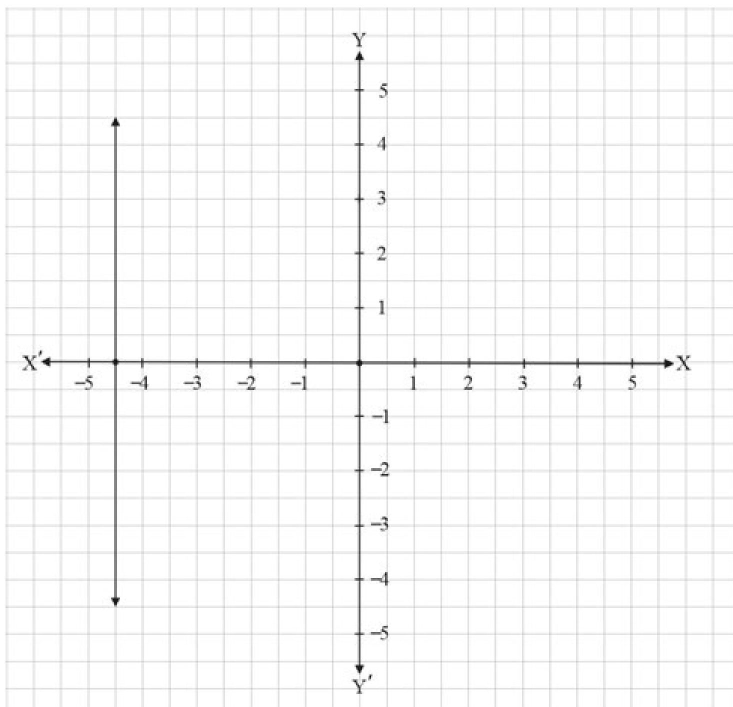
$X = -9-0y/2$

If $y = 0$

Then $X = -9/2 = -4.5$

If $y = 1$

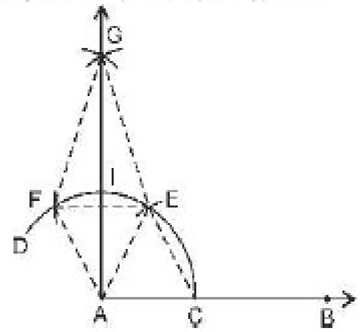
Then $X = -9/2 = -4.5$



11 | CONSTRUCTIONS

EXERCISE 10.1

Q.1. *Construct an angle of 90° at the initial point of a given ray and justify the construction.*



Steps of Construction

(i) Let us take a ray AB with initial point A.

(ii) Taking A as centre and some radius, draw an arc of a circle, which intersects AB at C.

(iii) With C as centre and the same radius as before, draw an arc, intersecting the previous arc at E.

(iv) With E as centre and the same radius, as before, draw an arc, which intersects the arc drawn in step (ii) at F.

(v) With E as centre and some radius, draw an arc.

(vi) With F as centre and the same radius as before, draw another arc, intersecting the previous arc at G.

(vii) Draw the ray AG.

Then $\angle BAG$ is the required angle of 90°.

Justification : Join AE, CE, EF, FG and GE

$AC = CE = AE$ [By construction]

$\Rightarrow \Delta ACE$ is an equilateral triangle

$\Rightarrow \angle CAE = 60°$ ... (i)

Similarly, $\angle AEF = 60°$ ... (ii)

From (i) and (ii), $FE \parallel AC$ ... (iii) [Alternate angles are equal]

Also, $FG = EG$ [By construction]

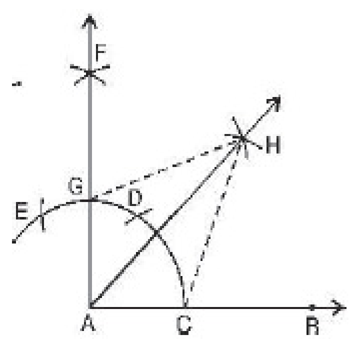
$\Rightarrow$ G lies on the perpendicular bisector of EF

$\Rightarrow \angle GIE = 90°$ ... (iv)

$\therefore \angle GAB = \angle GIE = 90°$ [Corresponding angles]

$GF = GE$ [Arcs of equal radii]

Q.2. *Construct an angle of 45° at the initial point of a given ray and justify the construction.*



Steps of Construction

(i) Let us take a ray AB with initial point A.

(ii) Draw $\angle BAF = 90°$, as discussed in Q. 1.

(iii) Taking C as centre and radius more than $\frac{1}{2}$ CG, draw an arc.

(iv) Taking G as centre and the same radius as before, draw another arc, intersecting the previous arc at H.

(v) Draw the ray AH. Then $\angle BAH$ is the required angle of 45°.

Justification : Join GH and CH.

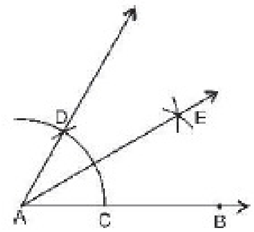
In ΔAHG and ΔAHC, we have

$HG = HC$	[Arcs of equal radii]
$AG = AC$	[Radii of the same arc]
$AH = AH$	[Common]
$\therefore \Delta AHG \cong \Delta AHC$	[SSS congruence]
$\Rightarrow \angle HAG = \angle HAC$	[CPCT] ... (i)
But $\angle HAG + \angle HAC = 90°$	[By construction] ... (ii)
$\Rightarrow \angle HAG = \angle HAC = 45°$	[From (i) and (ii)]

Q.3. *Construct the angles of the following measurements.*

(i) 30° *(ii)* $22\frac{1}{2}°$ *(iii)* 15°

(i) Steps of Construction



(a) Draw a ray AB, with initial point A.

(b) With A as centre and some convenient radius, draw an arc, intersecting AB at C.

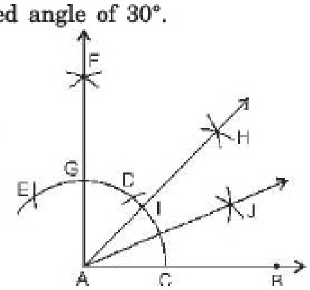
(c) With C as centre and the same radius as before, draw another arc, intersecting the previously drawn arc at D.

(d) Draw ray AD.

(e) Now, taking C and D as centres and with the radius more than $\frac{1}{2}$DC, draw arcs to intersect each other at E.

(f) Draw ray AE. Then $\angle BAE$ is the required angle of 30°.

(ii) Steps of Construction



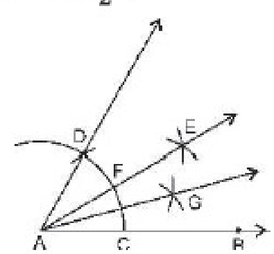
(a) Draw a ray AB with initial point A.

(b) Draw $\angle BAH = 45°$ as discussed in Q. 2.

(c) Taking I and C as centres and with the radius more than $\frac{1}{2}$CI, draw arcs to intersect each other at J.

(d) Draw ray AJ. Then $\angle BAJ$ is the required angle of $22\frac{1}{2}°$.

(iii) Steps of Construction



(a) Draw $\angle BAE = 30°$ as discussed in part (i).

(b) Taking C and F as centres and with the radius more than $\frac{1}{2}$ CF, draw arcs to intersect each other at G.

(c) Draw ray AG. Then $\angle BAG$ is the required angle of 15°.

Q.4. *Construct the following angles and verify by measuring them by a protractor.*

(i) 75° *(ii)* 105° *(iii)* 135°

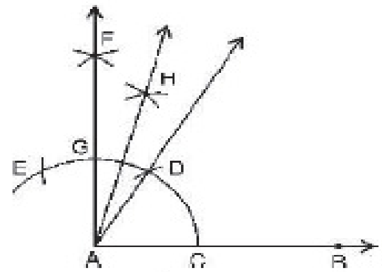
(i) Steps of Construction

(a) Draw a ray AB with initial point A.

(b) With A as centre and any convenient radius, draw an arc, intersecting AB at C.

(c) With C as centre and the same radius, draw an arc, cutting the previous arc at D.

(d) With D as centre and the same radius, draw another arc, cutting the arc drawn in step (b) at E.



(e) With D and E as centres and some radius, draw arcs to intersect each other at F.

(f) Draw ray AF and AD.

(g) With D and G as centres, and radius more than $\frac{1}{2}$GD, draw arcs to intersect. each other at H.

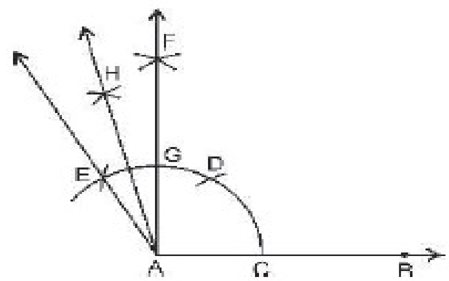
(h) Draw ray AH. Then $\angle$BAH is the required angle of 75°.
On measuring using a protractor, we find that $\angle$BAH = 75°.

(ii) Steps of Construction

(a) At A, draw an $\angle$BAF = 90°, as discussed in Q. 1.

(b) With A as centre and some convenient radius, draw an arc, intersecting AB at C.

(c) With C as centre and the same radius, draw an arc, which cuts the precious arc at D.



(d) With D as centre and the same radius, draw an arc, which cuts the arc drawn in step (b) at E.

(e) Draw ray AE.

(f) With G and E as centres and radius more than $\frac{1}{2}$GE, draw arcs to intersect each other at H.

(g) Join AH. Then $\angle$BAH is the required angle of 105°.
On measuring using a protractor, we find that $\angle$BAH = 105°.

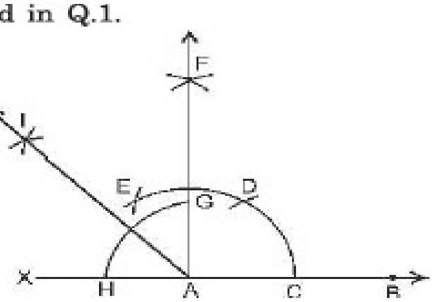
(iii) Steps of Construction

(a) At A, draw angle BAF = 90°, as discussed in Q.1.

(b) Produce BA to X.

(c) With A as centre and some convenient radius, draw an arc, which cuts AF and AX at G and H respectively.

(d) With G and H as centres and radius more than $\frac{1}{2}$GH, draw arcs to intersect each other at I.



(e) Draw ray AI. Then $\angle$BAI is the required angle of 135°.
On measuring using a protractor, we find that $\angle$BAI = 135°.

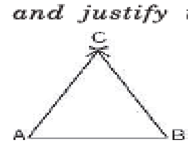
Q.5. *Construct an equilateral triangle, given its side and justify the construction.*

(i) Steps of Construction

(i) Draw a line segment AB of given length.

(ii) With A and B as centres and radius equal to AB, draw arcs to intersect each other at C.

(iii) Join AC and BC. Then ABC is the required equilateral triangle.



Justification : AB = AC [By construction]

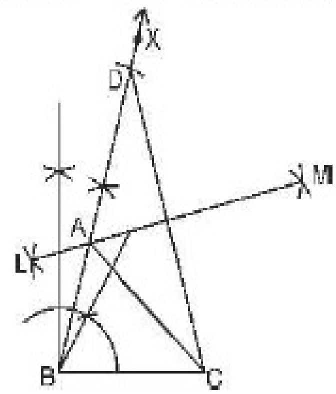
AB = BC [By construction]

$\Rightarrow$ AB = AC = BC

Hence, ΔABC is an equilateral triangle.

EXERCISE 10.2

Q.1. *C triangle ABC in which BC = 7 cm, ∠B = 75° and AB + A*



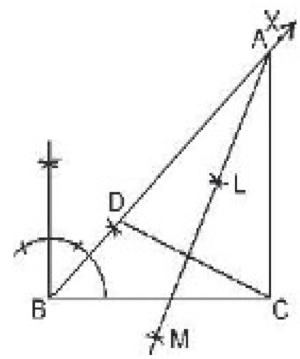
Steps of Construction

(i) Draw a line segment BC = 7 cm.
(ii) At B, draw ∠CBX = 75°.
(iii) Cut a line segment BD = 13 cm from BX.
(iv) Join DC
(v) Draw the perpendicular bisector LM of CD, which intersects BD at A.
(vi) Join AC. Then ABC is the required triangle.

Justification : In ΔACD, we have

AC = AD [A lies on the perpendicular bisector of DC.]
AB = BD – AD
= BD – AC
⇒ AB + AC = BD

Q.2. *Construct a triangle ABC, in which BC = 8 cm, ∠B = 45° and AB – AC = 3.5 cm.*



Steps of Construction

(i) Draw a line segment BC = 3.5 cm
(ii) At B, draw ∠CBX = 45°.
(iii) From BX, cut off BD = 3.5 cm.
(iv) Join DC.
(v) Draw the perpendicular bisector LM of DC, which intersects BX at A. (vi) Join AC. Then ABC is the required triangle.

Justification : In ΔADC,

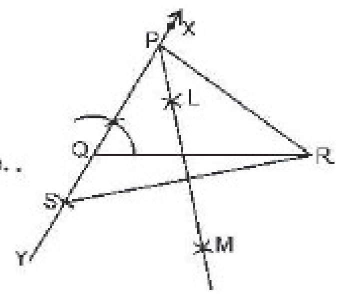
AD = AC [A lies on the perpendicular bisector of DC]
BD = AB – AD
⇒ BD = AB – AC

Q.3. *Construct a triangle PQR in which QR = 6 cm, ∠Q = 60° and PR – PQ = 2 cm.*

Steps of Construction

(i) Draw a line segment QR = 6 cm
(ii) At Q, draw ∠RQX = 60°.

(iii) Produce XQ to Y.
(iv) Cut off QS = 2 cm from QY.
(v) Join SR.
(vi) Draw the perpendicular bisector LM of SR, which intersect QX at P.
(vii) Join PR. Then PQR is the required triangle..



Justification : In ΔPSR, we have

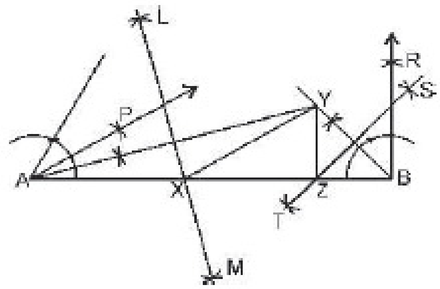
SP = PR [P lies on the perpendicular bisector of SR]

QS = PS − PQ
= PR − PQ

Q.4. *Construct a ΔXYZ in which $\angle X = 30°$, $\angle Z = 90°$ and $XY + YZ + ZX = 11$ cm.*

Steps of Construction

(i) Draw a line segment AB = 11 cm
(ii) At A, draw $\angle BAP = 30°$ and at B, draw $\angle ABR = 90°$
(iii) Draw the bisector of $\angle BAP$ and $\angle ABR$, which intersect each other at Y.
(iv) Join AY and BY.
(v) Draw the perpendicular bisectors LM and ST of AY and BY respectively. LM and ST intersect AB at X and Z respectively.
(vi) Join XY and YZ. Then XYZ is the required triangle.



Justification : In ΔAXY, we have

AX = XY [X lies on the perpendicular bisector of AY] ...(i)

Similarly, ZB = YZ ... (ii)

$\therefore$ XY + YZ + ZX = AX + ZB + ZX [From (i) and (ii)]
= AB

From (i), AX = AY

$\Rightarrow$ $\angle XAY = \angle XYA$ [Angles opposite to equal sides are equal] ... (iii)

In ΔAXY, $\angle YXZ = \angle XAY + \angle XYA$ [Exterior angle is equal to sum of interior opposite angles]

$\Rightarrow$ $\angle YXZ = 2\angle XAY$ [From (iii)]

$\Rightarrow$ $\angle YXZ = \angle XAP$ [$\because$ AY bisects $\angle XAP$]

Similarly, $\angle YZX = \angle ZBR$.

Q.5. *Construct a right triangle whose base is 12 cm and sum of its hypotenuse and other side is 18 cm.*

Steps of Construction

(i) Draw a line segment AB = 12 cm.
(ii) At A, draw $\angle BAX = 90°$.
(iii) From AX, cut off AD = 18 cm.
(iv) Join DB.
(v) Draw the perpendicular bisector LM of BD, which intersects AD at C.
(vi) Join BC. Then ΔABC is the required triangle.